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Jun Ma · J. A. Tenreiro Machado
Gabor Stepan *Editors*

New Trends in Nonlinear Dynamics

Proceedings of the First International
Nonlinear Dynamics Conference
(NODYCON 2019), Volume III



Springer

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Preface

This volume is part of three volumes collecting the *Proceedings of the First International Nonlinear Dynamics Conference (NODYCON 2019)* held in Rome, February 17–20, 2019. NODYCON was launched to foster the tradition of the conference series originally established by Prof. Ali H. Nayfeh in 1986 at Virginia Polytechnic Institute and State University (Virginia Tech), Blacksburg, VA, USA, as the Nonlinear Vibrations, Stability, and Dynamics of Structures Conference. With the passing in 2017 of Prof. Nayfeh, who was also the founder of the Springer journal *Nonlinear Dynamics* in 1990, NODYCON 2019 was organized as a collective tribute of the community to Prof. Nayfeh for being one of the most influential leaders of nonlinear dynamics. NODYCON 2019 was also established to look to and dream about the future. The call for papers attracted contributions dealing with established nonlinear dynamics research topics as well as with the latest trends and developments. At the same time, to reflect the rich spectrum of topics covered by the journal *Nonlinear Dynamics*, the call included diverse and multidisciplinary topics, to mention a few, multi-scale dynamics, experimental dynamics, dynamics of structures/industrial machines/equipment/facilities, dynamics of adaptive, multifunctional, metamaterial structures, dynamics of composite/nanocomposite structures, reduced-order modeling, nonsmooth dynamics, fractional-order system dynamics, nonlinear interactions and parametric vibrations, computational techniques, nonlinear system identification, dynamics of NEMS/MEMS/nanomaterials, multibody dynamics, fluid/structure interaction, influence of nonlinearities on vibration control systems, human–machine interaction, nonlinear wave propagation in discrete and continuous media, chaotic map-based cryptography, ecosystem dynamics, social media dynamics, complexity in engineering, and network dynamics.

For NODYCON 2019, the organizers received 450 two-page abstracts and based on 467 reviews from the Program Committee, the Steering and Advisory Committees, and external reviewers, 391 papers and 17 posters were accepted, published in the Book of Abstracts (NODYS Publications, Rome, ISBN 978-88-944229-0-0), and presented by nearly 400 participants from 68 countries. The diverse topics covered by the papers were organized along four major themes to organize the technical sessions:

- (a) Concepts and methods in nonlinear dynamics
- (b) Nonlinear dynamics of mechanical and structural systems
- (c) Nonlinear dynamics and control
- (d) Recent trends in nonlinear dynamics

The authors of a selection of approximately 60 papers were invited to publish in the Special Issue of *Nonlinear Dynamics* entitled “NODYCON 2019 First International Nonlinear Dynamics Conference.” Over 200 full papers were submitted to the *Proceedings of the First International Nonlinear Dynamics Conference* (NODYCON 2019) and only 121 of them were accepted. These papers have been collected into three volumes, which are listed below together with a sub-topical organization.

Volume 1: Nonlinear Dynamics of Structures, Systems, and Devices

- (a) Methods for nonlinear dynamics
- (b) Bifurcations and nonsmooth systems
- (c) Nonlinear phenomena in mechanical systems and structures
- (d) Experimental dynamics, system identification and monitoring
- (e) Fluid–structure interaction, multibody system dynamics
- (f) Turning processes, rotating systems, and systems with time delays

Volume 2: Nonlinear Dynamics and Control

- (g) Vibration absorbers and isolators
- (h) Control of nonlinear systems
- (i) Sensors and actuators
- (j) Network synchronization

Volume 3: New Trends in Nonlinear Dynamics

- (k) Smart materials, metamaterials, composite and nanocomposite materials, and structures
- (l) MEMS/NEMS and energy harvesters
- (m) Nonlinear phenomena in bio- and ecosystem dynamics
- (n) Chaos in electronic systems
- (o) Fractional-order systems

I wish to acknowledge the work of the Co-Editors of the NODYCON 2019 Proceedings: Prof. Balakumar Balachandran (University of Maryland, College Park, MD, USA), Prof. Jun Ma (Lanzhou University of Technology, China), Prof. J. A. Tenreiro Machado (Instituto Superior de Engenharia do Porto, Portugal), Prof. Gabor Stepan (Budapest University of Technology and Economics, Hungary).

The success of NODYCON 2019 relied primarily on the efforts, talent, energy, and enthusiasm of researchers in the field of nonlinear dynamics who wrote and submitted these papers. Special praise is also deserved for the reviewers who invested significant time in reading, examining, and assessing multiple papers, thus ensuring a high standard of quality for this conference proceedings.

Rome, Italy
August 2019

Walter Lacarbonara

Preface for Volume 3: New Trends in Nonlinear Dynamics

Volume 3 of the NODYCON 2019 Proceedings includes 33 papers on new trends in nonlinear dynamics and is organized in five parts.

The first part includes 12 papers and explores the mechanical properties and dynamical response of smart materials, metamaterials, composite and nanocomposite materials. A. Elhady, M. Basha, and E. Abdel-Rahman present an electric permittivity sensor based on the Bleustein–Gulyaev waves. The modeling suggests that under a bias of only a few volts, the sensor can be driven into a nonlinear regime where its sensitivity can be tuned to match that of aqueous solutions, thus making it suitable for biomedical applications. M. Bukhari and O. Barry examine the role of stiffness nonlinearity on a periodic one-dimensional chain with multiple local resonators. The dispersion equation for the system is derived analytically by the method of multiple scales. The nonlinearity shows enhancement in the bandgap regions, especially with increasing number of local resonators. S. Zhu, J. Li, J. Zhou, and T. Quan investigate the nonlinear dynamic response of a simply supported concave hexagonal honeycomb sandwich plate with negative Poisson's ratio. The results provide theoretical guidance towards nonlinear vibration control for the metamaterial honeycomb sandwich structures. F. Mezzani, A. Rezaei, and A. Carcaterra provide a general method to deal with nonlinear integro-differential equations by using the statistical linearization and Fredholm's approach. The elastic metamaterial is characterized by long-range nonlocal interactions besides a nonlinear short-range constitutive relationship. The analytical results are obtained to unveil the onset of unconventional propagation. F. Coppo, F. Mezzani, S. Pensalfini, and A. Carcaterra present a numerical investigation of a metamaterial in the form of a one-dimensional elastic waveguide, equipped with nonlocal, nonlinear interactions. Numerical simulations, comparing the linear vs. nonlinear models, unveil wave-stopping and backward propagation phenomena. M. Taló, B. Carboni, G. Formica, G. Lanzara, M. Snyder, and W. Lacarbonara investigate the nonlinear dynamic response of carbon nanotube (CNT) nanocomposite cantilevers, both experimentally and theoretically. Nanocomposite cantilevers made of a thermoplastic polymer and high aspect ratio CNTs are subject to a primary resonance base excitation. The CNT/polymer frictional sliding hysteresis described by a

hysteretic restoring force in the context of a nonlinear beam theory is shown to be responsible for a peculiar softening-hardening frequency response trend. L. Leonetti, G. Formica, D. Magisano, M. Taló, G. Garcea, and W. Lacarbonara suggest an efficient continuation strategy based on the Riks method to describe the stable and unstable branches of the response of CNT/polymer nanocomposite shells. The equilibrium paths and the static bifurcations of CNT nanocomposites are investigated and numerically described highlighting the effects of material parameters such as the CNT orientation and weight fraction. K. Lidiya, S. Tetyana, and J. Awrejcewicz study the buckling behavior and parametric vibrations of sandwich plates with arbitrary forms and made of isotropic and functionally graded materials (FGM). To calculate the mechanical characteristics for different lamination schemes, the analytical expressions are obtained in the context of first-order shear deformation theory. F. Silva, P. Rodrigues, and P. Gonçalves explore the influence of the elastic foundation discontinuities on the nonlinear response of a functionally graded cylindrical shell with internal flowing fluid. Different discontinuities of elastic foundation and axial fluid flow velocities are considered, and the influence of these parameters on the stable branches of the resonance curves is confirmed. V. Burlayenko, T. Sadowski, and S. Dimitrova investigate double cantilever beam interlaminar fracture toughness sandwich specimens under different kinds of dynamic loading and loading rates. Cohesive finite elements are used to simulate the dynamic fracturing of the specimens. The influence of inertia on interfacial crack propagation in the specimens is evaluated as a direct outcome of the finite element analysis. A. Nabarrete, E. Araujo, J. Balthazar, and A. Tuset discuss the dynamic buckling of a sandwich plate to identify the response signals and to find the relevant frequencies and amplitudes. The nonlinear variation of the response for the out-of-plane displacements is analyzed with the continuous wavelet transform method for characterizing the observed behavior. J. Zhang, S. Chen, and L. Chen study torsional stress waves, buckling of functionally graded material cylindrical shells under torsional impact load by the symplectic method. The influence of the material gradient and the geometric parameters on buckling loads is analyzed and discussed.

The second part includes four papers dealing with model setting and estimation of micro-electromechanical systems and energy harvesters. S. Arakelian, I. Chestnov, A. Istratov, T. Khudaiberganov, and O. Butkovskiy study laser-induced nanocluster structures of different types. The problem of optical response and high temperature superconductivity, due to topological surface structures with correlated states, is considered in the frame of nonlinear dynamic modeling. The quantum mobility of electrons over different trajectories in the spatially inhomogeneous structures/nanocluster systems is presented in accordance with the path integral theory approach. K. Khorkov, D. Kochuev, R. Chkalov, V. Prokoshev, and S. Arakelian adopt a nonstationary technique for the laser-induced functional elements synthesis based on micro- and nanostructures in graphite samples. Carbon nanostructures such as graphene, nanopeaks, and crystals are obtained. The nonlinear formation mechanisms of nanostructures and microcrystals under femtosecond laser radiation for graphite in liquid nitrogen are analyzed. K. Chinnam, A. Casalotti, E. Bemporad,

and G. Lanzara estimate the influence of the electrical and mechanical coupling on the dynamic response of an electrospun piezoelectric microfiber. Particular focus is given to the piezoelectric response dependence on the applied voltage. The results highlight the possibility of making materials characterized by tunable stiffness and resonance frequency. R. Mohamed, A. El-Badawy, A. Moustafa, A. Kirolos, M. Soliman, and E. Abdel-Rahman obtain an analytical model of an electromagnetic levitation energy harvester and validate it by comparing its result with FEM simulations and experimental measurements. The model is based on dipole moment approximations of magnetic fields and interaction forces. The level of agreement of both models with measurements is discussed.

The third part of this volume includes nine papers and analyzes biological and ecological systems. P. Feng, R. Wang, and Y. Wu study the role of regular waves in cortical information processing in the mammalian neocortex. A variety of spatiotemporal patterns can be induced for selecting plane waves, spiral and irregular waves, and even chaotic spatial patterns by changing the coupling strength between neurons connected by chemical synapses. The results indicate that the stability of neural network depends on the coupling function and type. E. Kaslik and R. Muresan analyze the dynamical behavior of the interplay of homeostatic regulation and coupling time delay in a pair of reciprocally coupled Wilson–Cowan networks. The occurrence of rich dynamical behavior is explored both theoretically and numerically. X. Mao, X. Zhou, T. Shi, and L. Qiao evaluate the effects of the autaptic connection on the behaviors of ring-coupled FitzHugh–Nagumo neurons. Different transmission time delays between neurons and one autaptic time delay on the self-connection are considered. Several examples reveal phenomena such as stability switches, different multi-periodic oscillation patterns, chaotic motions, and coexisting attractors. E. Kaslik, M. Neamtu, and A. Radulescu present a nonlinear model of dopamine-modulated prefrontal–limbic interactions in schizophrenia, including discrete time delays. A stability and bifurcation analysis is carried out in the neighborhood of the system positive equilibrium. The results reveal the importance of time delays in modulating dopamine reactivity. E. Kaslik, E. Kokovics, and A. Radulescu generalize the Wilson–Cowan model of excitatory and inhibitory interactions in localized neuronal populations by taking into consideration distributed time delays. The stability region in the characteristic parameter plane is determined and a comparison is given for several types of delay kernels. Important differences are also highlighted by comparing the generalized model with the original Wilson–Cowan model without time delays. T. Trifonova, S. Arakelian, D. Trifonov, S. Abrakhin, V. Koneshov, A. Nikolaev, and M. Arakelian propose a scheme to explain and predict the process of a flood and/or mudflow (debris) formation and spreading out over the river beds in mountain conditions taking into account nonlinear dynamics. The approach can enable more reasonable forecast and early warning for the natural water hazard/disaster taking into account the groundwater nonlinear flow contribution as a dominant factor under some conditions to the land surface water. H. Zhao, J. Yu, J. Cao, and W. Liao address the complexity quantification of human gait and physiologic signals. The study calculates a refined weighted-permutation entropy by assigning fewer weights to outliers and more

weights to regular spiky patterns according to the normal distribution function. The human gait and ECG experimental data are analyzed by the proposed method. J. Simonović analyzes the bone cell communication dynamics using the bone cell population model by means of a system of coupled ordinary differential equations with power-law nonlinearities. The work explores several *in silico* experiments and provides more realistic approaches for interpreting the development of interventions for patients with bone trauma and diseases. S. George, R. Misra, and G. Ambika consider the nonlinear dynamics of RRc Lyrae variable stars, using intensity data from the Kepler space telescope. The results support the existence of two distinct subcategories of RRc Lyrae stars and indicate a link between the nonlinear and astrophysical properties of RRc Lyrae variable stars.

The fourth part includes three papers with focus on resonance phenomena, the emergence of chaos in circuits with finite variables, and the multistability property that can be induced in circuits with memristor. M. Bucolo, A. Buscarino, C. Famoso, L. Fortuna, and M. Frasca describe the existence of multi-jump resonance for the driven Chua circuit. Multi-jump resonance is investigated to characterize its onset as a function of system parameter values. A physical implementation of the driven Chua circuit in which jump and multi-jump resonances occur is presented and discussed. S. Seth studies robust chaos occurring in piecewise smooth dynamical systems. The first experimental observation of this phenomenon in a 3D electronic switching system is reported and the region of its parameter space is determined experimentally. C. Li selects a chaotic system with amplitude/frequency parameter that controls the scale and speed of oscillations without changing its basic feature of chaos. By introducing a memristor into the feedback for amplitude/frequency control, a special regime of homogenous multistability emerges, where the initial condition of the internal variable only determines the amplitude of the variables without changing the essential chaotic oscillation.

The fifth and last part includes five papers dealing with fractional-order and nonsmooth systems. Here, researchers analyze dynamical effects, such as stability, bifurcations, and complex vibrations. M. Shitikova and B. Ajarmah investigate nonlinear damped vibrations of a cylindrical shell embedded into a fractional derivative medium. They consider the case of the combinational internal resonance, resulting in modal interactions, using two different numerical methods. The damping properties of the surrounding medium are described by the fractional derivative Kelvin–Voigt model adopting the Riemann–Liouville fractional derivatives. O. Brandibur, E. Kaslik, D. Mozyrska, and M. Wyrwas discuss the stability of the Caputo-type linear fractional variable-order discrete-time equations known as biquadratic equations. Linear equations with constant coefficients and variable-order differences defined by functions with values from the interval $(0, 2]$ are considered, and some sufficient conditions for the asymptotic stability are presented. O. Brandibur, E. Kaslik, D. Mozyrska, and M. Wyrwas find the necessary and sufficient conditions for the asymptotic stability and instability of two-dimensional linear autonomous noncommensurate systems of fractional-order Caputo difference equations. The results are applied to the fractional-order version of the Rulkov neuronal model. K. Hedrih addresses the topic of independent fractional-type modes

of free oscillations and hybrid modes of forced vibrations. The paper describes the hybrid forced fractional-type vibration modes and the corresponding analytical approximate solution expressed by convolution integral. Y. Yu and Z. Wang propose two fractional-order Chua's memristive circuits. The first is a fractional-order memristive circuit with only the memristor described by a fractional-order derivative due to the memory loss which is observed experimentally. The second is a direct fractional-order generalization of integer-order Chua's memristive circuit without considering the physical background. Numerical simulations show that both models exhibit multistability and different steady states switch via grazing bifurcation.

We hope that the multifaceted state-of-the-art contributions collected in this volume will provide fruitful inspiration for new advances in the relevant challenging areas of research.

Rome, Italy
College Park, MD, USA
Lanzhou, China
Porto, Portugal
Budapest, Hungary
August 2019

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Part I
Smart Materials, Metamaterials,
Composite and Nanocomposite Materials,
and Structures

Tunable Bleustein–Gulyaev Permittivity Sensors



Alaa Elhady , Mohamed Basha, and Eihab M. Abdel-Rahman

Abstract We present a novel electric permittivity sensor based on Bleustein–Gulyaev (BG) waves. We also demonstrate a mechanism by which biasing can be used to modulate the sensitivity of permittivity sensors to match different electric permittivity ranges. We formulate the nonlinear electromechanical differential equations governing the dynamics of BG waves. Model results suggest that under a bias of only a few volts, the sensor can be driven into a nonlinear regime where its sensitivity can be tuned to match that of aqueous solutions, thereby allowing for biomedical applications.

Keywords Bleustein–Gulyaev waves · Permittivity sensors

1 Introduction

This work proposes a novel miniature permittivity sensor with a large dynamic range that can operate in the MHz range. Dielectric permittivity sensors are widely used in industrial applications, such as oil characterization [1], environmental applications such as soil testing [2] and sea water salinity testing [3], and biological applications, such as DNA and tissue discrimination [4, 5], cancer cell identification [6], and blood analysis [7, 8]. They are typically RF devices with operating frequencies set in excess of 100 GHz in order to limit the sensor size [5]. The use of high frequency signals introduces additional complexity and the attendant challenges of cost, reliability, and overall device size. Indeed, operating sensors at lower frequencies inflate their size to a few centimeters [9].

Bleustein–Gulyaev (BG) waves [10, 11] are coupled acoustic-electromagnetic waves that propagate exclusively along the surface of shear poled piezoelectric

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materials at acoustic speeds. These waves have been used in viscosity sensors [12, 13] and telecommunication filters [14]. We exploit the wave's acoustic speed to develop sensors that detect disturbances to the electromagnetic field at the shorter wavelength of BG waves, thereby reducing the device size and operating frequency by several orders of magnitude. Prototypes ranging in size from a few hundred micrometers to a few millimeters are under fabrication. Further, we present a novel technique to deploy the nonlinear properties of dielectrics to tune the sensors' sensitivity on-the-fly, thereby extending its dynamic range.

2 Model

BG waves couple an electromagnetic component, made of a transverse electric field and a magnetic field perpendicular to it, with a surface acoustic shear wave. It propagates along the surface of shear poled piezoelectrics.

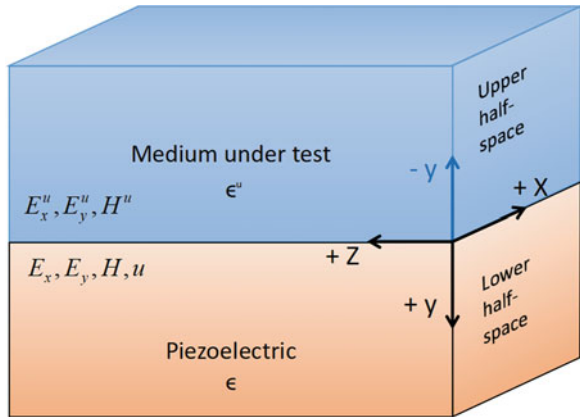
The wave constituent electric, magnetic, and displacement fields, Fig. 1, can be written as:

$$\mathbf{u}(x, y, t) = \begin{bmatrix} 0 \\ 0 \\ u \end{bmatrix}, \quad \mathbf{E}(x, y, t) = \begin{bmatrix} E_x \\ E_y \\ 0 \end{bmatrix}, \quad \mathbf{H}(x, y, t) = \begin{bmatrix} 0 \\ 0 \\ H_z \end{bmatrix}. \quad (1)$$

The wave is governed by the equation of motion for a piezoelectric element in the lower half-space

$$\nabla \cdot \boldsymbol{\sigma} = \rho \ddot{\mathbf{u}}; \quad y > 0 \quad (2)$$

Fig. 1 Propagation of BG waves



where ρ is the piezoelectric density. Further, Maxwell's equations in the lower and upper half-spaces [10] are

$$\nabla \times \mathbf{E} = -\mu \dot{\mathbf{H}} \quad y > 0, \quad \nabla \times \mathbf{E}^u = -\mu \dot{\mathbf{H}}^u \quad y < 0 \quad (3)$$

$$\nabla \times \mathbf{H} = \dot{\mathbf{D}} \quad y > 0, \quad \nabla \times \mathbf{H}^u = \dot{\mathbf{D}}^u \quad y < 0. \quad (4)$$

These equations are subject to a compatibility condition on the electromagnetic and mechanical fields:

$$\nabla \cdot \mathbf{D} = 0; \quad y > 0 \quad (5)$$

and boundary conditions at the interface plane ($y = 0$) between the two half-spaces, as well as reflector boundary conditions placed at both ends of the propagation direction (x -axis) to create a standing wave.

In the upper half-space, the relevant relations are only the linear Maxwell equations. In the lower half-space, we adopt a binomial expansion of the crystal's Gibbs free energy (PE) to capture the piezoelectric and electrostrictive material nonlinearities.

$$PE = \alpha_1 E^2 + \alpha_2 \sigma^2 + \alpha_3 \sigma E + \alpha_4 E^3 + \alpha_5 \sigma^3 + \alpha_6 \sigma^2 E + \alpha_7 E^2 \sigma + \alpha_8 \sigma^4 + \alpha_9 E^4 + \alpha_{10} E^3 \sigma + \alpha_{11} \sigma^3 E + \alpha_{12} \sigma^2 E^2 + H.O.T. \quad (6)$$

To represent the relative orders of the shear deformation and electric fields, we scale the electric field \mathbf{E} at $O(\hat{\epsilon})$ and the stress tensor $\boldsymbol{\sigma}$ at $O(\hat{\epsilon}^2)$, where $\hat{\epsilon}$ is a small bookkeeping parameter. Retaining terms up to order $O(\hat{\epsilon}^4)$, Gibbs free energy reduces to:

$$PE = \alpha_1 E^2 + \alpha_2 \sigma^2 + \alpha_3 \sigma E + \alpha_4 E^3 + \alpha_7 \sigma E^2 + \alpha_9 E^4 + O(\hat{\epsilon}^5). \quad (7)$$

The mechanical strain and electric displacement can then be derived as:

$$S = \frac{\partial PE}{\partial \sigma} = 2\alpha_2 \sigma + \alpha_3 E + \alpha_7 E^2 \quad (8)$$

$$D = \frac{\partial PE}{\partial E} = 3\alpha_1 E + \alpha_3 \sigma + 3\alpha_4 E^2 + 2\alpha_7 \sigma E + 4\alpha_9 E^3. \quad (9)$$

Using the field definitions in Eq. (1), the material definitions for the coefficients in Eqs. (8) and (9) for $y > 0$, and rearranging we obtain:

$$\sigma_{yz} = G \frac{\partial u}{\partial y} - \xi E_y - GME_x^2 + GME_y^2 \quad (10)$$

$$\sigma_{xz} = G \frac{\partial u}{\partial x} - \xi E_x - 2GME_x E_y \quad (11)$$

$$D_x = \epsilon E_x + \xi \frac{\partial u}{\partial x} + 2GM \frac{\partial u}{\partial y} E_x + 4GM \frac{\partial u}{\partial x} E_y - 6\xi M E_x E_y - 6GM^2 E_x E_y^2 - 2GM^2 E_x^3 \quad (12)$$

$$D_y = \epsilon E_y + \xi \frac{\partial u}{\partial y} - 4\xi M E_x^2 + 2\xi M E_y^2 - 2GM \frac{\partial u}{\partial y} E_y + 4GM \frac{\partial u}{\partial x} E_x - 6GM^2 E_x^2 E_y - 2GM^2 E_y^3, \quad (13)$$

where G is the shear modulus, ξ is the shear piezoelectric coefficient, M is the electrostrictive coefficient, and ϵ is the zero strain electric permittivity of the piezoelectric material.

3 Permittivity Sensor

Substituting with Eqs. (10)–(13) into Eqs. (2), (3), and (4), we obtain the governing system of equations. We set $M = 0$ to obtain the linear wave equations and solve the corresponding eigenvalue problem for the fundamental natural frequency:

$$\omega = \frac{m\pi}{L} \sqrt{\frac{G + \frac{\xi^2}{\epsilon}}{\rho} \left(1 - \frac{\xi^4}{(G\epsilon + \xi^2)^2} \frac{1}{(1+r)^2}\right)},$$

where m is the number of interdigitated transducer (IDT) fingers along the propagation direction, r is the ratio of the permittivity of the material-under-test ϵ'' to that of the piezoelectric material ϵ , and L is the length of the sensor. Since all of these parameters are known except for ϵ'' , the natural frequency can be used as a measure of the sample electric permittivity.

The sensitivity of the sensor can be expressed as the shift in natural frequency for a unit change in permittivity:

$$S_\omega = \frac{\partial \omega}{\partial \epsilon''}. \quad (14)$$

The black colored curve in Fig. 2 shows the sensitivity of 400 μm long permittivity sensor made of shear-poled Lead-Zirconium-Titanate Navy Type-I (PZT4) with $m = 10$ actuation fingers along the propagation direction as a function of the relative permittivity of the material-under-test ϵ''/ϵ_0 . Sensitivity reaches an optimal value for materials with relative permittivity similar to ~ 400 . On the other hand, aqueous media have a relative permittivity similar to ~ 80 . It is impractical to address this mismatch by changing the sensor material, rather we explore the possibility of addressing it by exploiting the electrostrictive properties of piezoelectrics.

For piezoelectrics where the electrostrictive constant is not negligible ($M \neq 0$), we can estimate the effective permittivity $\tilde{\epsilon}$ by linearizing Eq. (12) around a given

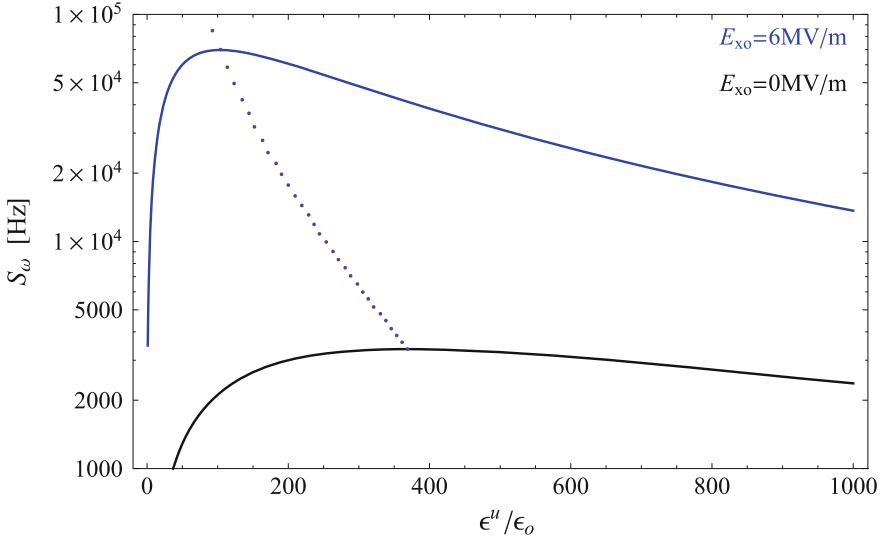


Fig. 2 Sensitivity of 400 μm long permittivity sensor made of PZT4 with $m = 10$ actuation fingers under no bias (black) and a DC electric field of $E_{x_0} = 6\text{ MV/m}$ and $E_{y_0} = 0$ (blue). The dotted line tracks the maximum position

DC electric field (E_{x_0}, E_{y_0}) and zero pre-strain ($S_0 = 0$), by taking the derivative of electric displacement D_x with respect E_x :

$$\tilde{\epsilon} = \left. \frac{\partial D_x}{\partial E_x} \right|_{(E_{x_0}, E_{y_0})} = \epsilon - 6\xi M E_{y_0} - 6GM^2(E_{y_0}^2 + E_{x_0}^2).$$

Therefore, it is possible to tune the permittivity of a piezoelectric material with non-negligible electrostriction by applying a DC electric field to it. The blue colored curve in Fig. 2 shows the sensitivity of the sensor described above as a function of the relative permittivity of the material-under-test where an electrostrictive constant of $M = 9.2 \times 10^{-18} \text{ m}^2/\text{V}^2$ was introduced under a biasing electric field of $E_{x_0} = 6\text{ MV/m}$ and $E_{y_0} = 0$. We note that the applied field has resulted in tuning the optimal sensitivity to a similar range of that for aqueous media and increased the sensor sensitivity, thereby opening the door to potential biomedical applications.

4 Nonlinear System Analysis

The right-hand side of Eqs. (3) is of order $O(\hat{\epsilon}^4)$ and, therefore, negligible. As a result, we adopt a quasi-static representation of the magnetic component of the field:

$$H_z(x, y, t) = H_0(x, y) \quad (15)$$

and reduce Eq. (3) to:

$$\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} = 0.$$

The electric field can, thus, be expressed purely in terms of the scalar potential $\psi(x, y)$ representing the voltage:

$$E_x = \frac{\partial \psi}{\partial x}, \quad E_y = \frac{\partial \psi}{\partial y}. \quad (16)$$

Using Eqs. (10), (11), (15), and (16) to substitute for the field variables in Eq. (2), we obtain the governing equation of motion as:

$$\ddot{u} + \beta_1 \nabla^2 u + \beta_2 \nabla^2 \psi + \beta_3 \psi_{,y} (\psi_{,xx} - \psi_{,yy}) - 2\beta_3 \psi_{,x} \psi_{,xy} = 0, \quad (17)$$

where the i subscript indicates partial derivative with respect to the i coordinate. The corresponding magnetic field is found from Eq. (4) as the time derivative of the electric displacement vector \mathbf{D} :

$$H_{o,x} = \frac{\partial}{\partial t} \left(\beta_4 u_{,y} + \beta_5 \psi_{,y} + \beta_6 u_{,x} \psi_{,x} + \beta_7 u_{,y} \psi_{,y} + \beta_8 \psi_{,x}^2 \psi_{,y} \right. \\ \left. + \beta_9 \psi_{,x}^2 + \beta_{10} \psi_{,y}^2 + \beta_8 \psi_{,y}^3 \right) \quad (18)$$

$$H_{o,y} = -\frac{\partial}{\partial t} \left(\beta_4 u_{,x} + \beta_5 \psi_{,x} + \beta_6 u_{,x} \psi_{,y} - \beta_7 u_{,y} \psi_{,x} + \beta_8 \psi_{,x} \psi_{,y}^2 \right. \\ \left. + \beta_{11} \psi_{,y} \psi_{,x} + \beta_8 \psi_{,x}^3 \right). \quad (19)$$

Differentiating equation (18) with respect to y and Eq. (19) with respect to x and subtracting the result, we eliminate the magnetic field to obtain:

$$\frac{\partial}{\partial t} (\nabla \cdot \mathbf{D}) = \frac{\partial}{\partial t} \left(\beta_4 \nabla^2 u + \beta_5 \nabla^2 \psi + 2\beta_6 u_{,x} \psi_{,xy} + \beta_7 u_{,y} (\psi_{,yy} - \psi_{,xx}) \right. \\ \left. + (\beta_6 - \beta_7) u_{,xy} \psi_{,x} + (\beta_6 u_{,xx} + \beta_7 u_{,yy}) \psi_{,y} + \beta_{11} \psi_{,xx} \psi_{,y} \right. \\ \left. + 2\beta_{10} \psi_{,y} \psi_{,yy} + (2\beta_9 + \beta_{11}) \psi_{,xy} \psi_{,x} + \beta_8 (3\psi_{,xx} + \psi_{,yy}) \psi_{,x}^2 \right. \\ \left. + \beta_8 (\psi_{,xx} + 3\psi_{,yy}) \psi_{,y}^2 + 4\beta_8 \psi_{,y} \psi_{,xy} \psi_{,x} \right) = 0. \quad (20)$$

We rewrite the displacement and electric fields as the summation of static and dynamic components:

$$u(x, y, t) = u^s(x, y) + u^d(x, y, t) \quad (21)$$

$$\psi(x, y, t) = \psi^s(x, y) + \psi^d(x, y, t). \quad (22)$$

The static displacement field $u^s(x, y)$ can be found by setting the time derivative in Eq. (17) equal to zero and solving the equilibrium equation:

$$\beta_1 \nabla^2 u^s + \beta_2 \nabla^2 \psi^s + \beta_3 \psi_{,y}^s (\psi_{,xx}^s - \psi_{,yy}^s) - 2\beta_3 \psi_{,x}^s \psi_{,xy}^s = 0 \quad (23)$$

subject to $\psi^s(x, y)$ the spatially distributed potential field imposed by the DC voltage applied to the metallic electrodes patterned along the sensor surface. As a first approximation, it is represented by an empirically fitted function in the propagation x -direction and an exponentially decaying function in the y -direction:

$$\begin{aligned} \psi^s(x, y) = e^{-\kappa_v^2 y} & \left(1.115 \cos\left(\frac{\pi mx}{L}\right) - 0.103 \cos\left(\frac{3\pi mx}{L}\right) \right. \\ & \left. - 0.019 \cos\left(\frac{5\pi mx}{L}\right) + 0.005 \cos\left(\frac{7\pi mx}{L}\right) \right). \end{aligned}$$

We substitute with Eqs. (21) and (22) in Eqs. (17) and (20) and use Eq. (23) in the result to obtain the equations of motion around the static equilibrium (u^s, ψ^s). In order to solve this dynamic system, we use a Galerkin expansion to express the system variables in terms of its mode shapes $\phi_j(x)$ as:

$$u^d(x, y, t) = \sum_{j=1}^N q_j(t) \kappa_{u_j} e^{-\kappa_{u_j}^2 y} \phi_j(x) \quad (24)$$

$$\psi^d(x, y, t) = \sum_{j=1}^N p_j(t) \kappa_{\psi_j} e^{-\kappa_{\psi_j}^2 y} \phi_j(x) + \frac{V(t)}{2} e^{-\kappa_v^2 y} \phi_1(x) \quad (25)$$

where $V(t)$ is the AC component of the applied voltage. We found the mode shapes by solving the eigenvalue problem of the system (Eqs. (17) and (20)) as:

$$\phi_j(x) = e^{ijm\pi \frac{x}{L}} + e^{-ijm\pi \frac{x}{L}}, \quad (26)$$

where j is an integer mode number.

In this work, the number of modes was taken to be $N = 2$. Multiplying the resulting Galerkin residuals with the corresponding left eigenfunctions and setting the integral over the domain equal to zero, we obtained a reduced-order model of the system in terms of the modal amplitudes as follows:

$$\ddot{q}_1 + \gamma_4 q_1 + \gamma_1 p_1 + \gamma_2 p_2 + \gamma_3 p_1 p_2 + \gamma_6 p_2 V = \gamma_5 V \quad (27)$$

$$\ddot{q}_2 + \gamma_{10} q_2 + \gamma_7 p_1 + \gamma_9 p_2 + \gamma_8 p_1^2 + \gamma_{12} p_1 V = \gamma_{11} V + \gamma_{13} V^2 \quad (28)$$

$$\begin{aligned} & \gamma_{14} \dot{p}_1 + \gamma_{17} \dot{p}_2 + \gamma_{21} \dot{q}_1 + \gamma_{23} \dot{q}_2 + 2\gamma_{15} \dot{p}_1 p_1 + 3\gamma_{16} \dot{p}_1 p_1^2 + \gamma_{18} \dot{p}_2 p_1 \\ & + \gamma_{18} \dot{p}_1 p_2 + 2\gamma_{19} \dot{p}_2 p_2 + \gamma_{20} \dot{p}_1 p_2^2 + 2\gamma_{20} \dot{p}_2 p_1 p_2 + \gamma_{22} p_2 \dot{q}_1 + \gamma_{22} \dot{p}_2 q_1 \end{aligned}$$