

Franz Herz
Rainer Nordmann

Vibrations of Power Plant Machines

A Guide for Recognition of Problems and
Troubleshooting

 Springer

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and Troubleshooting

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*I dedicate this book to my sons Harald,
Johann and Stefan.*
—Franz Herz

Preface

Coming to a power plant as an expert, because of a vibration problem, you will hear in the most cases from the maintenance people: When do we have to shut down for balancing? Rebalancing seems to be the only generally known countermeasure for high vibrations at a turbomachine. This book shows that a mechanical unbalance covers only a small section in the big line of all the possible vibration problems.

The aim of the book is to provide practical people working in the field, at power plants and/or as service engineers, with a guideline in case there are vibration problems. In the books available on vibration, there are usually a lot of theories, equations and high mathematics. This book does not make great demands on scientific perfection, it is meant for the practical “hands on” man.

Many of the illustrations and sketches in this book come from old reports that are no longer available. Therefore, the quality of the illustrations does not always meet today’s standards.

However, I have decided to include these images in this book, because they are valuable, given that they are the only images of their kind that still exist.

In Chap. 1—“Basics of Vibrations”—some fundamental theories are explained, which is necessary to understand the vibration events mentioned in Chap. 3—“Fault Analysis: Vibration Causes and Case Studies.”

This book does not have the usual references as it includes mainly real-life case studies originating from the 50-year experience about power plant events. These real-life case studies could serve as a reference, regarding the possible problems at jobsite.

It is tried to describe the case studies in three steps, explaining the:

1. identification of the problem,
2. explanation of the problem and
3. practical solution to the problem.

Also, some knowledge about measurement and presentation of results is not avoidable; therefore, see Chap. 2—“Instrumentation, Measurement.”

The rotors of a machine are balanced individually by the manufacturer. They will be coupled to a shaft train at first at jobsite. So, a new unbalance distribution is created and might require a balance correction. On top of that, the newly installed rotors will see operational parameters (like temperature, torque, etc.) the first time. This might be another reason for a necessary balance correction. Therefore, Chap. 4—“Jobsite Balancing”—has been added. It is not meant as a general balancing instruction, but as a guideline for trim balance corrections for the above-mentioned occurrences.

There are a lot of books on vibration on the market, but not in one of them we could find a comprehensive collection of case studies of real cases. This book had been written in English, because according to our experience, especially at the East Asian region, there seems to be a need for comprehensive practical machinery vibration primer.



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Chapter 1

Basics of Vibrations



This chapter mainly deals with the relationship of vibration excitation and vibration response of systems. Firstly, we analyze the different vibration signal types and the different vibration measurements units. This leads us to the Fast Fourier Transformation (FFT). We now can explain the resonance frequency: $\omega_0 = \sqrt{\frac{c}{m}}$. The vibrations of a single degree of freedom (SDOF) are explained then and finally the rotating shaft is explained by means of the Laval shaft. The last chapter deals with the practical behavior of turbomachine rotors.

In Chap. 3: “Fault Analysis: Vibration Causes and Case Studies”—there are various vibration problems specified, as they may appear in power plant machines. To understand and interpret these vibration phenomena, we need to understand the basic of vibrations.

At first, we must ask how we can describe vibrations of mechanical systems in terms of deflections, velocities and accelerations as a function of time. This consideration of the kinematic of vibrations is independent from the vibration system and from the source of vibrations. Important quantities to describe, for example, a simple harmonic vibration are the time period and the frequency of the vibration (see Sect. 1.1.1). However, very often vibrations cannot be described by one frequency only, but they consist of a superposition of time signals with different frequencies. In order to recognize these different frequencies involved in a vibration event, the Fourier analysis is a powerful tool (see Sect. 1.1.2). The important relations between vibration deflections, vibration velocities and vibration accelerations are derived in Sect. 1.1.3 for the simple case of a harmonic time signal, expressed by amplitude, frequency and phase.

The second important question is: What causes the vibrations of mechanical systems? The theory of mechanical vibrations shows that vibrations depend on some kind of excitation on one side and the dynamic characteristics of the vibrating mechanical system itself on the other side. Excitations can be time-dependent forces and/or moments, but movements of the ground or other boundaries are also possible. Excitations may be different in the time domain, where the time functions can be harmonic, periodic or non-periodic (transient). An excitation can be of very short time or may act permanently.

The other important influences on vibrations are the dynamic characteristics of the mechanical system itself. The physical system parameters of mass, damping and stiffness values determine how a vibration system reacts to excitations (disturbances). The dynamic characteristics of a mechanical system can also be expressed by its eigenvalues (natural frequencies, damping) and mode shapes or by frequency response functions (FRF). From a more practical view, mechanical systems react very sensitive with respect to vibrations, when they are excited in a resonance condition (exciter frequency equal to a natural frequency). In the case of rotating machinery, such resonance conditions are called critical speeds, where the rotational shaft frequency of an unbalance excitation is equal to one of the natural frequencies of the mechanical system.

Therefore, at each vibration problem two fundamental aspects must be considered:

1. the excitation forces like the unbalance forces due to rotation of the shaft and
2. the consequence of those forces to mechanical systems upon which they are acting like in critical speeds and in resonances.

The engineer in charge must decide what is the most promising way to overcome a problem: Is it 1 or 2 or perhaps both.

A very simple mechanical system to explain the basics of vibrations is the single degree of freedom (SDOF) system (see Sect. 1.2), consisting of the two parameters mass m and spring c . For this basic system, the equations of motion, the natural frequency, the free vibrations and the forced vibrations in case of a ground excitation are derived and discussed. The additional effect of damping on free vibrations (see Sect. 1.2.1) and forced vibrations (see Sect. 1.2.2) are investigated. From the simple mass-spring system, we lead to the rotating shaft since it also follows similar fundamental laws.

The transfer from the above SDOF mechanical system to a similar system with a flexible rotating shaft is shown in Sect. 1.3. This basic mechanical system is the Laval rotor, where the mass m is represented by a disc in the center of the shaft and the spring c by the bending stiffness of the rotating shaft. Extensions of this very basic system are possible, e.g., by the flexibility of the bearings and the support system, which may include additional springs and masses as well. The effects of rotation, e.g., unbalance and gyroscopic, must be considered.

The vibration system of a complete power plant machine, e.g., a turbine shaft train consisting of the shaft train, the oil film bearings, the pedestals and the foundation, is much more complicated and must be considered as a multi-degree of freedom (MDOF) vibration system (see Sect. 1.4).

1.1 Kinematic of Vibrations

We discuss the kinematic of vibrations independent from the fact which sources the vibrations have and at which locations of a system they occur. We will concentrate on periodic vibrations and the special case of harmonic vibrations, because of their dominance in vibration problems of power plant machines.

1.1.1 Periodic and Harmonic Vibration Signals in the Time Domain

Figures 1.1 and 1.2 describe three typical periodic oscillations $s(t)$: triangular, sinusoidal and rectangular. An important example of them is the harmonic sinusoidal time function:

$$s(t) = s_{\max} \sin \omega t \tag{1.1}$$

with the amplitude s_{\max} and the angular frequency ω .

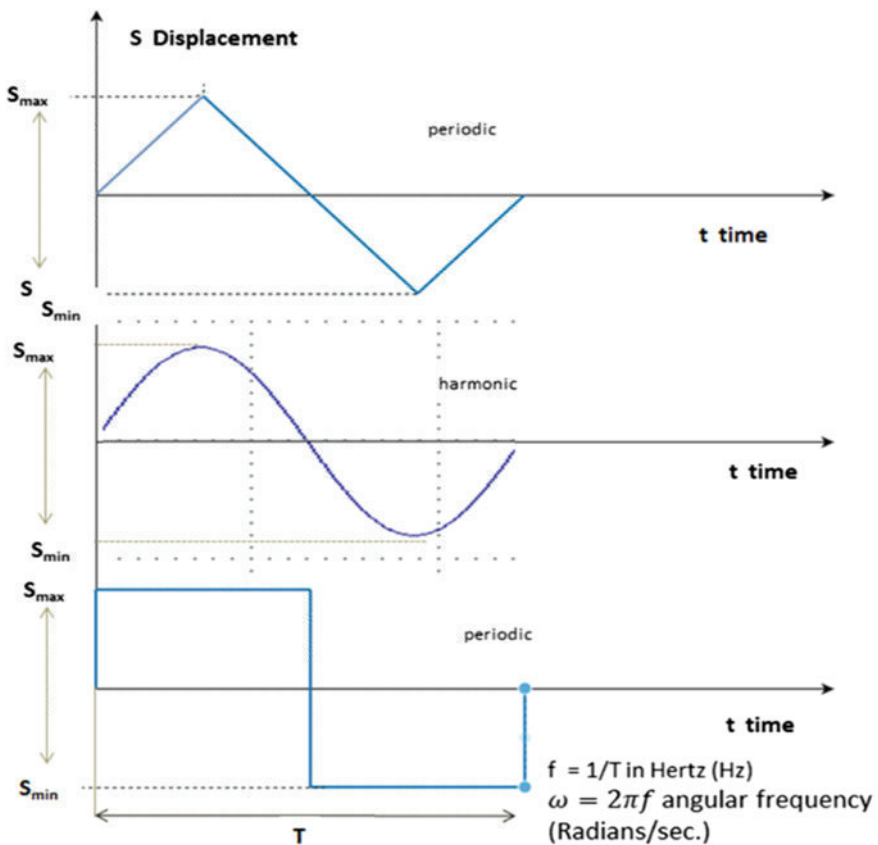


Fig. 1.1 Periodic vibrations

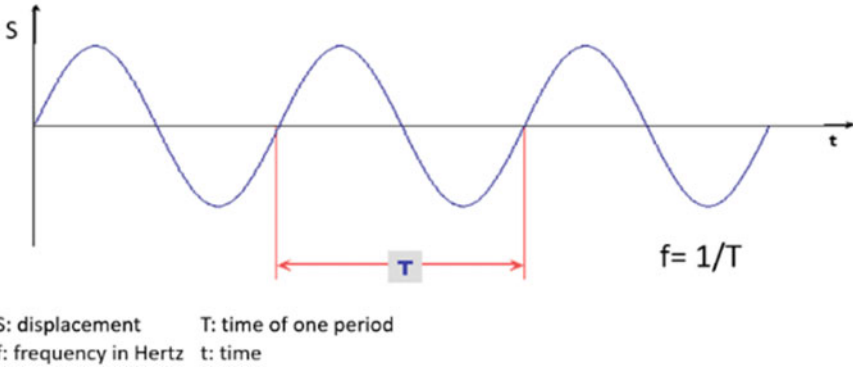


Fig. 1.2 Harmonic vibration

After elapse of the time period T , all three time function conditions which govern the changes repeat themselves. We can now determine the oscillatory (vibration) frequency f from the period T , as follows:

$$f = 1/T \quad (1.2)$$

The unit of the frequency f is Hertz: f is equal to the number of events (periods) per second. In vibration calculations, one often uses the angular frequency, i.e., the number of oscillations in 2π seconds:

$$\omega = 2\pi f \quad (1.3)$$

In the analysis of vibration, the subject of “harmonic motion” is of particular importance. This is when the variables which apply change in accordance with a sinusoidal curve (see Figs. 1.1, 1.2 and 1.3). The harmonic motion is characterized by one frequency only. As an example, Fig. 1.3 shows the superposition of two harmonic functions S_1 and S_2 leading to the combined periodic vibration S_3 with two frequencies of S_1 and S_2 .

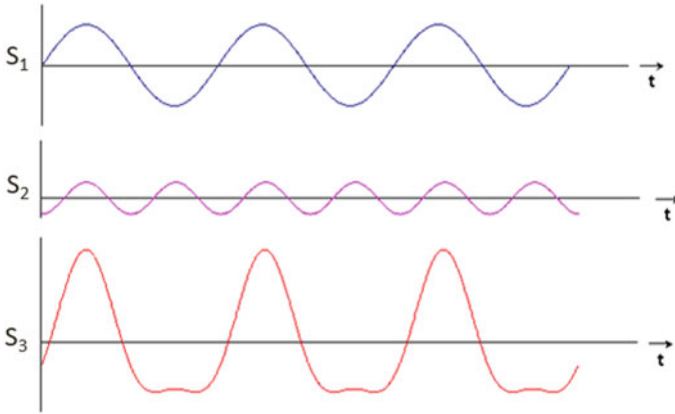


Fig. 1.3 Combined vibrations

1.1.2 Vibrations in the Time and Frequency Domain (Fourier Analysis)

The Fourier analysis is applied, if the different frequencies being involved in a vibration event. *The vibration event should be individually recognized.* We speak about frequency analyzers or Fourier analyzers. These instruments have a special importance at modal analysis applications like in structural resonance problems. They enable a transition from the time domain into the frequency domain.

Every periodic oscillation can be traced back to a combination of harmonic (i.e., sinusoidal) oscillations. Considering as example a rectangular form (see Fig. 1.4) and using the Fourier transform method, this time function $s = f(t)$ can be transformed from the time domain to the frequency domain $s = f(f)$ and we obtain a spectrum. Figure 1.4 indicates the Fourier transformation as a transition from time to frequency domain:

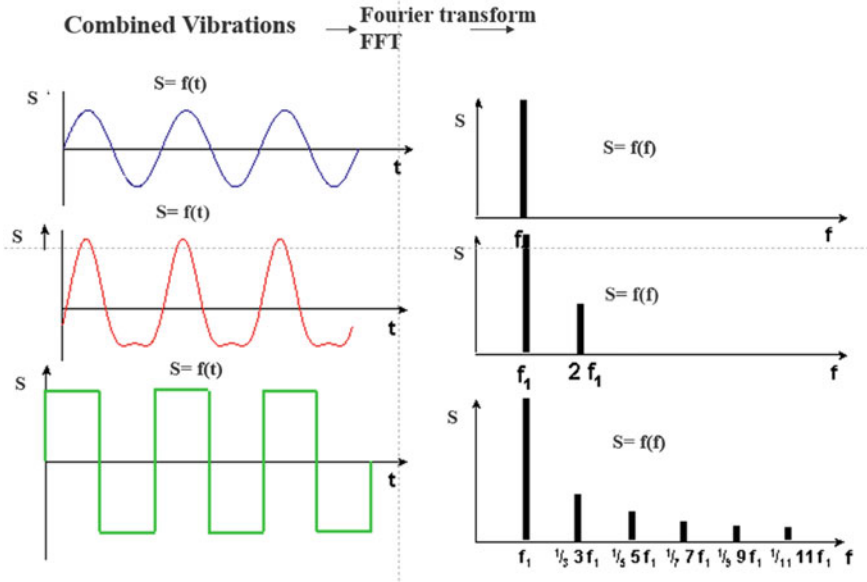


Fig. 1.4 Time-frequency functions

If the rectangular oscillation is given by the time period T and the basic frequency $f = f_0 = 1/T$ in the time domain, one finds in the frequency domain that this is a combination of basic frequency f_0 and an infinite number of harmonic components $f_1, f_2, f_3, \dots, f_\infty$.

Harmonic frequency components f_1 to f_∞ are odd-numbered multiples of the fundamental f_0 :

$$f_1 = 1f_0, f_2 = 0, f_3 = 3f_0, f_4 = 0, f_5 = 5f_0, \dots$$

If we look at a triangular oscillation, we will find even-numbered frequency components:

$$f_1 = f_0, f_2 = 2f_0, f_3 = 0, f_4 = 4f_0, f_5 = 0, f_6 = 6f_0, \dots$$

The Fourier cube is a visualization of the time domain and the frequency domain in a 3-dimensional presentation (see Fig. 1.5):

- From the frequency view, the different frequency lines are visible as a spectrum.
- From the time view, the superimposed time functions of these frequencies are visible.

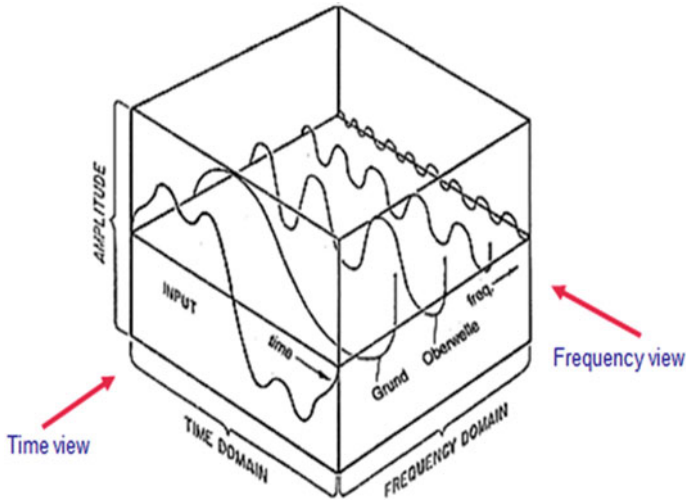


Fig. 1.5 Fourier cube

The following Figs. 1.6, 1.7, 1.8, 1.9 and 1.10 demonstrate how a periodic function is built from harmonics.

Fig. 1.6 Fundamental (Brüel and Kjær Vibro 1995)

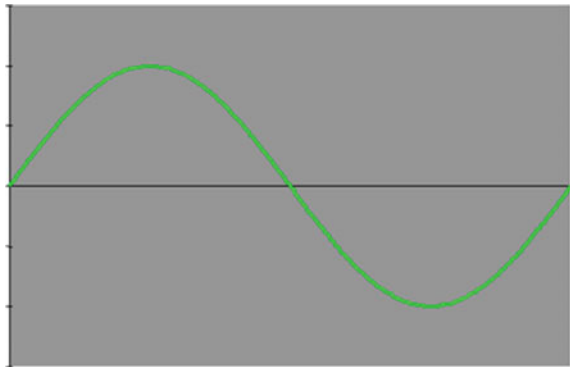


Fig. 1.7 Third harmonic added (Brüel and Kjær Vibro 1995)

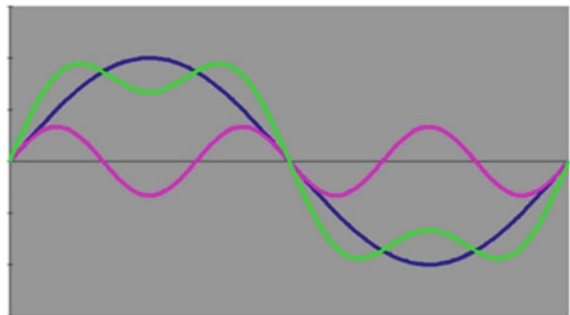


Fig. 1.8 Fifth harmonic added (Brüel and Kjær Vibro 1995)

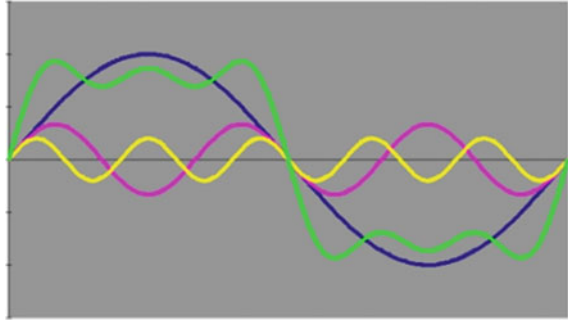


Fig. 1.9 Seventh harmonic added (Brüel and Kjær Vibro 1995)

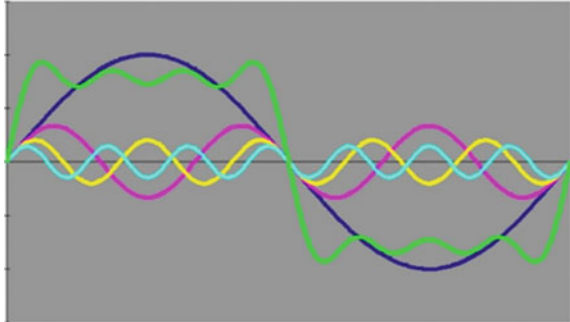
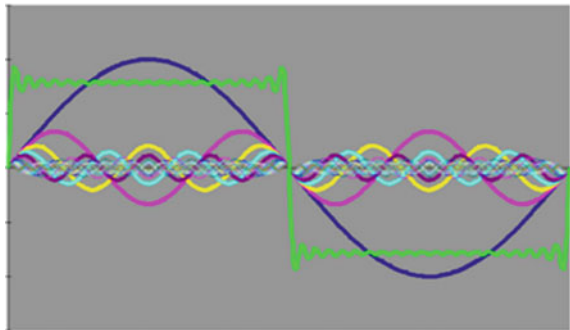


Fig. 1.10 Fifth to ninth harmonic added (Brüel and Kjær Vibro 1995)



We are now already very close to the rectangular shape. As more harmonics are added, the closer we get to the rectangular shape. Adding an indefinite number of harmonics will result in an ideal rectangular signal.

The Fourier rule says: All periodic and quasi-periodic signals are a combination of several harmonic signals.

1.1.3 Relations Between Deflections, Velocities and Accelerations

We consider a simple mass-spring vibration system as shown in Fig. 1.11 and assume that the point-mass m performs a harmonic up and down movement, where the deflection $s(t)$ follows a time function as described in Eq. (1.4). It describes a motion whereby its distance from the zero position varies according to a sinusoidal time function. Such a harmonic motion of the SDOF system can occur either as a free motion after a short disturbance or as a forced motion due to some harmonic excitation. Solutions can be found from the equations of motion for the SDOF system as it will be derived in the next Sect. 1.2. From a kinematic point of view, the harmonic time function shown in Fig. 1.2 is determined by the projection of an arrow, rotating in a polar diagram with the angular velocity ω (see Fig. 1.11 for the definition quantities of a spring pendulum).

The origin of the polar graph is 0, but we define an arbitrary chosen starting point $t = 0$ for our considerations. This because of the balancing phase reference, which will be explained later.

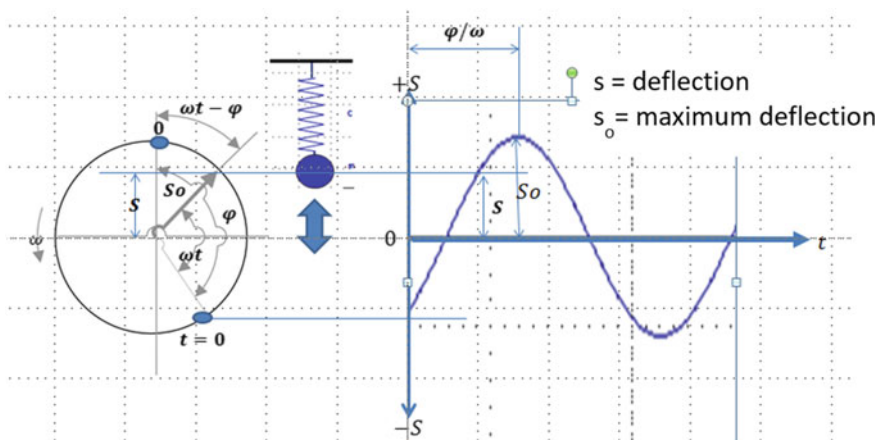


Fig. 1.11 Mass on a spring

If we consider the period starting at $t = 0$, the relationship between s and t is given by the following equation:

$$s = s_0 \cos(\omega t - \varphi) \tag{1.4}$$

where s_0 is the deflection or vibration amplitude and φ is the phase angle. Both quantities are very important, particularly when different time signals must be superimposed.

We will need the phase angle later for balancing, which will be described in more detail in Chap. 4. The phase angle φ is determined from the point of time zero. With most of the vibration measurements made on turbo-sets, time zero is set by a reference signal, generated by a photo-electric or magnetic pickup from a mark on the shaft, so that one impulse peak is given for each revolution.

The balancing phase reference will be explained later.

So far, we have been considering the deflection s which is also referred to as the vibration amplitude. If the deflection s is now differentiated with respect to time, we obtain the velocity v :

$$v = \frac{ds}{dt} = -s_0\omega \sin(\omega t - \varphi) = v_0 \sin(\omega t - \varphi) \quad (1.5)$$

If Eq. (1.4) is now differentiated a second time with respect to time, we obtain the vibration acceleration a :

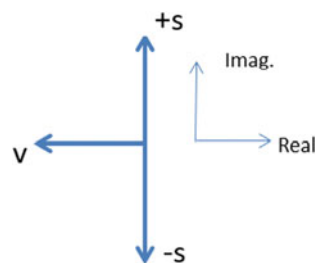
$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = -s_0\omega^2 \cos(\omega t - \varphi) = a_0 \cos(\omega t - \varphi) \quad (1.6)$$

From Eqs. (1.4), (1.5) and (1.6), the following relationship becomes clear:

- s is a function of $\cos(\omega t - \varphi)$ with factor s_0
- v is a function of $-\sin(\omega t - \varphi)$ with factor $s_0\omega$
- a is a function of $-\cos(\omega t - \varphi)$ with factor $s_0\omega^2$.

For a given deflection amplitude s_0 , the velocity amplitude v_0 rises linearly with the frequency ω while the acceleration amplitude a_0 rises quadratic with ω . The vibration parameters mostly used in power plant applications, s and v , have a phase displacement of 90° to each other (see Fig. 1.12).

Fig. 1.12 Phase relation between s and v



In practice, oscillation or vibration takes place when a mass is subject to forces under resilient conditions. Mass and spring elements are the requisite components of a system which is capable of vibration or oscillation. The vibration may either be free (i.e., natural) or forced vibration.

During the motion, spring forces, mass inertia forces and external forces are acting, and at each instant are in a state of equilibrium. In actual practice, another element is also present in the form of frictional forces, which act in opposition to the direction of motion and which must be included in the conditions of equilibrium. The effect of friction is called damping. If its effect is so small that it can be neglected for the considered case, we have an undamped (or weakly damped) system, otherwise it is damped (see Fig. 1.13).

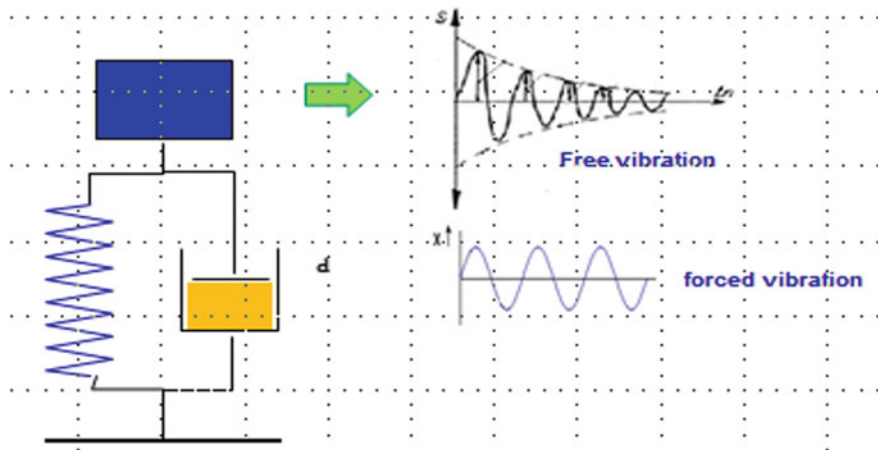


Fig. 1.13 Damped vibration

In contrary to the free, undamped vibration, the damping causes that the system will restore the standstill according to an exponential function. How long that takes depends on the amount of damping. A mass-spring system comes into free vibration if the mass is once displaced from its resting position and then allowed to move on its own; forced vibration takes place if the system is continuously kept in motion by an external force.

In Fig. 1.13, we see the difference between a free and a forced vibration. In practice, a free vibration will decay because of the damping d . A forced vibration will be kept up, because of an external driving force.

1.2 Vibrations of a Single Degree of Freedom (SDOF) System

A very simple mechanical system to explain the basics of vibrations is the single degree of freedom (SDOF) system, consisting of the two parameters mass m and spring c . For this basic system, the equations of motion, the natural frequency, the

free vibrations and the forced vibrations will be derived and discussed in this chapter. The additional effect of damping on free vibrations and forced vibrations will also be investigated.

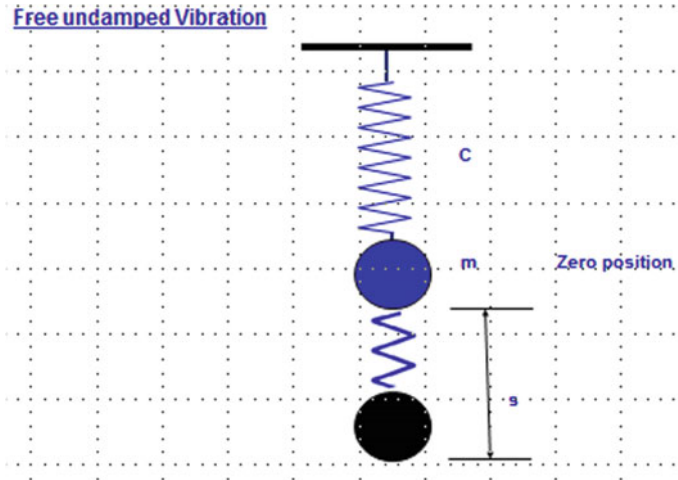


Fig. 1.14 Free vibration of a SDOF system

Free undamped vibrations: A ball of mass m as shown in Fig. 1.14 is suspended from a linear spring having the spring constant c . The ball is next displaced by amount s and then allowed to move freely. We now want to find the natural frequency of the oscillation of the mass-spring system, in other words the frequency of the system, at which “resonance” appears. In case of the free vibration, the forces acting on the ball are:

- the spring restoring force $F = -cs$ and
- the inertia force of the mass m .

The minus sign indicates that F acts in the opposite direction to the deflection s . In accordance with the fundamental law of dynamics, expressed by Newton’s law, these two forces must be in equilibrium:

$$-cs = m \frac{d^2s}{dt^2} \Rightarrow \frac{d^2s}{dt^2} + \frac{c}{m}s = 0 \quad (1.7)$$

For this equation of motion for the free vibrations, the time solution can be obtained by $s = s_0 \sin \omega t$ and with the derived acceleration:

$$\frac{d^2s}{dt^2} = -\omega^2s = a \tag{1.8}$$

we obtain the angular natural frequency ω of the SDOF system:

$$\omega^2s = \frac{c}{m}s \Rightarrow \omega = \omega_0 = \sqrt{\frac{c}{m}} \tag{1.9}$$

The natural frequency can also be expressed in Hertz. This natural frequency is a function of c and m .

$$f = \frac{1}{2\pi} \sqrt{\frac{c}{m}} \tag{1.10}$$

Forced undamped vibration: If the mass-spring system is caused to vibrate by external means, the ball will be subject to forced vibration. We consider the special case of a forced ground excitation as shown in Fig. 1.15.

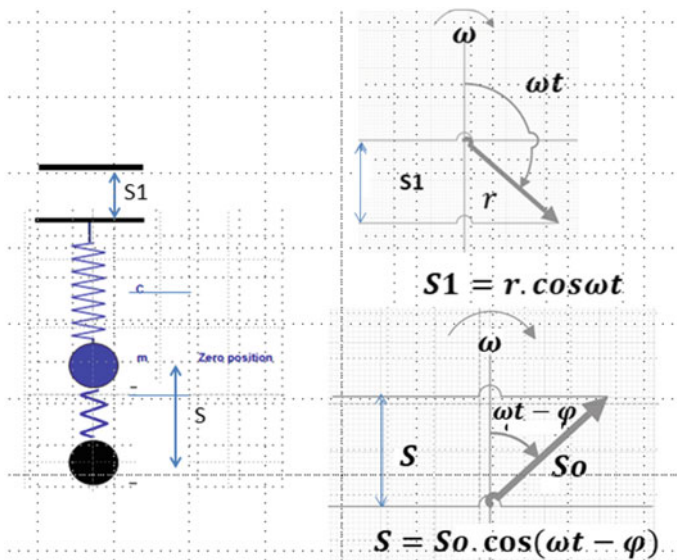


Fig. 1.15 Forced vibration due to ground excitation

We concentrate on the “vibrating condition” after all initiating processes have died out. If the point of suspension (or of excitation) has moved by amount s_1 , the spring has been lengthened by amount $s - s_1$ and as per the basic law of dynamics, we find the equations of motion for the forced vibration of the SDOF:

$$m \frac{d^2s}{dt^2} + cs = cs_1 \tag{1.11}$$

Substituting s_1 , we obtain the equation of motion:

$$m \frac{d^2s}{dt^2} + cs = cr \cos(\omega t) \tag{1.12}$$

By solving Eq. (1.12) for the deflection s , we obtain:

$$s = \frac{cr}{c - m\omega^2} \cos(\omega t) \tag{1.13}$$

Thus, the deflection depends on the frequency ω . At very low values of ω , the maximum value becomes practically equal to r , and the forced vibration hardly differs from the exciting oscillation. We could consider that as the “rigid state.” When ω reaches the value $\omega = \omega_0$ (the case of resonance), the denominator becomes zero and s approaches infinity. In practice, infinite deflection does not occur, because it is damped by damping effects which are always present. When ω is increased further, above, the denominator becomes negative; this means that the excitation point reverses direction in relation to the mass. Thus, the mass experiences, as it passes through the resonance frequency, a “phase jump” of 180° in relation to the movement of the suspension point. This transition does not take place suddenly, as the damping results in a continuous changeover (see Fig. 1.16).

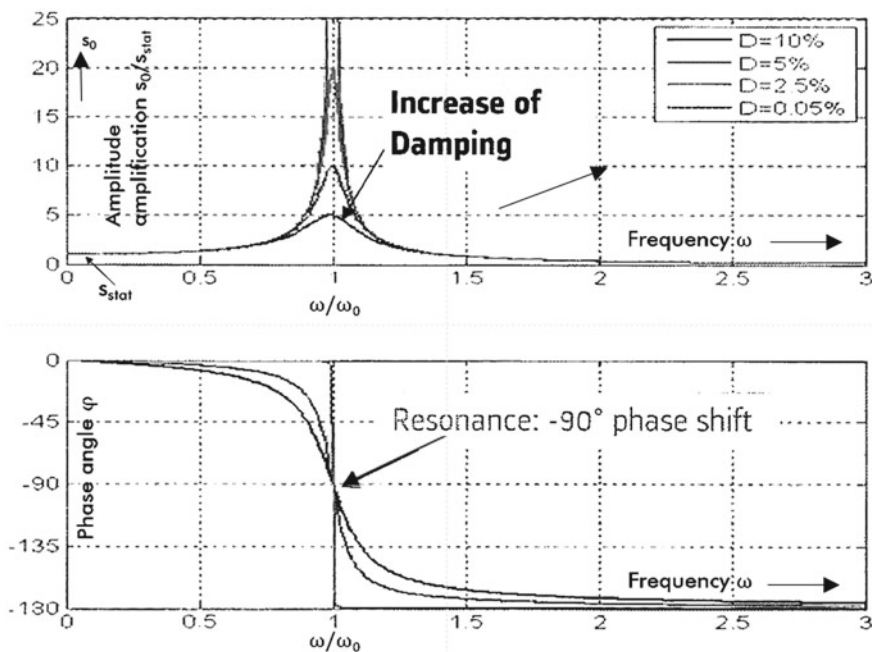


Fig. 1.16 Resonance, phase shift, damping

Figure 1.16 demonstrates the effect of damping, expressed by the damping factor D . Damping sources can be

- material,
- joints,
- journal bearings,
- seals and
- electromagnetic interaction.

Below its resonance frequency, a flexible system is governed by its spring properties $1/c$. At the high point of the resonance, the damping will limit the response $1/\omega d$. After having passed the resonance, the inertia mass will govern the response.

The effect of damping on the free as well the forced vibrations will be derived in the following (Sects. 1.2.1 and 1.2.2).

1.2.1 Effect of Damping on Free Vibration

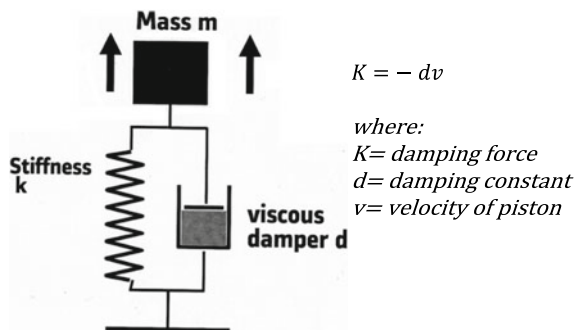
Up to now, the damping effect was neglected in the equations of motion, as shown again for the free vibrations

$$m \frac{d^2 s}{dt^2} + cs = 0 \quad (1.14)$$

where $m \frac{d^2 s}{dt^2}$ represents the mass inertia force and cs represents the spring restoring force.

As described earlier, in real cases there are always forces present which have a damping effect. The damping forces perform work and reduce the content of kinetic energy in the system. To illustrate this, we will add a damping cylinder to the simple model, where the piston follows the same motion as the mass (see Fig. 1.17).

Fig. 1.17 Damped spring-mass system



The sign of the damping force is negative because the damping force acts against the motion and slows it down. Again, the dynamic forces are in equilibrium for the free vibrations, so that:

$$m \frac{d^2s}{dt^2} + d \frac{ds}{dt} + cs = 0 \quad (1.15)$$

In the cylinder, frictional forces K occur which act in the opposite direction to the vibration movement and which have a magnitude proportional to the velocity v of the piston (in this case one speaks of a “linear damping,” which is the usual situation in our work).

Equation (1.15) is the equation of motion for the damped, free vibration. The terms $m \frac{d^2s}{dt^2}$ and cs are known from the previous discussion, and the term $d \frac{ds}{dt}$ now represents the damping force.

When solving this equation for the displacement s by means of a mathematical set up, we obtain

$$s = s_0 e^{-Dt} \cos(\omega t - \varphi) \quad (1.16)$$

where D is the constant for decay time, in the technical literature known as damping measure:

$$D = \frac{d}{2\sqrt{cm}} \quad (1.17)$$

D is a dimensionless number, which in our practice lies between 0 and 1 (or between 0 and 100%). A good way to determine D is the “logarithmic decrement δ ,” by consideration of the process of decay.

The initially undamped time function $s = s_0 \cos(\omega t - \varphi)$ will be enveloped by the damping $s = s_0 e^{-Dt}$ and decays logarithmically. When the values of the vibration maxima are plotted on a vertically scaled logarithmic graph, each maximum of the points plotted must lie on a straight line (see Fig. 1.18).