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Filtered Repetitive Control with Nonlinear Systems

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Foreword

Since its inception in 1981, repetitive control (RC) has become a major chapter of control theory, with applications as diverse as power supplies, robotic manipulators, and quadcopters. These may have in common the requirement that the system track a periodic reference signal or reject a periodic disturbance or do both.

This book, by two well-known control researchers at the Beijing University of Aeronautics and Astronautics, aims to provide state-of-the-art coverage of RC, with due attention to theoretical precision combined with a strong emphasis on engineering design. The basic design challenge is to achieve an appropriate trade-off between the mutually conflicting goals of steady-state tracking accuracy and robust internal stability.

As their starting point, the authors introduce the familiar internal model principle of linear regulation, but now for a generic, not necessarily continuous, periodic reference signal. This infinite-dimensional extension raises new issues of stabilizability resolved by filtered repetitive control (FRC). FRC lays the groundwork for an extensive treatment of alternative design approaches to both linear and nonlinear systems, including the technique (original with the authors) of "additive state decomposition".

The book is well suited to a course on engineering design for readers with some preparation in ordinary differential-delay equations and Lyapunov stability. I recommend it as a timely and significant contribution to the current literature on RC.

September 2019

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Preface

Repetition is the mother of all learning
—A Latin Phrase

In nature, numerous examples of periodic phenomena are found and observed, ranging from the orbital motion of celestial bodies to heart rate. In practice, many control tasks are often of periodic nature as well. Industrial manipulators are often required to track or reject periodic exogenous signals when performing operations such as picking, dropping, and painting. Besides, special applications include magnetic spacecraft attitude control, helicopter vibration active control and vertical landing on oscillating platform, aircraft power harmonic elimination, satellite formation, LED light tracking, control of hydraulic servo mechanism, and lower limb exoskeleton control. For these periodic control tasks, repetitive control (RC, or repetitive controller, also specified RC) enables high-precision control performance. RC is derived from the internal model principle and contains a special structure with time-delay components which play the memory role. RC is, at the root, based on the compensation control or the predictive control that uses the additional memory. RC was originally developed on continuous single-input, single-output linear time-invariant (LTI) systems for high-precision tracking of periodic signals within a known period. Later, RC extended to multiple-input multiple-output LTI systems. Since then, RC has been propelled to the forefront of research and development in control theory. However, previous studies focused on theories and applications that use frequency-domain methods in relation to LTI systems, while RC for nonlinear systems received limited attention. What is more, RC often faces robustness problem, including stability robustness against uncertain parameters of systems and performance robustness against uncertain or time-varying period-time of external signals. For these problems, filter design with the frequency-domain analysis is the main tool, which then develops into filtered RCs. But it is difficult to apply them, if possible, to nonlinear systems. Therefore, we write this book for the utilization of filtered RC with nonlinear systems.

As an outcome of a course developed at Beihang University (Beijing University of Aeronautics and Astronautics, BUAA), this book aims at providing more methods and tools for the students and researchers in the field of RC to explore the potential of RC. In this book, commonly used methods like the feedback linearization method and adaptive-control-like method are summarized and further modified to be filtered RC. However, feedback linearization or error dynamics derived is often difficult to perform due to various reasons. To solve this problem, three new methods parallel to the two methods mentioned above are also proposed: the additive-state-decomposition-based method, the actuator-focused design method, and the contraction mapping method. To be specific, an introduction (Chap. 1) and preliminaries (Chaps. 2–4) are presented in the first four chapters, where the preliminaries consist of mathematics preliminary (Chap. 2), a brief introduction to RC for linear systems (Chap. 3), and the robustness problem of RC system (Chap. 4) that will serve to be an illustration of what RC is and why filtered RC must be used. After that, this book will give basic but new methods to solve RC problems for some special nonlinear systems: commonly used methods like linearization method (Chap. 5) and adaptive-control-like method (Chap. 6) will be summarized. They consist of both previous research findings and authors' contributions. In addition, three new methods parallel to the two methods mentioned above will be proposed: the additive-state-decomposition-based method in Chaps. 7–8 that will bridge the LTI systems and nonlinear systems so that the linear RC methods can be used in nonlinear systems; the actuator-focused design method in Chap. 9 derived from another viewpoint of the internal model principle proposed by the authors; and the contraction mapping method (Chap. 10) being another attempt of the authors to solve the RC problems for nonlinear systems without the need of corresponding Lyapunov functions.

Beijing, China

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Symbols

$=$	Equality
$\triangleq$	Definition. $x \triangleq y$ means that x is defined to be another name for y, under certain assumption
$\equiv$	Congruence relation
$\in$	Belong to
$\times$	Cross product
$*$	The convolution of f and g is written $(f*g)(t) \triangleq \int_{-\infty}^{\infty} f(s)g(t-s)\mathrm{d}s$, where $f(t), g(t) \in \mathbb{R}$
$\mathscr{B}$	$\mathscr{B}(\mathbf{o},\delta) \triangleq \{\boldsymbol{\xi} \in \mathbb{R}^n \mid \lVert\boldsymbol{\xi}-\mathbf{o}\rVert \le \delta\}$, and the notation $\mathbf{x}(t) \to \mathscr{B}(\mathbf{o},\delta)$ means $\min\limits_{\mathbf{y}\in\mathscr{B}(\mathbf{o},\delta)} \lVert\mathbf{x}(t)-\mathbf{y}\rVert \to 0$
$\mathbb{C}$	Set of complex numbers
$\mathbb{R}, \mathbb{R}^n, \mathbb{R}^{n\times m}$	Set of real numbers, Euclidean space of dimension n, and Euclidean space of dimension $n \times m$
$\mathbb{R}_+$	Set of positive real numbers
$\mathbb{C}, \mathbb{C}^n, \mathbb{C}^{n\times m}$	Set of complex numbers, complex vector of dimension n, complex matrix of dimension $n \times m$
$\mathbb{Z}$	Integer
$\mathbb{Z}_+$	Positive integer
$\mathbb{N}$	Nonnegative integers
$\mathscr{L}_{2e}$	$\mathscr{L}_{2\mathrm{e}} \triangleq \{\mathbf{f} \in \mathscr{L}_2[0,T], \text{for all } T<\infty\}$
$\mathscr{L}_2(-\infty,\infty)$	$\mathscr{L}_2(-\infty,\infty) \triangleq \left\{\lVert\mathbf{f}\rVert_2 < \infty\right\}$
$\mathscr{L}_2[0,\infty)$	$\mathscr{L}_2[0,\infty) \triangleq \{\mathbf{f} \in \mathscr{S} : \mathbf{f}(t) = 0 \text{ for all } t<0\} \cap \mathscr{L}_2(-\infty,\infty)$
$\mathscr{L}_\infty[a,b]$	$\mathscr{L}_\infty[a,b] \triangleq \left\{\mathbf{f} \mid \sup_{t\in[a,b]} \lVert\mathbf{f}(t)\rVert < \infty\right\}$
$\mathbf{I}_n$	Identify matrix of dimension $n \times n$
$\mathbf{0}_{n\times m}$	Zero matrix of dimension $n \times m$
x	Scale
$\mathbf{x}$	Vector, x_i represents the ith element of vector $\mathbf{x}$
$\mathbf{X}$	Matrix, x_{ij} represents the element of matrix $\mathbf{X}$ at the ith row and the jth column

$\dot{\mathbf{x}}, \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t}$	The first derivative with respect to time t
$\hat{\mathbf{x}}$	An estimate of $\mathbf{x}$
$\tilde{\mathbf{x}}$	An estimate error of $\mathbf{x}$
$\mathbf{A}^{\mathrm{T}}$	Transpose of $\mathbf{A}$
$\mathbf{A}^{-\mathrm{T}}$	Transpose of inverse $\mathbf{A}$
$\det(\mathbf{A})$	Determinant of $\mathbf{A}$
$\mathrm{tr}(\mathbf{A})$	Trace of a square matrix $\mathbf{A}$, $\mathrm{tr}(\mathbf{A}) \triangleq \sum_{i=1}^{n} a_{ii},\ \mathbf{A} \in \mathbb{R}^{n\times n}$
$\sigma_{\max}(\mathbf{C}), \sigma_{\min}(\mathbf{C})$	The maximum singular value and the minimum singular of matrix $\mathbf{C} \in \mathbb{C}^{n\times n}$, respectively
$\sigma(\mathscr{A})$	is the spectrum and $r_{\mathscr{A}} = \sup_{z\in\sigma(\mathscr{A})}\lvert z\rvert$ the spectral radius, where $\mathscr{A}$ be a linear compact operator
$\lambda(\mathbf{A}), \lambda_{\max}(\mathbf{A}), \lambda_{\min}(\mathbf{A})$	The eigenvalue, maximum eigenvalue, and the minimum eigenvalue of a matrix $\mathbf{A} \in \mathbb{R}^{n\times n}$, respectively
sup	The least upper bound of a set
inf	The greatest lower bound of a set
$(\cdot)^{(k)}$	The kth derivative with respect to time t
∇	Gradient
$\partial a/\partial \mathbf{x}$	$\partial a/\partial \mathbf{x} \triangleq [\,\partial a/\partial x_1 \quad \partial a/\partial x_2 \quad \ldots \quad \partial a/\partial x_n\,] \in \mathbb{R}^{1\times n}$
$\partial \mathbf{a}/\partial \mathbf{x}$	$(\partial \mathbf{a}/\partial \mathbf{x})_{ij} \triangleq \partial a_i/\partial x_j \in \mathbb{R}^{m\times n}$, where $\mathbf{a} = [\,a_1 \quad \ldots \quad a_m\,]^{\mathrm{T}}$ and $\mathbf{x} = [\,x_1 \quad \ldots \quad x_n\,]^{\mathrm{T}}$
$\mathscr{C}(\mathbf{A}, \mathbf{B})$	Controllability matrix of pairs $(\mathbf{A}, \mathbf{B})$, $\mathscr{C}(\mathbf{A}, \mathbf{B}) = \left[\mathbf{B}\ \mathbf{AB} \ldots \mathbf{A}^{n-1}\mathbf{B}\right]$
$L_{\mathbf{f}}h$	The Lie derivative of the function h with respect to the vector field $\mathbf{f}$
$L_{\mathbf{f}}^{i}h$	The ith-order derivative of $L_{\mathbf{f}}h$, $L_{\mathbf{f}}^{i}h = \nabla\left(L_{\mathbf{f}}^{i-1}h\right)\mathbf{f}$
$\mathscr{O}(\mathbf{A}, \mathbf{C}^{\mathrm{T}})$	Observability matrix of pairs $(\mathbf{A}, \mathbf{C})$, $\mathscr{O}\left(\mathbf{A}, \mathbf{C}^{\mathrm{T}}\right) = \begin{bmatrix} \mathbf{C}^{\mathrm{T}} \\ \mathbf{C}^{\mathrm{T}}\mathbf{A} \\ \vdots \\ \mathbf{C}^{\mathrm{T}}\mathbf{A}^{n-1} \end{bmatrix}$
$\mathrm{Re}(s)$	Real part of the complex number s
$\mathscr{L}, \mathscr{L}^{-1}$	Laplace transform and inverse Laplace transform, respectively
$\mathscr{Z}, \mathscr{Z}^{-1}$	Z-transform and inverse Z-transform, respectively
$\lvert\cdot\rvert$	Absolute value
$\lvert s\rvert$	The modulus of a complex number s is defined as $\lvert s\rvert \triangleq \sqrt{a^2 + b^2}$, where $s = a + ib$, $a, b \in \mathbb{R}$
$\lVert\cdot\rVert$	Euclidean norm, $\lVert\mathbf{x}\rVert \triangleq \sqrt{\mathbf{x}^{\mathrm{T}}\mathbf{x}}, \mathbf{x} \in \mathbb{R}^n$
$\lVert\mathbf{C}\rVert$	The norm of a complex matrix $\lVert\mathbf{C}\rVert \triangleq \sigma_{\max}(\mathbf{C}),\ \mathbf{C} \in \mathbb{C}^{n\times n}$
$\lVert\cdot\rVert_\infty$	Infinity norm, $\lVert\mathbf{x}\rVert_\infty = \max\{\lvert x_1\rvert, \ldots, \lvert x_n\rvert\}, \mathbf{x} \in \mathbb{R}^n$

$\Vert\mathbf{f}\Vert_{2,[0,T]}, \Vert\mathbf{f}\Vert_2$	$\Vert\mathbf{f}\Vert_{2,[0,T]} \triangleq \left\{\int_0^T \Vert\mathbf{f}(t)\Vert^2 \mathrm{d}t\right\}^{\frac{1}{2}}, \Vert\mathbf{f}\Vert_2 \triangleq \left\{\int_{-\infty}^{\infty} \Vert\mathbf{f}(t)\Vert^2 \mathrm{d}t\right\}^{\frac{1}{2}}, \mathbf{f} : \mathbb{R} \to \mathbb{R}^n$
$\Vert\mathbf{f}\Vert_\infty$	$\Vert\mathbf{f}\Vert_\infty \triangleq \sup_{t\in[0,\infty)} \Vert\mathbf{f}(t)\Vert$
$\Vert\mathbf{f}\Vert_{[a,b]}$	$\Vert\mathbf{f}\Vert_{[a,b]} \triangleq \sup_{t\in[a,b]} \Vert\mathbf{f}(t)\Vert$
$\mathbf{A} \geq \mathbf{0}, \mathbf{A} > \mathbf{0}$	Represent that $\mathbf{A} \in \mathbb{R}^{n\times n}$ is a positive semidefinite or positive definite matrix, respectively
$\bar{s}$	The conjugate of the complex number $s = a + ib$ is $\bar{s} \triangleq a - ib$
$\mathbf{C}^*$	The conjugate transpose is formally defined by $(\mathbf{C}^*)_{ij} \triangleq \left(\bar{\mathbf{C}}\right)_{ji}$, $\mathbf{C} \in \mathbb{C}^{n\times m}$
$\mathscr{C}([a,b], \mathbb{R}^n)$	The space of continuous n-dimension function vector on $[a,b]$
$\mathscr{C}^m_{PT}([0,\infty), \mathbb{R}^n)$	The space of mth-order continuously differentiable functions $\mathbf{f} : [0,\infty) \to \mathbb{R}^n$ which are T-periodic, i.e., $\mathbf{f}(t+T) = \mathbf{f}(t)$
$\Vert\mathbf{x}_t\Vert_{[a,b]}$	$\Vert\mathbf{x}_t\Vert_{[a,b]} \triangleq \sup\limits_{\theta\in[a,b]} \Vert\mathbf{x}(t+\theta)\Vert$, where $\mathbf{x}_t \triangleq \mathbf{x}_t(\theta) = \mathbf{x}(t+\theta), \theta \in [a,b]$
$\mathscr{K}$	A continuous function $\alpha : [0,a) \to [0,\infty)$ is said to belong to class $\mathscr{K}$ if it is strictly increasing and $\alpha(0) = 0$
$\mathscr{K}_\infty$	A continuous function α is said to belong to class $\mathscr{K}_\infty$ if $a = \infty$ and $\alpha(r) \to \infty$ as $r \to \infty$
$\mathscr{KL}$	A continuous function $\beta : [0,a) \times [0,\infty) \to [0,\infty)$ is said to belong to class $\mathscr{KL}$ if, for each fixed s, the mapping $\beta(r,s)$ belongs to $\mathscr{K}$ with respect to r and, for each fixed r, the mapping $\beta(r,s)$ is decreasing with respect with s and $\beta(r,s) \to 0$ as $s \to \infty$
$\Vert\mathbf{f}\Vert_a$	$\Vert\mathbf{f}\Vert_a \triangleq \limsup\limits_{t\to\infty} \Vert\mathbf{f}(t)\Vert$, where $\mathbf{f} \in \mathscr{L}_\infty[0,\infty)$
$\mathscr{D}^+\mathbf{x}$	The upper right Dini derivative of a function and is defined by $\mathscr{D}^+\mathbf{x}(t_0) \triangleq \left(\lim\limits_{t\to t_0+0} \sup \frac{x_1(t)-x_1(t_0)}{t-t_0}, \ldots, \lim\limits_{t\to t_0+0} \sup \frac{x_n(t)-x_n(t_0)}{t-t_0}\right)^{\mathrm{T}}$
$\vert\cdot\vert$	Modulus of a complex number
$O(\mathbf{x})^n$	A function $\boldsymbol{\delta}(\mathbf{x})$ is said to be $O(\mathbf{x})^n$ if $\lim\limits_{\Vert\mathbf{x}\Vert\to 0} \frac{\Vert\boldsymbol{\delta}(\mathbf{x})\Vert}{\Vert\mathbf{x}\Vert^n}$ exists and is $\neq 0$. In particular, a function $\boldsymbol{\delta}(\mathbf{x})$, said to be $O(\mathbf{x})^0$ or $O(1)$, implies $\lim\limits_{\Vert\mathbf{x}\Vert\to 0} \Vert\boldsymbol{\delta}(\mathbf{x})\Vert$ exists and is $\neq 0$.
$\circ$	Function composition, $\mathbf{f}_1 \circ \mathbf{f}_2 : \mathbb{R}^n \to \mathbb{R}^m$ implies that $(\mathbf{f}_1 \circ \mathbf{f}_2)(\mathbf{x}) = \mathbf{f}_1(\mathbf{f}_2(\mathbf{x})), \forall \mathbf{x} \in \mathbb{R}^n$, where $\mathbf{f}_1 : \mathbb{R}^p \to \mathbb{R}^m, \mathbf{f}_2 : \mathbb{R}^n \to \mathbb{R}^p$

Chapter 1
Introduction

There are several examples of periodic phenomena, such as the orbital motion of heavenly bodies and heartbeats, that can be observed in nature. In practice, many control tasks are often considered to exhibit periodic behavior. Industrial manipulators are often used to track or reject periodic exogenous signals when performing picking, placing, or painting operations. Moreover, some special applications of these periodic exogenous signals include magnetic spacecraft attitude control, active control of helicopter vibrations, autonomous vertical landing in an oscillating platform, elimination of harmonics in aircraft power supply, satellite formation, light-emitting diode tracking, control of hydraulic servomechanisms, and control of lower limb exoskeletons. High-precision control performance can be realized for such periodic control tasks using repetitive control (RC, or repetitive controller, which is also designated as RC). RC was initially developed for continuous single-input, single-output (SISO) linear time-invariant (LTI) systems in [1] to accurately track periodic signals with a known period. RC was then extended to multiple-input multiple-output (MIMO) LTI systems in [2]. Since then, RC has been the subject of increasing attention, and applications that employ RC have become a special subject of focus in control theory. Recent developments concerning RC have not been consistent, with limited research on RC in nonlinear systems. However, the use of frequency-based methods has significantly aided the development of theories and applications pertaining to LTI systems. This chapter aims to answer the following question:

What are the challenges in employing repetitive control for nonlinear systems?

To answer this question, it is essential to introduce the basic idea of RC and provide a brief overview of RC for linear and nonlinear systems. This chapter presents a revised and extended version of a paper that was published earlier [3].

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1.1 Basic Idea of Repetitive Control

1.1.1 Basic Concept

Before discussing RC, the concept of iterative learning control (ILC, or iterative learning controller, which is also designated as ILC) must be introduced to avoid confusion between these two similar control methods.

1.1.1.1 Iterative Learning Control

ILC is used for repetitive tasks with multiple execution times. It focuses on improving task results by learning from previous executions [4–10]. This control method performs a repetitive task and can utilize past control (delayed) information for generating present control action, which makes it different from most existing control methods. The classic ILC comprises three steps for each trial: (i) storing past control information; (ii) suspending the plant and resetting to the initial state condition; and (iii) controlling the plant using stored past control information and current feedback. For example, a remote pilot practices the take-off movements of a multicopter from the ground to a predetermined height. During each take-off, the remote pilot observes the trajectory of the multicopter (first step). If the trajectory is not satisfactory, the remote pilot will land the multicopter and then start it again by setting the initial rotation speed of the propellers to the previously recorded values (second step). Finally, the remote pilot adjusts the operation based on previous data. As the pilot continues to practice, the correct operation is learned and ingrained into the muscle memory of the pilot so that the skill of the pilot can be improved iteratively, which is the principle of the ILC learning method.

1.1.1.2 Repetitive Control

RC is used for periodic signal tracking and rejection. It aims to improve task results by learning from previous executions. RC is a special tracking method intended for a class of special problems. The classic RC comprises two steps for each trial: (i) storing past control information and (ii) controlling the plant using stored past control information (the last trial) and current feedback. For example, a pilot attempts to land a helicopter on a periodic oscillating deck at sea. Given the periodicity of the deck, the pilot can adjust his or her operation based on the previous trajectory of the helicopter, which is the principle of the RC learning method. The most significant difference between RC and ILC is that during RC, the initial state of the current trial cannot be reset to the final state of the previous trial. The entire process is continuous without any interruption at the end of each trial.

Table 1.1 Comparison between ILC and RC

	ILC	RC
Desired trajectory	$\mathbf{y}_{\mathrm{d}} : [0, T]$, $\mathbf{y}_{\mathrm{d}}(0) \neq \mathbf{y}_{\mathrm{d}}(T)$ or $\mathbf{y}_{\mathrm{d}}(0) = \mathbf{y}_{\mathrm{d}}(T)$	$\mathbf{y}_{\mathrm{d}} : [0, \infty)$, $\mathbf{y}_{\mathrm{d}}(t + T) = \mathbf{y}_{\mathrm{d}}(t)$, $t \geq 0$
Initial condition	$\mathbf{x}_{k+1}(0)$ can be reset manually	$\mathbf{x}_{k+1}(0) = \mathbf{x}_k(T)$ automatically
Controller form	$\mathbf{u}_k = \mathbf{u}_{k-1} + \mathbf{L}(\mathbf{y}_{k-1} - \mathbf{y}_{\mathrm{d}})$	$\mathbf{u}_k = \mathbf{u}_{k-1} + \mathbf{L}(\mathbf{y}_{k-1} - \mathbf{y}_{\mathrm{d}})$

1.1.1.3 Comparison

Let us consider a class of linear systems as follows:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)\\ \mathbf{y}(t) &= \mathbf{C}^{\mathrm{T}}\mathbf{x}(t),\end{aligned}$$

where $\mathbf{A} \in \mathbb{R}^{n\times n}$, $\mathbf{B}, \mathbf{C} \in \mathbb{R}^{n\times m}$, $\mathbf{x} \in \mathbb{R}^n$, and $\mathbf{y}(t), \mathbf{u}(t) \in \mathbb{R}^m$. The control objective is to design $\mathbf{u}$ that enables $\mathbf{y}$ to track the desired trajectory $\mathbf{y}_{\mathrm{d}}$. For simplicity, resetting the time is ignored for ILC and the control variable $\mathbf{u}$ is defined as

$$\mathbf{u}_k(t) \triangleq \mathbf{u}(kT + t), t \in [0, T],$$

where $T > 0$ denotes the interval time of a trial for ILC and the period for RC, $k = 0, 1, 2, \ldots$. A comparison is presented in Table 1.1, where the two controller forms are found to be the same but exhibit a major difference in terms of the initial condition setting.

1.1.2 Internal Model Principle

The basic concept of RC originates from the internal model principle (IMP); this principle states that if any exogenous signal can be regarded as the output of an autonomous system, then the inclusion of this signal model, namely, *internal model*, in a stable closed-loop system can assure asymptotic tracking and asymptotic rejection of the signal [11]. If a given signal is composed of a certain number of harmonics, then a corresponding number of neutrally stable internal models (one for each harmonic) should be incorporated into the closed-loop based on the IMP to realize asymptotic tracking and asymptotic rejection. To further explain the IMP, the *zero-pole cancelation viewpoint* of the IMP is used to explain the role of the internal models in step signals, sine signals, and T-periodic signals.

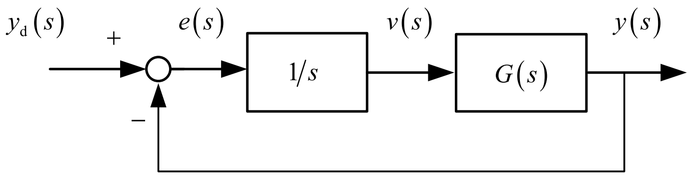

Fig. 1.1 Step signal tracking

1.1.2.1 Step Signal

It is well known that integral control can track and reject any external step signal; this can be explained using the IMP given that the models of an integrator and a step signal are the same, namely, $1/s$. Based on the IMP, inclusion of the internal model $1/s$ into a stable closed-loop system can assure asymptotic tracking and asymptotic rejection of a step signal.

According to Fig. 1.1, the transfer function from the desired signal to the tracking error can be written as follows:

$$\begin{aligned} e(s) &= \frac{1}{1+\frac{1}{s}G(s)}y_d(s) = \frac{1}{s+G(s)}\left(s\frac{a}{s}\right) \\ &= \frac{a}{s+G(s)}, \end{aligned}$$

where $y_d(s) = a/s$ is the Laplace transformation of a step signal with amplitude $a \in \mathbb{R}$. Therefore, it is sufficient to merely verify whether the roots of the equation $s + G(s) = 0$ are all in the left s-plane, which confirms the stability of the closed-loop system. If all roots are in the left s-plane, then the tracking error tends to zero as $t \to \infty$. Therefore, the tracking problem is converted into a stability problem of the closed-loop system.

1.1.2.2 Sine Signal

Suppose that the external signal is in the form $a_0 \sin(\omega t + \varphi_0)$, where a_0, φ_0 are constants, and the Laplace transformation model of $a_0 \sin(\omega t + \varphi_0)$ is $(b_1 s + b_0)/\left(s^2+\omega^2\right)$, where $b_0, b_1 \in \mathbb{R}$. Precise tracking or complete rejection can then be achieved by incorporating the model $1/\left(s^2+\omega^2\right)$. into the closed-loop system.

Figure 1.2 demonstrates that the transfer function from the desired signal to the tracking error can be expressed as follows:

$$\begin{aligned} e(s) &= \frac{1}{1+\frac{1}{s^2+\omega^2}G(s)}y_d(s) \\ &= \frac{1}{s^2+\omega^2+G(s)}\left(\left(s^2+\omega^2\right)\frac{b_1 s+b_0}{s^2+\omega^2}\right) \end{aligned}$$

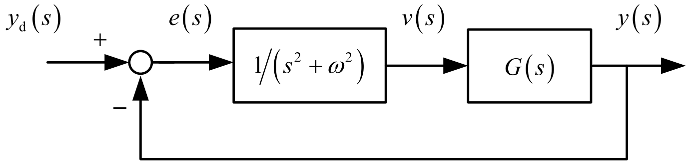

Fig. 1.2 Sine signal tracking

$$= \frac{b_1 s + b_0}{s^2 + \omega^2 + G(s)}.$$

Thus, it is adequate to only verify whether the roots of the equation $s^2 + \omega^2 + G(s) = 0$ are all in the left s-plane, which confirms the stability of the closed-loop system. Therefore, the tracking problem is converted into a stability problem of the closed-loop system.

Based on the IMP, designing a tracking controller for a general periodic signal is challenging because any periodic signal may be a summation of finite or infinite harmonics with period T. The harmonics of a general periodic signal must first be analyzed. However, obtaining accurate harmonics is difficult or time-consuming. Second, according to the IMP, the controller will contain more neutrally stable internal models (one for each harmonic) as the number of harmonics increases. For example, a T-periodic signal consists of harmonics with frequencies $0, 2\pi/T, \ldots, 2\pi N/T$, where $N \in \mathbb{Z}_+$. The corresponding internal model can then be written as

$$I_{M,\text{fin}} = \frac{1}{s \prod_{k=1}^{N} \left(1 + \frac{T^2 s^2}{4\pi^2 k^2}\right)}. \tag{1.1}$$

This will result in an extremely complex controller structure. Moreover, it will be time-consuming to solve these neutrally stable internal models (differential equations) to obtain the control output. However, these two drawbacks can be overcome by using the following internal model for the T-periodic signal.

1.1.2.3 *T*-Periodic Signal

The Laplace transformation of a signal $y_d(t)$ delayed by T is expressed as follows:

$$\mathscr{L}(y_d(t-T)) = e^{-sT}\mathscr{L}(y_d(t)).$$

Suppose that the external signal is of the form $y_d(t) = y_d(t-T)$, which can represent any T-periodic signal. Its Laplace transformation is $1/\left(1-e^{-sT}\right)$ with an initial condition on the interval $[-T, 0]$. Based on the IMP, asymptotic tracking and asymptotic rejection can be achieved by incorporating the internal model $1/\left(1-e^{-sT}\right)$ into

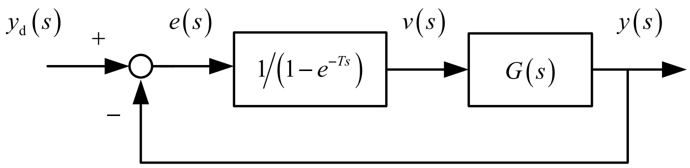

Fig. 1.3 T-periodic signal tracking

the closed-loop system. The internal model for any T-periodic signal can be rewritten as follows [12]:

$$I_{M,\mathrm{inf}} \triangleq \frac{1}{1-e^{-sT}} = \lim_{N\to\infty} \frac{1}{s\prod_{k=1}^{N}\left(1+\frac{T^2s^2}{4\pi^2k^2}\right)}. \tag{1.2}$$

This internal model contains the internal models of all harmonics with period T, including the step signal. However, it is interesting to note the simple structure of RC with an infinite number of harmonics. This observation validates the Chinese proverb that states "things will develop in the opposite direction when they become extreme."

Figure 1.3 shows that the transfer function from the desired signal to the tracking error can be written as follows:

$$\begin{aligned} e(s) &= \frac{1}{1+\frac{1}{1-e^{-sT}}G(s)} y_\mathrm{d}(s) \\ &= \frac{1}{1-e^{-sT}+G(s)}\left(\left(1-e^{-sT}\right)\frac{1}{1-e^{-sT}}\right) \\ &= \frac{1}{1-e^{-sT}+G(s)}. \end{aligned}$$

Therefore, it is sufficient to merely verify whether the roots of the equation $1 - e^{-sT} + G(s) = 0$ are all in the left s-plane. Consequently, the tracking problem is converted into a stability problem of the closed-loop system.

Based on the RC presented in Table 1.1, the Laplace transformation of the controller is expressed as follows:

$$\mathbf{u}(s) = \frac{1}{1-e^{-sT}}\mathbf{L}\left(\mathbf{y}_{k-1}(s) - \mathbf{y}_\mathrm{d}(s)\right),$$

where the internal model of T-periodic signals is also incorporated. A controller that includes the internal model $I_{M,\mathrm{inf}}$ in (1.2) is called an RC, and a system that employs this controller is called an RC system [2].

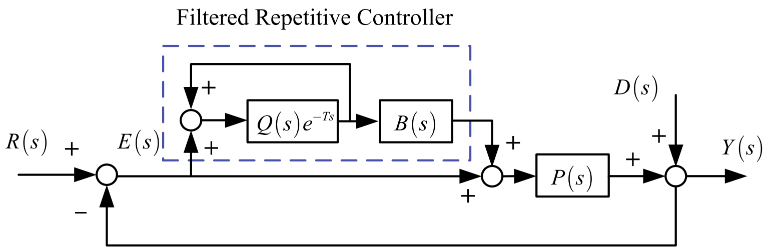

Fig. 1.4 Plug-in RC system diagram

1.2 Brief Overview of Repetitive Control for Linear System

RC is an internal model-based control method where the infinite-dimensional internal model $I_{M,\text{inf}}$ gives rise to an infinite number of poles on the imaginary axis. [2] proved that for a class of general linear plants, the exponential stability of RC systems can be achieved only when the plant is proper, but not strictly proper. Moreover, the system may be destabilized by the internal model $I_{M,\text{inf}}$. A linear RC system is a neutral-type system in a critical case [13, 14]. Consider the following simple RC system:

$$\begin{aligned} \dot{x}(t) &= -x(t) + u(t) \\ u(t) &= u(t-T) - x(t), \end{aligned}$$

where $x(t), u(t) \in \mathbb{R}$. The RC system expressed above can also be written as a neutral-type system in a critical case as follows:

$$\dot{x}(t) - \dot{x}(t-T) = -2x(t) + x(t-T).$$

The above system is a neutral-type system in a critical case [13, 14]. Additional information is presented in Chap. 3.

To enhance stability, a suitable filter is introduced, as shown in Fig. 1.4, forming a filtered repetitive controller (FRC, or filtered repetitive control, which is also designated as FRC) where the loop gain is reduced at high frequencies.[1] Stable results can only be achieved by compromising on high-frequency performance. However, using an appropriate design, FRC can often achieve an acceptable trade-off between tracking performance and stability. This trade-off broadens the practical applications of RC. The plug-in RC system shown in Fig. 1.4 is a widely used structure. The objective of this structure is to design and optimize the filter $Q(s)$ and compensator $B(s)$.

Given the developments over the past 30 years, it is evident that significant research efforts have been devoted toward developing theories and applications regarding RC for linear systems. Additional information on RC for linear systems has been

[1] In this book, the term "modified" in [2] is replaced with a more descriptive term "filtered".

included as part of [6, 10, 15–17] and the references therein. Current research mainly focuses on robust RC [18–21], spatial-based RC [22], or a combination of both [24]. Robust RC mainly includes two aspects: robustness against uncertain parameters of the considered systems [18–20] and robustness against uncertain or time-varying periods [21, 23–25]. Researchers are currently attempting to design better RCs to satisfy the increasing practical requirements.

1.3 Repetitive Control for Nonlinear System

For nonlinear systems, the concept of FRC is not difficult to follow because the relevant theories have been derived in the frequency domain; these can be applied with difficulty, if at all, only to nonlinear systems. Currently, RCs for nonlinear systems are designed using two methods, namely, the feedback linearization method and adaptive-control-like method.

1.3.1 Major Repetitive Controller Design Method

1.3.1.1 Linearization Method

One of the design methods involves transforming a nonlinear system into a linear system and then applying the existing design methods to the transformed linear system. Earlier, researchers often considered the following nonlinear system:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \boldsymbol{\phi}(t, \mathbf{x}) \\ \mathbf{y}(t) &= \mathbf{C}^{\mathrm{T}}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t). \end{aligned} \tag{1.3}$$

This method is related to the early stages of research on nonlinear systems. RC design is often restricted on the nonlinear term $\boldsymbol{\phi}(t, \mathbf{x})$, including Lipschitz conditions [26] or sector conditions [27, 28]. Along with the development of feedback linearization and backstepping, RC design for nonlinear systems was further developed [29–33]. Using these new techniques, some nonlinear systems can be transformed into (1.3) with some restrictions, while some existing design methods can also be used directly.

Differential geometric techniques are combined with the IMP, resulting in a nonlinear RC strategy. A formulation is presented for the case of input–state linearizable and input–output linearizable systems in continuous time [29]. Using the input–output linearized method and the approximate input–output linearized method, the applicability of the finite-dimensional RC to nonlinear tracking control problems is investigated for three different classes of nonlinear systems: (1) systems with a well-defined relative degree, (2) systems without a well-defined relative degree, and (3) linear plants with small actuator nonlinearity [30]. Using feedback linearization