Walter Lacarbonara
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Jun Ma · J. A. Tenreiro Machado
Gabor Stepan *Editors*

Nonlinear Dynamics and Control

Proceedings of the First International Nonlinear Dynamics Conference (NODYCON 2019), Volume II

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Preface

This volume is part of three volumes collecting the *Proceedings of the First International Nonlinear Dynamics Conference (NODYCON 2019)* held in Rome, February 17–20, 2019. NODYCON was launched to foster the tradition of the conference series originally established by Prof. Ali H. Nayfeh in 1986 at Virginia Polytechnic Institute and State University (Virginia Tech), Blacksburg, VA, USA, as the Nonlinear Vibrations, Stability, and Dynamics of Structures Conference. With the passing in 2017 of Prof. Nayfeh, who was also the founder of the Springer journal *Nonlinear Dynamics* in 1990, NODYCON 2019 was organized as a collective tribute of the community to Prof. Nayfeh for being one of the most influential leaders of nonlinear dynamics. NODYCON 2019 was also established to look to and dream about the future. The call for papers attracted contributions dealing with established nonlinear dynamics research topics as well as with the latest trends and developments. At the same time, to reflect the rich spectrum of topics covered by the journal *Nonlinear Dynamics*, the call included diverse and multidisciplinary topics, to mention a few, multi-scale dynamics, experimental dynamics, dynamics of structures/industrial machines/equipment/facilities, dynamics of adaptive, multifunctional, metamaterial structures, dynamics of composite/nanocomposite structures, reduced-order modeling, nonsmooth dynamics, fractional-order system dynamics, nonlinear interactions and parametric vibrations, computational techniques, nonlinear system identification, dynamics of NEMS/MEMS/nanomaterials, multibody dynamics, fluid/structure interaction, influence of nonlinearities on vibration control systems, human–machine interaction, nonlinear wave propagation in discrete and continuous media, chaotic map-based cryptography, ecosystem dynamics, social media dynamics, complexity in engineering, and network dynamics.

For NODYCON 2019, the organizers received 450 two-page abstracts and based on 467 reviews from the Program Committee, the Steering and Advisory Committees, and external reviewers, 391 papers and 17 posters were accepted, published in the Book of Abstracts (NODYS Publications, Rome, ISBN 978-88-944229-0-0), and presented by nearly 400 participants from 68 countries. The diverse topics covered by the papers were organized along four major themes to organize the technical sessions:

(a) Concepts and methods in nonlinear dynamics
(b) Nonlinear dynamics of mechanical and structural systems
(c) Nonlinear dynamics and control
(d) Recent trends in nonlinear dynamics

The authors of a selection of approximately 60 papers were invited to publish in the Special Issue of *Nonlinear Dynamics* entitled "NODYCON 2019 First International Nonlinear Dynamics Conference." Over 200 full papers were submitted to the *Proceedings of the First International Nonlinear Dynamics Conference* (NODYCON 2019) and only 121 of them were accepted. These papers have been collected into three volumes, which are listed below together with a sub-topical organization.

Volume 1: Nonlinear Dynamics of Structures, Systems, and Devices

(a) Methods for nonlinear dynamics
(b) Bifurcations and nonsmooth systems
(c) Nonlinear phenomena in mechanical systems and structures
(d) Experimental dynamics, system identification and monitoring
(e) Fluid–structure interaction, multibody system dynamics
(f) Turning processes, rotating systems, and systems with time delays

Volume 2: Nonlinear Dynamics and Control

(g) Vibration absorbers and isolators
(h) Control of nonlinear systems
(i) Sensors and actuators
(j) Network synchronization

Volume 3: New Trends in Nonlinear Dynamics

(k) Smart materials, metamaterials, composite and nanocomposite materials, and structures
(l) MEMS/NEMS and energy harvesters
(m) Nonlinear phenomena in bio- and ecosystem dynamics
(n) Chaos in electronic systems
(o) Fractional-order systems

I wish to acknowledge the work of the Co-Editors of the NODYCON 2019 Proceedings: Prof. Balakumar Balachandran (University of Maryland, College Park, MD, USA), Prof. Jun Ma (Lanzhou University of Technology, China), Prof. J. A. Tenreiro Machado (Instituto Superior de Engenharia do Porto, Portugal), Prof. Gabor Stepan (Budapest University of Technology and Economics, Hungary).

The success of NODYCON 2019 relied primarily on the efforts, talent, energy, and enthusiasm of researchers in the field of nonlinear dynamics who wrote and submitted these papers. Special praise is also deserved for the reviewers who invested significant time in reading, examining, and assessing multiple papers, thus ensuring a high standard of quality for this conference proceedings.

Rome, Italy
August 2019

Walter Lacarbonara

Preface for Volume 2: Nonlinear Dynamics and Control

Volume 2 of the NODYCON 2019 Proceedings is composed of 33 papers, in which different kinds of control are applied for nonlinear dynamical systems. The first section of this volume groups together ten papers; in these studies, the aim of the applied control is to absorb and/or isolate the vibrations of a physical system. The second section presents the results of 17 research papers, where the quality of the applied control strategies is assessed, or sophisticated nonlinear control strategies are implemented to achieve the desired behavior of a given dynamical system. The third section involves two papers in which the effects of nonlinearities within sensors and/or actuators are discussed. Finally, the four papers of the fourth section investigate synchronization phenomena in networks of nonlinear dynamical systems and coupled oscillators.

The primary view point of the grouping of the papers was the goal of the research and not the applied methodology. Independently from the actual placement of a paper in a given section, the authors make use of a wide range of experimental, analytical, and numerical techniques for study of nonlinear dynamics.

In the work of S. Mohanty and S. K. Dwivedy, an active nonlinear vibration absorber is analyzed for a harmonically excited beam system. Z. Lu, D.-H. Gu, Y.-W. Zhang, H. Ding, W. Lacarbonara, and L.-Q. Chen compare linear and nonlinear damping effects in case of a ring vibration isolator. T. Lebrun, M. Wijnand, T. Hélie, D. Roze, and B. d'Andréa-Novel numerically examine electroacoustic absorbers based on the passive finite-time control of the loudspeakers. The seismic response of multiple base-isolated structures is predicted by F. Potenza, V. Gattulli, and S. Nagarajaiah for monitoring purposes. H. S. Kizilay and E. Cigeroglu analyze liquid-filled column dampers by means of nonlinear modeling in the frequency domain. M. E. Dogan and E. Cigeroglu achieve vibration reduction by means of two tuned mass dampers with dry friction. The nonlinear behavior of pendulum-tuned mass dampers is examined by K. Xu, X. Hua, and Z. Chen for vibration control. Resonance behavior is studied by Y. Mikhlin and Anton Onizhuk in a non-ideal system containing a snap-through truss absorber. A. Salvatore, B. Carboni, L.-Q. Chen, and W. Lacarbonara experimentally study the dynamic response of a nonlinear wire rope isolator. In the work of A. Boccamazzo, B. Carboni, G. Quaranta, and W. Lacarbonara, optimization strategies of hysteretic tuned mass dampers are discussed for seismic control.

A. Younespour and S. Cheng examine sliding mode control of nonlinear systems under nonstationary random vibrations via the equivalent linearization method. G. Stefani, M. De Angelis, and U. Andreaus discuss the experimental dynamic response of a harmonically excited SDOF oscillator constrained by two symmetrically arranged deformable and dissipative bumpers. Active sling load stabilization is considered by A. Morock, A. Arena, M. Lanzerotti, J. Capps, B. Huff, and W. Lacarbonara. In the work of D. Jing, J.-Q. Sun, C.-B. Ren, and X.-H. Zhang, the multi-objective optimization is studied for control of an active vehicle suspension system. P. Domański and M. Ławryńczuk assess the quality of nonlinear model predictive control by means of fractal and entropy measures. M. Gidlewski, L. Jemioł, and D. Żardecki analyze the impact of the controller algorithm on the motor vehicle steering during a lane-change manoeuver. The application of fractional-order impedance control is considered by G. Chen, S. Guo, B. Hou, J. Wang, and X. Wang. In the study of I. Krzysztofik and Z. Koruba, the quadcopter dynamics are analyzed during programmed movement and under external disturbance. The same authors study the nonlinear model of quadrotor dynamics during observation and laser target illumination. D. Li, C. Xu, M. Gola, and D. Botto consider the problem of reduced-order modeling of friction in case of the line contact in a turbine blade damper system. M. Galicki examines the finite-time control of omnidirectional mobile robots. L. Nesi, D. Antonelli, G. Pepe, and A. Carcaterra apply the feedback local optimality principle for rocket vertical landing. Time-delayed feedback control is applied by A. M. Tusset, J. M. Balthazar, R. T. Rocha, M. A. Ribeiro, W. B. Lenz, and F. C. Janzen for a non-ideal system with chaotic behavior. The distributed event-triggered output feedback control is used for semilinear time fractional diffusion systems by F. Ge and Y.-Q. Chen. Control performance assessment of the disturbance with fractional-order dynamics is carried out by K. Liu, Y.-Q. Chen, and P. Domański. The work of W. Tang, Y. Qi, and H. Gao addresses model correction-based multivariable predictive functional control for uncertain nonlinear systems.

J. Yuan, S. Fei, and Y.-Q. Chen investigate compensation strategies for actuator rate limit effect on first-order time-delay systems. The work of C.-E. Park, N. K. Kwon, and P.-G. Park examines the reliability of output feedback control for Markovian jump descriptor systems with sensor failure and actuator saturation.

L.-X. Yang and X.-J. Liu discuss the synchronization of coupled oscillatory networks with different node arrangements. The work of G. Panovko and A. Shokhin considers the synchronization of unbalance vibration exciters near resonance. J. P. Ramirez and J. Alvarez examine the mixed synchronization in unidirectionally coupled chaotic oscillators. Finally, in this volume, the effect of synchronized hopping induced by the interplay of coupling and noise is studied by M. Aravind, K. Murali, and S. Sinha.

We hope that readers will benefit from the collection of works here reported on the interplay between control and nonlinear dynamics and that these efforts will inspire new ideas in the future.

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Lanzhou, China — Jun Ma
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Part I
Vibration Absorbers and Isolators

Active Nonlinear Vibration Absorber for a Harmonically Excited Beam System

S. Mohanty and S. K. Dwivedy

Abstract An active nonlinear vibration absorber (ANVA) using displacement, velocity, and acceleration feedback from the absorber mass is proposed to reduce the vibration of an Euler–Bernoulli beam subjected to a harmonical point force. The ANVA comprises mass, linear spring, cubic nonlinear spring and actuator. The steady-state equation of the system is obtained by solving the governing differential equation by harmonic balance method. From the steady state equations, the stability and vibration reduction of the beam are investigated by frequency responses, time responses and phase portraits using Newton's method and fourth-order Runge–Kutta method. The analysis is carried out by studying the effects of different feedback control gains and cubic nonlinear stiffness of the absorber to suppress the vibration of the beam for the first three modal frequencies under different boundary conditions, namely fixed-fixed, simply supported and cantilevered type. The performance of the absorber is found to be better with cubic nonlinear stiffness in the absorber which reduces the vibration of the beam more effectively than the linear passive or active vibration absorber.

Keywords Vibration absorber · Harmonic balance method · Feedback

1 Introduction

The tuned mass damper (TMD) is an auxiliary mechanical device consisting of spring, mass and damper with optimal configuration which is attached to the vibrating primary structure to suppress its vibration by absorbing its vibrational energy [1]. The energy transformed from the host vibrating structure to the TMDs makes it to vibrate at higher amplitude which leads to nonlinear response in the auxiliary mass [2, 3]. The passive TMDs are not useful for wider range of

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frequency of operation with lower mass ratio. To avoid these problems, linear and nonlinear TMDs are designed with various optimization techniques, along with the use of active control devices to suppress the vibration for a broader range of operating frequencies [4, 5]. Hua et al. [6] designed a beam-based dynamic vibration absorber (DVA) to suppress vibration of a cantilevered beam. They showed that the proposed DVA outperforms traditional DVAs under same mass ratio. Chatterjee [7] considered linear analysis and proportional displacement and velocity feedback to suppress the vibration of a fixed-fixed beam excited by a point load. The present work is an extension of [7] where nonlinear secondary system is considered by taking different boundary conditions of the beam with different feedback conditions.

In the proposed model, the auxiliary system consists of a nonlinear spring, mass and an actuator, which is attached to the beam to suppress its vibration. The beam that is modelled as an Euler–Bernoulli beam is excited by an external harmonic point force. Three different feedbacks, namely proportional displacement or velocity or acceleration or combination of these, are used by the actuator to suppress the first three modes of vibrations of the beam for various boundary conditions. In the following section, the mathematical modelling of the system is described.

2 System Description and Mathematical Formulations

The schematic diagram of a fixed-fixed beam subjected to a harmonic excitation $F \cos \omega t$ at distance b from left end is shown in Fig. 1. The parameters ρ, A, L, E and I denote density, elastic modulus, moment of inertia, cross-sectional area and length of the beam, respectively. The mass, spring and cubic nonlinear stiffness of the auxiliary system are denoted by m, k_1 and k_{13}, respectively. The auxiliary system is attached at a distance a from the left end of the beam. The active control force F_c by the actuator is considered to be proportional to displacement or velocity or acceleration feedback or combination of these taken from the absorber mass. The transverse displacement of the beam about the neutral axis at any point x and time t is denoted by $W(x, t)$. The governing differential equation of motion of the system is obtained by considering equilibrium of the forces and moments, which are given below.

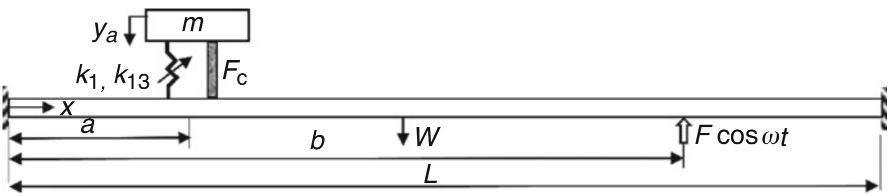

Fig. 1 Vibration control of a fixed-fixed beam subjected to harmonically excited point load by using an active nonlinear vibration absorber

$$\rho A\frac{\partial^2 W(x,t)}{\partial t^2}+EI\frac{\partial^4 W(x,t)}{\partial x^4}+W\,(x(a),t)\,(k_1\,(W\,(x(a),t)-y_{\mathrm{a}})) \\ +W\,(x(a),t)\left(k_{13}(W\,(x(a),t)-y_{\mathrm{a}})^3\right)=F\cos\omega t-F_{\mathrm{c}} \tag{1}$$

$$m\ddot{y}_{\mathrm{a}}+k_1\,(y_{\mathrm{a}}-W\,(a,t))+k_{13}(y_{\mathrm{a}}-W\,(a,t))^3=F_{\mathrm{c}} \tag{2}$$

$$F_{\mathrm{c}}=-\left(K_{\mathrm{p}}y_{\mathrm{a}}-K_{\mathrm{v}}\dot{y}_{\mathrm{a}}+K_{\mathrm{I}}\ddot{y}_{\mathrm{a}}\right) \tag{3}$$

where K_{p}, K_{v} and K_{I} are the control gains of the absorber mass, for displacement, velocity and acceleration feedback, respectively. The term y_{a} is the displacement of the absorber mass. Taking $W\,(x,t)=\sum_{i=1}^{3}\phi_i(x)q_i(t)$ in Eqs. (1) and (2), where $\phi_i(x)$ is the mode shape function and $q_i(t)$ is the time modulation of the ith mode of the beam vibration, the temporal equations of the system is obtained by using Galerkin's method. Here normalization criteria is taken as $\int_0^L\left(\phi_i{}^2(x)\right)dx=1$. Following non-dimensional parameters are used for further analysis. Modal displacement of the beam for ith mode: $w_i=q_i/\delta_{\mathrm{st}}$, displacement of the absorber: $y=y_a/\delta_{\mathrm{st}}$, mass ratio: $\mu=m/(\rho AL)\,,\omega_{\mathrm{a}}=\sqrt{k_1/m}/\omega_0,\omega_0=$ reference frequency, normalized natural frequency of ith mode the beam $=\omega_{ni}$, cubic nonlinear stiffness coefficient: $\alpha=k_{13}/k_1$, excitation force: $f=F/\left(\rho AL\omega_0^2\delta_{\mathrm{st}}\right)$, control gains: $k_{\mathrm{p}}=K_{\mathrm{p}}/\left(\rho AL\omega_0^2\right),k_{\mathrm{v}}=K_{\mathrm{v}}/\left(\rho AL\omega_0^2\right),k_{\mathrm{I}}=K_{\mathrm{I}}/\left(\rho AL\omega_0^2\right),\varOmega=\omega/\omega_0$ and non-dimensional time $\tau=\omega_0 t$. The obtained non-dimensional equations of motion are given below.

$$\ddot{w}_i\,(\tau)+\omega_{ni}^2w_i\,(\tau)+\mu\omega_{\mathrm{a}}^2\phi_i(a)\left\{\sum_{i=1}^{3}\phi_1(a)w_i\,(\tau)-y\,(\tau)\right\} \\ +\phi_i(a)\left\{k_{\mathrm{p}}y\,(\tau)-k_{\mathrm{v}}\dot{y}\,(\tau)\right\}+\alpha\mu\omega_{\mathrm{a}}^2\phi_i(a)\left\{\sum_{i=1}^{3}\phi_1(a)w_i\,(\tau)-y\,(\tau)\right\}^3 \\ +\phi_i(a)k_{\mathrm{I}}\ddot{y}\,(\tau)=\phi_i(b)f\cos\left(\varOmega\tau\right) \tag{4}$$

$$\mu\ddot{y}(t)+\mu\omega_{\mathrm{a}}^2\left\{y\,(\tau)-\sum_{i=1}^{3}\phi_i(a)w_i\,(\tau)\right\}+\alpha\mu\omega_{\mathrm{a}}^2\left\{y\,(\tau)-\sum_{i=1}^{3}\phi_i(a)w_i\,(\tau)\right\}^3 \\ -k_{\mathrm{p}}y(t)+k_{\mathrm{v}}\dot{y}\,(\tau)-k_{\mathrm{I}}\ddot{y}\,(\tau)=0 \tag{5}$$

It may be noted that Eqs. (4) and (5) are similar to Chatterjee [5], but here the cubic nonlinear stiffness in the absorber and different feedback forces are considered for various boundary conditions of the beam. Eqs. (4) and (5) are solved for the first three modal displacement of the beam, i.e. for $i=1$, 2 and 3 using harmonic balance method which is discussed in the following section.

2.1 Approximate Solution by Harmonic Balance Method

In this section, the harmonic balance method with slowly varying parameters are employed to Eqs. (4) and (5) to analyse the steady-state response of the system. The assumed solution are stated as follows:

$$w_n(\tau) = A_n(\tau)\cos(\Omega\tau + \varphi_n(\tau)) \text{ for } n = 1, 2, 3 \tag{6}$$

$$y(\tau) = B(\tau)\cos(\Omega\tau + \varphi_4(\tau)) \tag{7}$$

where $A_1(\tau), A_2(\tau), A_3(\tau), B(\tau),\ \phi_1(\tau),\ \phi_2(\tau),\ \phi_3(\tau)$ and $\phi_4(\tau)$ are slowly varying functions of time τ such that one can neglect the higher order or multiplication of derivatives. Substituting Eqs. (6) and (7) into Eqs. (4) and (5) and equating the coefficient of $\sin\Omega\tau$ and $\cos\Omega\tau$ terms separately to zero yield the following set algebraic equations in the matrix form.

$$\begin{bmatrix} a_{11} & \ldots & a_{18} \\ \vdots & \ddots & \vdots \\ a_{81} & \cdots & a_{88} \end{bmatrix} \left\{ \dot{A}_1\ \dot{\varphi}_1\ \dot{A}_2\ \dot{\varphi}_2\ \dot{A}_3\ \dot{\varphi}_3\ \dot{B}\ \dot{\varphi}_4 \right\}^{\mathrm{T}} = \left\{ b_1 \cdots b_8 \right\}^{\mathrm{T}} \tag{8}$$

From Eq. (8), the following amplitude and phase equations are obtained.

$$\dot{A}_n = f_n(A_1,\ A_2,\ A_3,\ B,\ \varphi_1,\ \varphi_2,\ \varphi_3,\ \varphi_4) \text{ for } n = 1, 2, 3 \tag{9}$$

$$\dot{\varphi}_n = f_n(A_1, A_2, A_3, B,\ \varphi_1,\ \varphi_2,\ \varphi_3,\ \varphi_4) \text{ for } n = 4, 5, 6 \tag{10}$$

$$\dot{B} = f_7(A_1, A_2, A_3, B,\ \varphi_1,\ \varphi_2,\ \varphi_3,\ \varphi_4) \tag{11}$$

$$\dot{\varphi}_4 = f_8(A_1, A_2, A_3, B,\ \varphi_1,\ \varphi_2,\ \varphi_3,\ \varphi_4) \tag{12}$$

The steady-state solutions of the equations are obtained by setting the first derivatives of the slowly varying amplitudes and phases equals to zero, i.e., $\dot{A}_1 = \dot{A}_2 = \dot{A}_3 = \dot{B} = \dot{\varphi}_1 = \dot{\varphi}_2 = \dot{\varphi}_3 = \dot{\varphi}_4 = 0$ in Eqs. (9–12), and the stability of the system is studied by obtaining the eigenvalues of the Jacobian matrix from Eq. (8). In the following section, the performance of the active nonlinear vibration absorber is discussed for various system parameters and control gains to suppress the vibration of the beam for the first three modal frequencies.

3 Results and Discussions

In this section, the performance of the active nonlinear vibration absorber on the beam is studied by the time responses, phase portraits and frequency responses of the system for the first three modal frequencies and also compared with the passive linear vibration absorber. The optimum absorber parameters and the control gains for displacement (k_p) and velocity feedback (k_v) are considered from Chatterjee [7]. The system parameters are considered as follows: mass ratio $\mu = 0.2$, amplitude of external excitation $f = 1$, absorber frequency $\omega_a = 2$ and the first three modal frequencies of the beam are $\omega_{n1} = 1$, 4 and 9. The parametric study is carried out by varying cubic nonlinear stiffness coefficient α and control gains k_p, k_v and k_I for various boundary conditions of the beam. The time responses and phase portraits of the system are obtained by solving Eqs. (4) and (5) using fourth order Runge–Kutta method, and the frequency responses of the system are obtained by solving the steady Eqs. (9–12) using Newton's method. The stability of a particular solution is ensured by the negative real part of the eigenvalues of the Jacobian matrix from the Eq. (8).

3.1 Time Responses Curves of the System

The time responses and phase portraits of the fixed-fixed beam and the absorber are shown in Fig. 2 for studying the effects of α and control gains in the vibration suppression of the beam. In Fig. 2a–d comparison of time responses at the first modal frequency ($\omega_{n1} = \Omega = 1$) of the beam for linear (red) and nonlinear (black) passive absorber are shown. From Fig. 2a one can observe that with cubic nonlinear stiffness, the response amplitude of the beam and the absorber decreases by 24% and 50%, respectively, than the linear absorber. The corresponding phase portraits of the system are shown in Fig. 2b, d where the beam and absorber show quasi periodic responses. In Fig. 3 responses of the fixed-fixed beam is studied for $\alpha = 0.1$ and 0.2 with different control gains. In Fig. 3a–d the black line shows the system response with only control gain k_v, and red lines shows with zero control gain for the first modal frequency ($\omega_{n1} = \Omega = 1$) of operation. It is observed from Fig. 3a, c that with $k_v = 0.3578$ the response of the beam and the absorber decreases by 52% and 28%, respectively, than the passive nonlinear absorber. In Fig. 3b, d phase portraits of the beam and the absorber show quasiperiodic responses. In Fig. 3e–f time responses are shown to control second modal frequency ($\omega_{n1} = \Omega = 4$) of the beam with $k_v = -1$, while all other system parameters are considered the same as in Fig. 3a–d. Here it is observed that the system amplitude reduces by the application of control gain in the feedback. In Fig. 3g, h comparison of time responses of the fixed-fixed beam system are shown, with acceleration feedback (continuous black line) and both displacement and velocity feedback (dotted black line) for $\alpha = 0.2$. From these figures, one can observe that with both k_p and k_v the system amplitude is more while

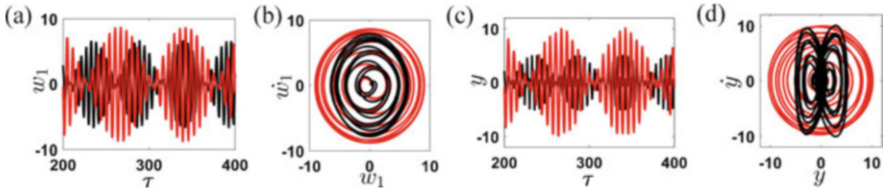

Fig. 2 Comparison of time responses (**a**, **c**) and phase portraits (**b**, **d**) of the passive fixed-fixed beam system at ($\omega_{n1} = \Omega = 1$) for linear ($\alpha = 0$) (red) and nonlinear absorber ($\alpha = 0.1$) (black)

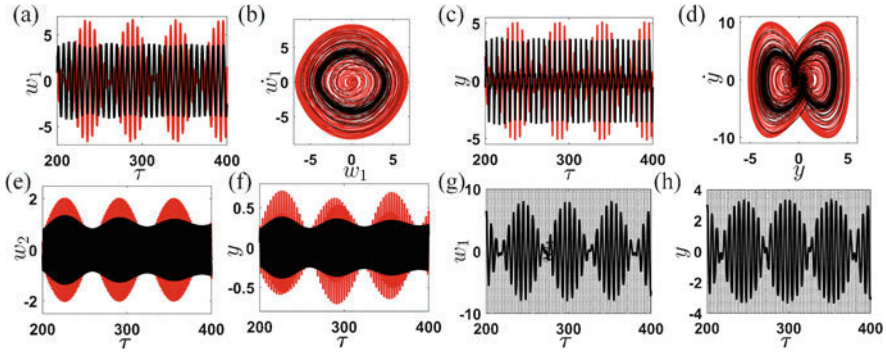

Fig. 3 Time responses and phase portraits of the system with fixed-fixed boundary conditions for different feedbacks and nonlinear stiffness in the absorber, where passive (red) and active (black). (**a**–**d**) $k_{\mathrm{v}} = 0.3578$ and $\alpha = 0.1$ at $\omega_{n1} = \Omega = 1$, (**e**, **f**) $k_{\mathrm{v}} = 1$ and $\alpha = 0.1$ at $\omega_{n1} = \Omega = 4$. (**g**, **h**) $k_{\mathrm{p}} = 0.6$, $k_{\mathrm{v}} = 0.3578$ (dotted lines) and $k_{\mathrm{I}} = 0.9$ (solid lines) for $\alpha = 0.2$ at $\omega_{n1} = \Omega = 1$

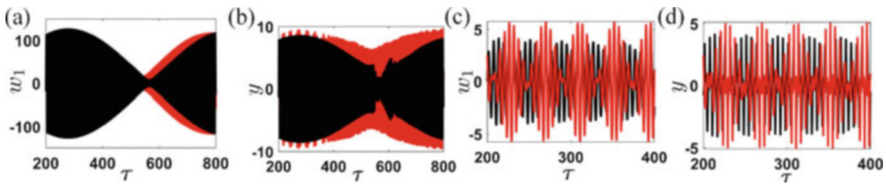

Fig. 4 Time responses of the (**a**, **b**) cantilevered beam and (**c**, **d**) simply supported beam at ($\omega_{n1} = \Omega = 1$ for $\alpha = 0.1$), where passive (red) and active ($k_{\mathrm{v}} = 0.3578$) (black)

with only k_{I} the system amplitude is lower. From Fig. 3 it is observed that for the nonlinear absorber, control gain with velocity feedback reduces the amplitude of the system, but when the nonlinearity in the absorber increases than acceleration feedback is more suitable to suppress vibrations of the fixed-fixed beam. In Fig. 4 the response of the system is studied for the simply supported beam (Fig. 4a, b) and the cantilevered beam (Fig. 4c, d) with same α and control gains as in Fig. 3a–d.

From Fig. 4a, b the same beating time responses are observed, but with higher amplitude, though with the control gain (black), the amplitude is lower. For the cantilever beam shown in Fig. 4c, d the vibration suppression is better than the simply supported beam with the same applied control gain.

3.2 Frequency Response Curves

In this section, parametric study is undertaken to analyses the frequency response of the system with different values of α, boundary conditions of the beam (fixed-fixed, simply supported, cantilevered) and control gains for displacement (k_p) and velocity feedback (k_v). In Fig. 5a, b, a comparison of frequency responses between the linear (black) and nonlinear active vibration absorber (blue) with k_p and k_v is shown to suppress the first modal frequency ($\omega_{n1} = \Omega = 1$) of vibration in the fixed-fixed beam. From these figures, it is observed that for $\alpha = 0.01$ the amplitude of the beam and the absorber decreases for the first three modal frequencies than the linear absorber. In Fig. 5c, d only velocity feedback is used to suppress the second modal vibration ($\omega_{n1} = \Omega = 4$) of the beam. It is observed from these figures that with only k_v the second mode of vibration of the beam is suppressed and also the same analogy can be interpreted for the absorber. It may be noted that Fig. 5a, c are similar to the results obtained by Chatterjee [7] where only linear analysis has been carried out. In Fig. 5e, f the same k_v and negative k_p are considered to suppress the vibration of the beam at the third mode ($\omega_{n1} = \Omega = 9$). It is observed from these figures that with the control gains both the third mode and the second mode of vibration of the beam decrease. In Fig. 5g for $\alpha = 0.05$ it is observed that the vibration of the beam further reduces in the first two modal frequencies than Fig. 5a. But for $\alpha = 0.1$ the beam vibration increases which is shown in Fig. 5h. In Fig. 5i, j the vibration suppression

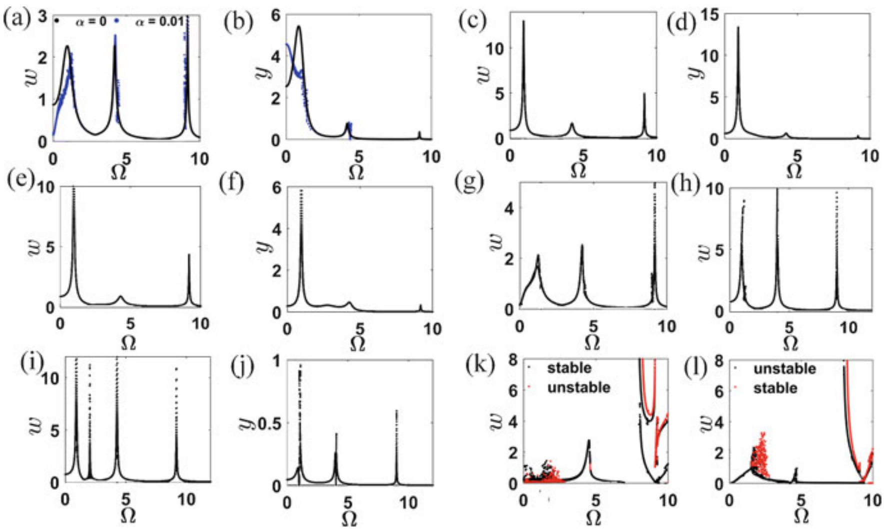

Fig. 5 Frequency responses of the fixed-fixed beam at the first three modal frequencies with $k_v = 0.3578$ and different k_p and α. (**a**, **b**) $k_p = 0.6$, at $\omega_{n1} = \Omega = 1$, (**c**, **d**) $k_p = 0$, $\alpha = 0$ at $\omega_{n1} = \Omega = 4$, (**e**, **f**) $k_p = -1$, $\alpha = 0.05$ at $\omega_{n1} = \Omega = 9$, (**g**) $k_p = 0.6$, $\alpha = 0.05$ at $\omega_{n1} = \Omega = 1$, (**h**) $k_p = -0.6$, $\alpha = 0.1$ at $\omega_{n1} = \Omega = 1$, (**i**, **j**) $k_p = -1$, $\alpha = 0.01$ at $\omega_{n1} = \Omega = 1$ and (**k**, **l**) $k_p = -1$, $\alpha = 1$ at $\omega_{n1} = \Omega = 1$

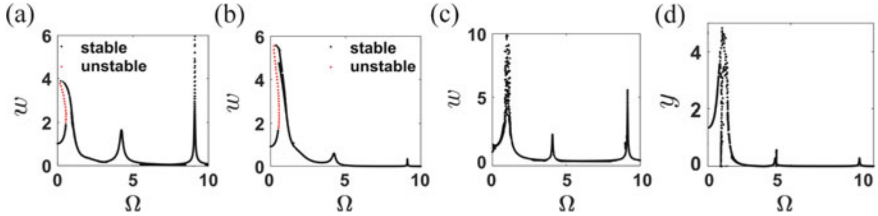

Fig. 6 Frequency responses of (**a**, **b**) the simply supported beam and (**c**, **d**) the cantilever beam for $k_p = 0.6$, $k_v = 0.3578$ and $\alpha = 0.01$ at $\omega_{n1} = \Omega = 1$

of the third mode is analysed for $\alpha = 0.01$ with $k_v$and negativek_p. It is observed that the vibration suppression of the beam in the third mode is not achieved with the applied control gains. In Fig. 5k, l, $\alpha = 1$ is considered to suppress the first mode of vibration ($\omega_{n1} = \Omega = 1$) of the beam. From Fig. 5k, l, high response amplitude and unstable regions are observed in the system. From Fig. 5 it is observed that for α in the range of 0 to 0.1 with the applied control gains, the response amplitude of the system reduces for the first three modal frequencies, but higher values of α, i.e. for α = 1, make the system unstable with high response amplitude. In Fig. 6a, d the simply supported beam and cantilever beam are considered with k_p, k_v and α, while all other parameters are same as in Fig. 3a–d. In simply supported beam (Fig. 6a, b), the unstable region is observed for $0.85 < \Omega < 0.95$ with jump-up and jump-down phenomena. For cantilever beam (Fig. 6c, d), the response amplitude is higher than in the fixed-fixed beam with the same control gains.

4 Conclusions

In the present chapter, it is observed that the proposed passive nonlinear vibration absorber reduces the vibration up to 24% than that of the corresponding linear vibration absorber. In the active linear and nonlinear vibration absorber, by suitably taking the optimized displacement and velocity gains, one may reduce the vibration of beams with fixed-fixed, simply supported and cantilevered beam for different modal frequencies. For higher nonlinear stiffness in the absorber, it is shown that the acceleration feedback is more useful than that of the displacement or velocity feedback. The results obtained by using the harmonic balance method is found to be in good agreement with those obtained by using Runge–Kutta method. Hence one may use the developed equations using harmonic balance method to study the passive and active vibration absorber to effectively suppress the vibration of the system with less computational time and memory space.

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