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K. Deergha Rao

Channel Coding Techniques for Wireless Communications

Second Edition

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मातृभ्यो नमः
पितृभ्यो नमः
गुरुभ्यो नमः

To
My Parents Boddu and Dalamma,
My Beloved Wife Sarojini,
and My Mentor Prof. M.N.S. Swamy

Preface

Lives of people have tremendously changed in view of the rapid growth of mobile and wireless communication. Channel coding is the heart of digital communication and data storage. Traditional block codes and conventional codes are commonly used in digital communications. To approach the theoretical limit for Shannon's channel capacity, the length of a linear block code or constant lengths of convolutional codes have to be increased, which in turn makes the decoder complexity higher and may render it physically unrealizable. The powerful turbo and LDPC codes approach the theoretical limit for Shannon's channel capacity with feasible complexity for decoding. MIMO communications is a multiple antenna technology which is an effective way for high-speed or high-reliability communications. The MIMO can be implemented by space-time coding. Recently, a new channel coding technique, namely polar codes, has emerged as one of the channel coding techniques for fifth-generation (5G) wireless communications, and it has been recommended by third-generation partnership project (3GPP) as a channel coding scheme for enhanced mobile broadband (eMBB) in 5G systems. However, the market lacks a book which can serve as a textbook for graduate and undergraduate students on channel coding techniques.

This book includes illustrative examples in each chapter for easy understanding of coding techniques. An attractive feature of this book is the inclusion of MATLAB-based examples with codes encouraging readers to implement them on their personal computers and become confident of the fundamentals by gaining more insight into coding theory. In addition to the problems that require analytical solutions, MATLAB exercises are introduced to the reader at the end of each chapter.

This book is divided into 13 chapters. Chapter 1 introduces the basic elements of a digital communication system, statistical models for wireless channels, capacity of a fading channel, Shannon's noisy channel coding theorem, and the basic idea of coding gain. Chapter 2 gives an overview of the performance analysis of different modulation techniques and also deals with the performance of different diversity combining techniques in a multichannel receiver. Chapter 3 introduces Galois fields and polynomials over Galois fields. Chapter 4 covers linear block codes including

RS codes because of their popularity in burst error correction in wireless networks. Chapter 5 discusses the design of a convolutional encoder and Viterbi decoding algorithm for the decoding of convolutional codes, as well as the performance analysis of convolutional codes over AWGN and Rayleigh fading channels. In this chapter, punctured convolutional codes, tail-biting convolutional codes, and their performance analysis are also discussed. Chapter 6 provides a treatment of the design of turbo codes, BCJR algorithm for iterative decoding of turbo codes, and performance analysis of turbo codes. In this chapter, enhanced turbo codes, enhanced list turbo decoding, and their performance evaluation are also described.

Chapter 7 focuses on the design and analysis of trellis-coded modulation schemes using both the conventional and turbo codes. Chapter 8 describes the design of low parity check codes (LDPC), quasi-cyclic (QC)-LDPC codes, decoding algorithms, and performance analysis of LDPC and QC-LDPC codes. The erasure correcting codes like Luby transform (LT) codes and Raptor codes are described in Chap. 9. The design of polar encoder and successive cancelation decoding (SCD), successive cancelation list decoding (SCLD), and multiple bit decision successive cancelation list decoding algorithms and their performance evaluation are provided in Chap. 10.

Chapter 11 provides an in-depth study of multiple-input multiple-output (MIMO) systems in which multiple antennas are used both at the transmitter and at the receiver. The advanced techniques for MIMO OFDM channel estimation are also described in this chapter. The design of space-time codes and implementations of MIMO systems are discussed in Chap. 12. Chapter 13 deals with the evolution of channel codes for 5G wireless communications.

The motivation in writing this book is to include modern topics of increasing importance such as turbo codes, LDPC codes, polar codes, LT and Raptor codes, and space-time coding in detail, in addition to the traditional RS codes and convolutional codes, and also to provide a comprehensive exposition of all aspects of coding for wireless channels. The text is integrated with MATLAB-based programs to enhance the understanding of the underlying theories of the subject.

This book is written at a level suitable for undergraduate and master students in electronics and communication engineering, electrical and computer engineering, computer science, and applied physics as well as for self-study by researchers, practicing engineers, and scientists. Depending on the chapters chosen, this text can be used for teaching a one- or two-semester course on coding for wireless channels. The prerequisite knowledge of the readers in principles of digital communication is expected.

Hyderabad, India K. Deergha Rao

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About the Author

K. Deergha Rao is Professor at the Department of Electronics and Communication Engineering, Vasavi College of Engineering, Osmania University, Hyderabad, India. He is former Director and Professor of Research and Training Unit for Navigational Electronics (NERTU), Osmania University. He was a postdoctoral fellow and part-time professor for four years at the Department of Electrical and Computer Engineering, Concordia University, Montreal, Canada. His teaching areas are signals and systems, digital signal processing, channel coding techniques, and MIMO communication systems. Professor Rao has executed research projects for premium Indian organizations such as DRDO, HAL, and BEL and is interested in research areas of wireless channel coding, MIMO-OFDM communications, image processing, cryptosystems, and VLSI signal processing. He has also served as the founder chairman of the joint chapter of IEEE Communications Society and IEEE Signal Processing Society in Hyderabad from 2010–2012 and communications track chair for IEEE INDICON 2011 held in Hyderabad. Five students have been so far awarded Ph.D. degrees under Professor Rao, while three are currently working towards their Ph.D.

An awardee of the IETE K. S. Krishnan Memorial Award for the best system oriented paper in 2013, Prof. Rao has presented papers at IEEE International conferences several times in the USA, Switzerland, Russia, and Thailand. He has more than 100 publications to his credit including more than 60 publications in the IEEE journals and conference proceedings. Professor Rao is the author of two books—*Channel Coding Techniques for Wireless Communications* (Springer, 2015) and *Signals and Systems* (Springer, 2018)—and has co-authored *Digital Signal Processing* (Jaico Publishing House, 2012) and *Digital Signal Processing: Theory and Practice* (Springer, 2018). He is an editorial board member for the *International Journal of Sustainable Aviation*.

Chapter 1
Introduction

In this chapter, a digital communication system with coding is first described. Second, various wireless communication channels, their probability density functions, and capacities are discussed. Further, Shannon's noisy channel coding theorem, channel coding principle and channel coding gain are explained. Finally, some application examples of channel coding are included.

1.1 Digital Communication System

A communication system is a means of conveying information from one user to another user. The digital communication system is one in which the data is transmitted in digital form. A digital communication system schematic diagram is shown in Fig. 1.1. The source coding is used to remove redundancy from source information for efficient transmission. The transmitted signal power and channel bandwidth are the key parameters in the design of digital communication system. Using these parameters, the signal energy per bit (E_b) to noise power spectral density (N_0) ratio is determined. This ratio is unique in determining the probability of bit error, often referred to as bit error rate (BER). In practice, for a fixed E_b/N_0, acceptable BER is possible with channel coding. This can be achieved by adding additional digits to the transmitted information stream. These additional digits do not have any new information, but they make it possible for the receiver to detect and correct errors, thereby reducing the overall probability of error.

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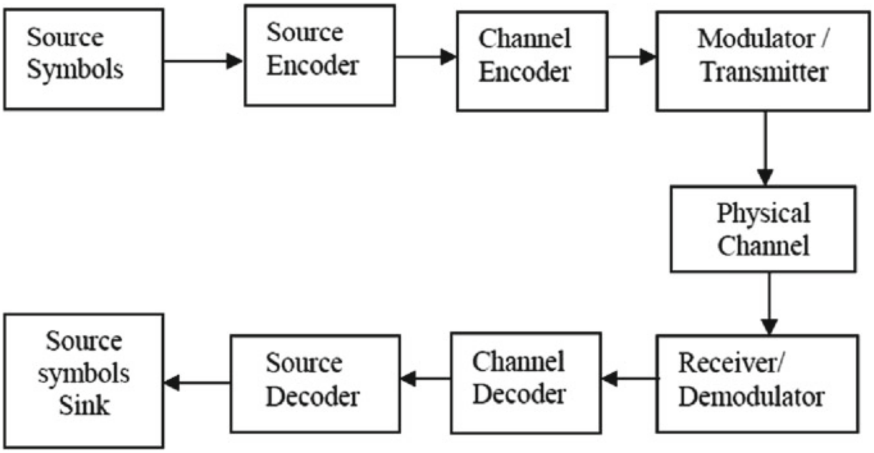


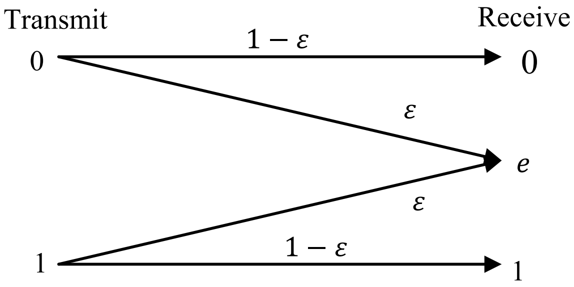
Fig. 1.1 Digital communication system with coding

1.2 Wireless Communication Channels

1.2.1 Binary Erasure Channel (BEC)

Erasure is a special type of error with known location. The binary erasure channel (BEC) transmits one of the two binary bits 0 and 1. However, an erasure "*e*" is produced when the receiver receives an unreliable bit. The BEC output consists of 0, 1, and *e* as shown in Fig. 1.2. The BEC erases a bit with probability ε, called the erasure probability of the channel. Thus, the channel transition probabilities for the BEC are:

Fig. 1.2 Binary erasure channel



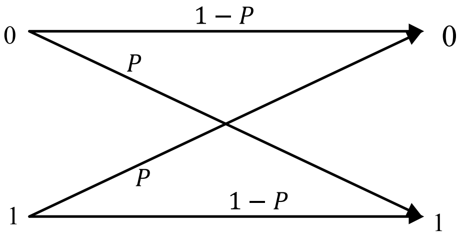


Fig. 1.3 Binary symmetric channel

$$\left.\begin{aligned}
&P(y=0|x=0)=1-\varepsilon,\\
&P(y=e|x=0)=\varepsilon,\\
&P(y=1|x=0)=0,\\
&P(y=0|x=1)=0,\\
&P(y=e|x=1)=\varepsilon,\\
&P(y=1|x=1)=1-\varepsilon.
\end{aligned}\right\}\tag{1.1}$$

1.2.2 Binary Symmetric Channel (BSC)

The binary symmetric channel (BSC) is a discrete memoryless channel that has binary symbols both in the input and output. It is symmetric because the probability for receiving 0 when 1 is transmitted is same as the probability for receiving 1 when 0 is transmitted. This probability is called the crossover probability of the channel denoted by P as shown in Fig. 1.3. The probability for no error, i.e., receiving the same as transmitted is $1 - P$. Hence, the channel transition probabilities for the BSC are:

$$\left.\begin{aligned}
&P(y=0|x=0)=1-P,\\
&P(y=0|x=1)=P,\\
&P(y=1|x=0)=P,\\
&P(y=1|x=1)=1-P,
\end{aligned}\right\}\tag{1.2}$$

1.2.3 Additive White Gaussian Noise Channel

In an AWGN channel, the signal is degraded by white noise η which has a constant spectral density and a Gaussian distribution of amplitude. The Gaussian distribution has a Probability Density Function (PDF) given by

$$P_{df}(\eta) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{\eta^2}{2\sigma^2}\right) \tag{1.3}$$

where σ^2 is the variance of a Gaussian random process.

1.2.4 Gilbert–Elliott Channel

For bursty wireless channels, the Gilbert–Elliott (GE) channel [1, 2] is one of the simplest and practical models. The GE channel is a discrete-time stationary model as shown in Fig. 1.4 with two states: one bad state or burst state “2” wherein a BSC resides with high-error probabilities $(1 - P_2)$ and the other state is a good state “1” wherein a BSC resides with low-error probabilities $(1 - P_1)$.

Another common GE Example is that the BEC resides in a bad state with ε close to unity and assigns erasures to all of the bits transmitted during the high-error-rate (bad) state.

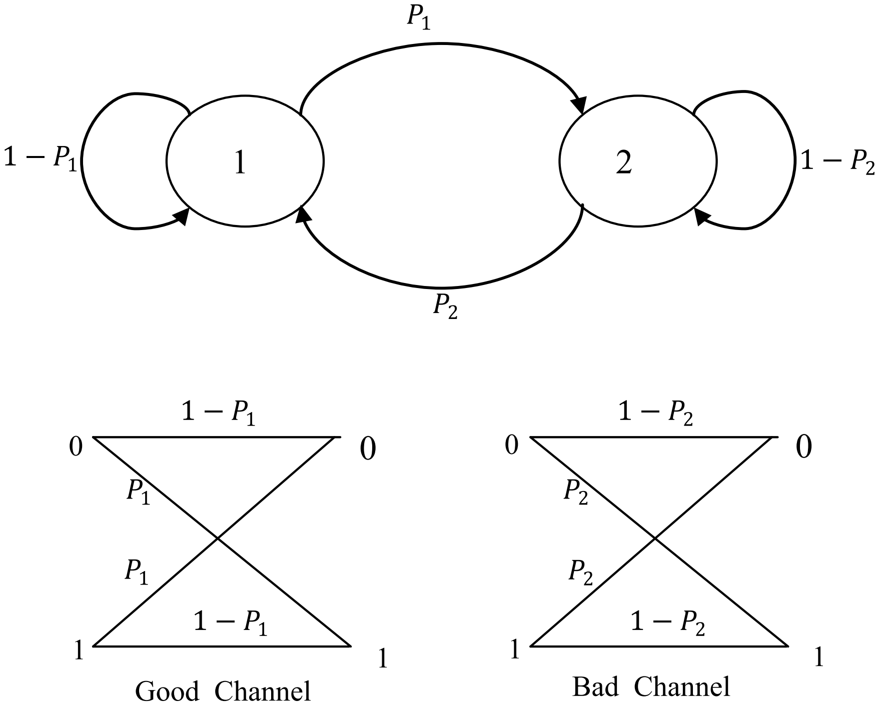


Fig. 1.4 Two-state channel

1.2.5 Fading Channel

In the radio channel, the received power is affected by the attenuations due to the combinations of the following effects

1. *The Path loss*: It is the signal attenuation. The power received by the receiving antenna decreases when the distance between transmitter and receiver increases. The power attenuation is proportional to (distance)$^{\alpha}$, where α values range from 2 to 4. When the distance varies with time, the path loss also varies.
2. *The Shadowing loss*: It is due to the absorption of the radiated signal by scattering structure. It is derived from a random variable with log-normal distribution.
3. *The Fading loss*: The combination of multipath propagation and the Doppler frequency shift produces the random fluctuations in the received power which gives the fading losses.

1.2.6 Fading

Fading gives the variations of the received power along with the time. It is due to the combination of multipath propagation and the Doppler frequency shift which gives the time-varying attenuations and delays that may degrade the communication system performance. The received signal is a distorted version of the transmitted signal which is a sum of the signal components from the various paths with different delays due to multipath and motion.

Let T_s be the duration of a transmitted signal and B_x be the signal bandwidth. The fading channel can be classified based on coherence time and coherence bandwidth of the channel. The coherence time and coherence bandwidth of a channel are defined as

Doppler spread:

Doppler shift is defined as the perceived change in the frequency of the electromagnetic wave due to the mobility of the mobile user.

If the mobile station is moving with a velocity v at an angle θ with the line joining mobile user and base station, the Doppler shift f_d is expressed by

$$f_d = \left(\frac{v}{c}\cos\theta\right)f_c \tag{1.4a}$$

where f_c is the carrier frequency, and c is the velocity of light.

From the above equation, it can be observed that the Doppler shift is maximum when $\theta = 0$ and the Doppler shift is zero when $\theta = 0 = \frac{\pi}{2}$ that is, when the mobile motion is perpendicular to the receive direction.

Thus, the maximum Doppler shift $f_{d\max}$ is given by

$$f_{d\max} = \left(\frac{v}{c}\right) f_c \tag{1.4b}$$

Then, the delay spread $B_D = 2f_{d\max}$

The coherence time of the channel T_c is

$$T_c \triangleq \frac{1}{2B_D} = \frac{1}{4f_{d\max}} \tag{1.4c}$$

The coherence time implies that the channel changes at every coherence time duration, and the channel is to be estimated at least once in every coherence time interval.

Example 1.1 Consider a mobile user is moving at 40 kmph with $f_c = 2.4$ GHz. Compute the coherence time.

Solution

$$\begin{aligned}
40\,\text{kmph} &= 40 \times \frac{5}{18} = \frac{2000}{18} = 11.1111 \text{ m/s} \\
f_{d\max} &= \frac{11.1111}{3 \times 10^8} \times 2400 \times 10^6 = 88.8888\,\text{Hz} \\
T_c &= \frac{1}{4f_{d\max}} = \frac{1}{4 \times 88.8888} = 22.5 \text{ ms}
\end{aligned}$$

Delay spread:

The maximum among the path delay differences, a significant change occurs when the frequency change exceeds the inverse of T_D, called the *delay spread* of the channel.

The channel bandwidth of the channel B_c is

$$B_c \triangleq \frac{1}{T_D} \tag{1.5}$$

For example, if a 4-multipath channel with the delays corresponding to the first and the last arriving are 0 μs and 6 μs, the maximum delay spread

$$T_D = 6\,\mu\text{s} - 0\,\mu\text{s} = 6\,\mu\text{s}$$

RMS Delay spread:

In a typical wireless channel, the later arriving paths are with lower power due to larger propagation distances and weaker reflections. In such a scenario, maximum delay spread metric is not reliable.

The RMS delay spread is defined by

$$T_{\mathrm{DRMS}} = \sqrt{\frac{\sum_{i=0}^{L-1} g_i (\tau_i - \tilde{\tau})^2}{\sum_{i=0}^{L-1} g_i}}$$

where

τ_i is the delay of the *i*th path.
g_i is the power corresponding to the *i*th path.
$\tilde{\tau}$ is the average delay given by

$$\tilde{\tau} = \frac{\sum_{i=0}^{L-1} g_i \tau_i}{\sum_{i=0}^{L-1} g_i}$$

Example 1.2 Consider a wireless channel consists of $L = 4$ multipath with the delays and power tabulated as follows

i	τ_i (μs)	g_i
0	0	1
1	1	0.1
2	3	0.1
3	5	0.01

Solution

$$\tilde{\tau} = \frac{1 \times 0 + 0.1 \times 1 + 0.1 \times 3 + 0.01 \times 5}{1 + 0.1 + 0.1 + 0.01}\,\mu\mathrm{s} = 0.3719\,\mu\mathrm{s}$$

$$T_{\mathrm{DRMS}} = \sqrt{\frac{1 \times (0 - 0.3719)^2 + 0.1 \times (1 - 0.3719)^2 + 0.1 \times (3 - 0.3719)^2 + 0.01 \times (5 - 0.3719)^2}{1 + 0.1 + 0.1 + 0.01}}\,\mu\mathrm{s}$$
$$= 0.9459\,\mu\mathrm{s}$$

1.2.6.1 Fading Channels Classification

The classification of fading channels is shown in Fig. 1.5.

The fast fading causes short burst errors which are easy to correct. The slow fading will affect many successive symbols leading to long burst errors. Due to energy absorption and scattering in physical channel propagation media, the transmitted signal is attenuated and becomes noisy. The attenuation will vary in mobile

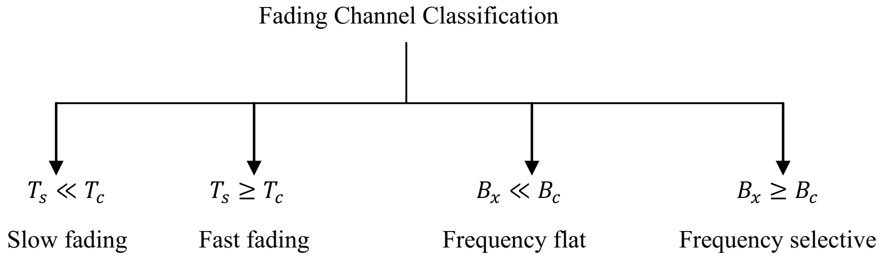


Fig. 1.5 Classification of fading channels

communications based on the vehicle speed, surrounding trees, buildings, mountains, and terrain. Based on the receiver location, moving receiver signals interfere with one another and take several different paths. As such, the wireless channels are called multipath fading channels. Hence, the additive white Gaussian noise (AWGN) assumption for wireless channels is not realistic. Thus, the amplitudes in the wireless channels are often modeled using Rayleigh or Rician probability density function.

The most common fading channel models are

1. Flat independent fading channel
2. Block fading channel.

In flat independent fading channel, the attenuation remains constant for one symbol period and varies from symbol to symbol. Whereas in block fading channel, the attenuation is constant over a block of symbols and varies from block to block.

1.3 Statistical Models for Fading Channels

1.3.1 Probability Density Function of Rician Fading Channel

When the received signal is made up of multiple reflective rays plus a significant line-of-sight (non-faded) component, the received envelope amplitude has a Rician Probability Density Function (PDF) as given in Eq. (1.6), and the fading is referred to as Rician fading.

$$\begin{aligned} P_{df}(x) &= \frac{x}{\sigma^2}\exp\left(-\frac{(x^2+A^2)}{2\sigma^2}\right)I_0\left(\frac{xA}{\sigma^2}\right) && ;\text{ for } x \geq 0, A \geq 0 \\ &= 0 && ;\text{ Otherwise} \end{aligned} \tag{1.6}$$

where x is the amplitude of the received faded signal, I_0 is the zero-order modified Bessel function of the first kind, and A denotes the peak magnitude of the non-faded signal component called the specular component. The Rician PDF for different values of sigma and $A = 1$ is shown in Fig. 1.6.

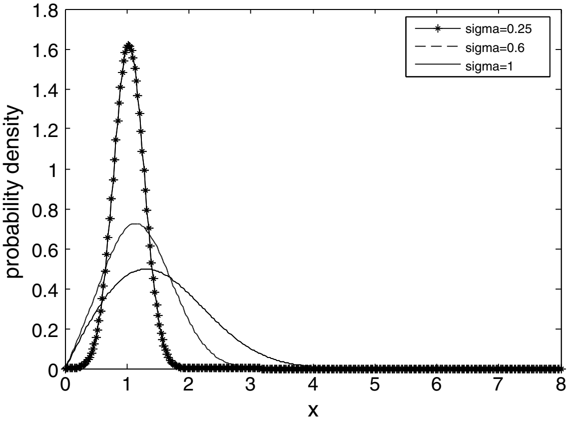


Fig. 1.6 Probability density of Rician fading channel

1.3.2 Probability Density Function of Rayleigh Fading Channel

Rayleigh fading occurs when there are multiple indirect paths between the transmitter and the receiver and no direct non-fading or line-of-sight (LOS) path. It represents the worst case scenario for the transmission channel. Rayleigh fading assumes that a received multipath signal consists of a large number of reflected waves with independent and identically distributed phase and amplitude. The envelope of the received carrier signal is Rayleigh distributed in wireless communications [3].

As the magnitude of the specular component approaches zero, the Rician PDF approaches a Rayleigh PDF expressed as follows:

$$\begin{aligned} P_{df}(x) &= \tfrac{x}{\sigma^2}\exp\left(-\tfrac{x^2}{2\sigma^2}\right) && \text{for } x \geq 0 \\ &= 0 && \text{Otherwise} \end{aligned} \tag{1.7}$$

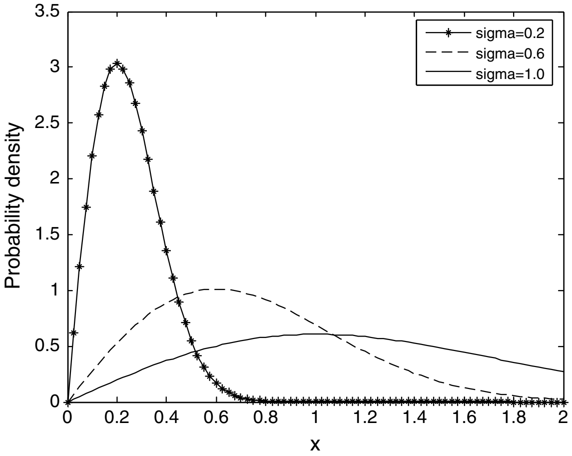
The Rayleigh PDF for different values of sigma is shown in Fig. 1.7.

Additive white Gaussian noise and Rician channels provide fairly good performance corresponding to an open country environment, while Rayleigh channel, which best describes the urban environment fading, provides relatively worse performance.

1.3.3 Probability Density Function of Nakagami Fading Channel

The Nakagami model is another very popular empirical fading model [4]

Fig. 1.7 Probability density of Rayleigh fading channel



$$P_{df}(r) = \frac{2}{\Gamma(m)}\left(\frac{m}{2\sigma^2}\right)^m r^{2m-1}\mathrm{e}^{-m\frac{r^2}{2\sigma^2}} \tag{1.8}$$

where

$\sigma^2 = \frac{1}{2}E\left[r^2\right]$, $\Gamma(.)$ is the gamma function.
$m \geq \frac{1}{2}$ is the fading figure.

The received instantaneous power r^2 satisfies a gamma distribution. The phase of the signal is uniformly distributed in $[0, 2\pi)$. The Nakagami distribution is a general model obtained from experimental data fitting and its shape very similar to that of the Rice distribution. The shape parameter "*m*" measures the severity of fading.

when

$m = 1$, it is Rayleigh fading.
$m \to \infty$, it is AWGN channel, that is, there is no fading.
$m > 1$, it is close to Rician fading.

However, due to lack of physical basis, the Nakagami distribution is not as popular as the Rician and Rayleigh fading models in mobile communications. Many other fading channel models are discussed in [5].

1.4 Channel Capacity

Channel capacity can be defined as the maximum rate at which the information can be transmitted over a reliable channel.

$$\text{Spectral or Bandwidth Efficiency} = \frac{\text{Transmission rate}}{\text{Channel Bandwidth}} = \frac{R_s\mathcal{H}}{B} \quad \text{bits/s/Hz} \tag{1.9}$$

where R_s is the symbol rate, $\mathcal{H}$ is the entropy.

The channel capacity is also known as Shannon's capacity which can be defined as the average mutual information for a channel with energy constraint.

1.4.1 Channel Capacity of Binary Erasure Channel

The channel capacity of binary erasure channel is

$$\mathbf{C}_{\mathrm{BEC}} = 1 - \varepsilon \tag{1.10}$$

ε is the probability of a bit erasure, which is represented by the symbol e.

1.4.2 Channel Capacity of Binary Symmetric Channel

The channel capacity of binary symmetric channel is

$$\mathbf{C}_{\mathrm{BSC}} = 1 - \mathcal{H}(P) \tag{1.11}$$

$\mathcal{H}(P)$ is the binary entropy function given by Ryan and Lin [6]

$$\mathcal{H}(P) = -P\log_2(P) - (1-P)\log_2(1-P) \tag{1.12}$$

P is the probability of a bit error.

1.4.3 Capacity of AWGN Channel

An additive white Gaussian noise (AWGN) channel can be expressed by the following input–output relationship

$$y = x + \eta \tag{1.13}$$

where x is the transmitted source signal, y denotes the output of the channel, η is a real Gaussian process with zero mean, variance $\sigma_\eta^2 = E\left[\eta^2\right]$, and two-sided power spectral density $\frac{N_0}{2}$. The mutual information $I(x;y)$ with constraint on the energy of the input signal can be expressed as