Marianne Nolte (Editor)

Including the Highly Gifted and Creative Students – Current Ideas and Future Directions

Proceedings of the 11th International Conference on Mathematical Creativity and Giftedness (MCG 11)

22.08.2019 – 24.08.2019

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INTRODUCTION

The 11th International Conference on Mathematical Creativity and Giftedness (MCG11) took place at the Universität Hamburg, Germany, during August 22th – 24th 2019.

In 1999, Prof. Dr. Hartwig Meissner invited to a conference on mathematical creativity and giftedness at the University of Münster in Germany. This conference became the starting point for an international group of researchers, teachers and practitioners whose shared concerns are on the one hand the development of mathematical creativity and giftedness and on the other hand the exploration of mathematical creativity and giftedness. In the meantime, a series of conference was held in different parts of the world and MCG, the international group for mathematical creativity and giftedness was founded (see www.igmcg.org).

The MCG conference aims at promoting mathematical creativity and giftedness in students of all ages and backgrounds, and supporting interested mathematics educators, mathematicians, psychologists, researchers, teachers and other in this topic interested people.

Nowadays, international exchange of research about creativity and giftedness, of concepts how to foster the development of creativity and high competences in mathematics is highly important. School systems and their development differ from country to country. Thus, we can learn from each other's approaches. We all have in common a rapid change of living conditions. The technical development is progressing rapidly while societies are changing continuously. Today we do not know which will be problems to be solved in the future. We teach our students about subjects, which are important now. Preparing them for an unknown future implies thinking about the cognitive competences but, also about personal traits and openness for changes.

From a societal perspective, we must provide opportunities and environments for individuals can develop adequately. This implies encouragement for creativity. The research about giftedness underlines the impact of environmental factors and the personal interplay with these conditions. We talk about promising children to point out that all students need an education to develop their capability and personality to the fullest. Nevertheless, the conditions of realization of this aim are settled differently in many countries.

We hope that international collaboration and discussion will contribute to preparing our prospective active and responsible members of society to cope well with their responsible tasks in their future professional life.

For further information about the activities of the International Group for Mathematical Creativity and Giftedness (MCG), please visit our website
http://www.igmcg.org/home.

Marianne Nolte, Universität Hamburg, Germany, July 2019

MAIN LECTURES

RESEARCH ON MATHEMATICAL GIFTEDNESS IN GERMANY – LOOKING BACK AND AHEAD

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Abstract. *Beginning with William Stern, giftedness research has a long tradition in Germany, whereby in the times of German Partition the developments in East and West were quite different. In addition to psychological studies, didactic research on mathematical giftedness has also been on the rise for about 40 years. The lecture presents important developments and results, especially of mathematics education research in Germany, in their interdisciplinary and pedagogical-practical references. On this basis, further research questions for possible future projects will be put up for discussion.*

Key words: mathematical giftedness, developing expertise.

WILLIAM STERN

William Stern (1871-1938) worked as Director of Psychological Laboratory (1916-1919) in Hamburg and his work gave an essential impulse for founding the University of Hamburg one hundred years ago (https://www.uni-hamburg.de/en/uhh/profil/geschichte.html). At the University of Hamburg, he worked until 1933. Due to the Nazi-regime, he left Germany and continued his work as professor at the Duke University, USA. He paid particular intention to questions about prerequisites for professions and related to that, he was very interested in the diagnostic of abilities. In 1912, he proposed a new way of interpreting the results of intelligence tests with calculation the IQ. Up to now, this is one of the most popular concepts in psychology.

Stern was also very interested in giftedness or talents and many of his considerations are still very important. To refer to only a few of them, we mention the conceptual distinction between intelligence and talent. Stern underlined the existence of *general intelligence* and *giftedness in special subjects* (talents). From his point of view, the specificity of talent arises from a directionality of the entire personality towards the corresponding area. Thus, talents include not only special abilities, but also other personality traits such as interest and perseverance.

For Stern, giftedness does not necessarily have to be cognitive in nature. In addition, he formulated already in 1916 that giftedness not in any case leads to achievement. In that regard, he emphasized the importance of environmental factors for the development of abilities taking into account the interplay between genetic aspects, environmental factors and personality.

> "Giftedness as such provides no more than an opportunity to perform, it is an inevitable precondition, but it does not imply performance itself" (Stern, 1916, p. 110, translated from German original quote by the authors).

In the first half of the 20th century, Stern was already able to lay the foundations for a modern concept of talent or giftedness with his considerations. What has happened in Germany since then – especially with regard to the domain of mathematics?

In order to be able to contextualize selected research efforts, we will first take a look at the school systems in Germany. On this basis, essential research approaches and results will then be presented as important elements of a broad spectrum.

SCHOOL SYSTEMS AND PERSPECTIVES ON GIFTEDNESS IN GERMANY

Developments in the Federal Republic of Germany (western part of Germany)

The school system in the former FRG separated, and still does so, students after 4 or 6 years of schooling (10 or 12 years olds) depending on their academic achievements. Thus, there were different schools and differences in curricula for low, average and high performing students beside school for students with special needs. In parallel, some schools offered curricula for all levels of performance – here, we do not go into detail. To separate students and not to overlook hidden talents one of the first big studies to identify potentials of students started in 1965 (Heller, 2014). The aim was to match individual profiles of giftedness with the appropriate school or possibilities of fostering.

However, until the 1980s there was a widespread rejection of research projects on giftedness in cognitive domains or even fostering efforts. Based on the experience of the the Third Reich, many argued that the formation of elites in the German society should be avoided. Instead, the focus was on overcoming social inequalities by supporting students with low performance. This was motivated by the conviction that under appropriate conditions every child can learn everything (Bloom, 1976, as cited in Weinert, 2000, p.7).

Research about giftedness increased with the beginning of the 1980s. The impulses of congresses like the congress about gifted children headed by the psychologist *Wieczerkowski* in 1980 ("The highly gifted child: medical, psychological and pedagogical perspectives"; cf. Wieczerkowski & Wagner, 1981) and the *Sixth World Conference on Gifted and Talented Children* in Hamburg resulted in establishing a group of interested and engaged researchers at the University of Hamburg, consisting of math educators, mathematicians and psychologists. Together they developed concepts for identification and fostering mathematically gifted students at secondary school level – exchanging their ideas with a working group at Johns-Hopkins-University in Baltimore (USA). Headed by *Kießwetter* a still running program ("Hamburger Model") was established. Later in 1985, the William-Stern-Society Hamburg was founded; a group that offers fostering programs for mathematically gifted students, counseling of parents and that supports research on giftedness. These activities made an important contribution to the gradually change of the social attitudes against giftedness in western Germany.

Developments in the German Democratic Republic (eastern part of Germany)

In the GDR, there was a uniform school system, i.e. a school for all, in which all students learned with the same textbooks and according to the same curricula. The aim was a high level of polytechnic general education and the development of the "socialistic personality".

In spite of the uniform school, there were no reservations against the promotion of special talents. On the contrary, the promotion of giftedness was seen as a social obligation (e.g. Dassow, 1983). From the beginning of the 1960s at the latest, this was especially true for mathematics,[1] which was regarded as fundamental to the entire field of natural science and technology because of its character as a structural science. Since the possibilities for promoting giftedness in the teaching of the uniform school were limited, a wide variety of extra-curricular activities was developed. This included systematically organised regional and supra-regional math clubs, correspondence circles, two math magazines for students ("alpha", "Die Wurzel")[2], an extensive book series for students in which nearly 150 volumes were published until 1990, and above all the Mathematics Olympiad as a state-wide multi-stage competition. This enjoyed very high media attention; especially in the first two stages, the participation rate was extremely high. However, this also led to the fact that the entire area of extra-curricular promotion of mathematics was very competition-oriented. In addition, in the GDR there were up to 14 special schools of mathematics, science and technology with their own curricula and final examinations.

Developments in reunited Germany

After reunification, the West German school system was initially adopted for the entire country. However, it was also possible to maintain a large number of special schools for mathematics and science in the eastern part of the reunified Germany.
In many regions, there have been more and more community or comprehensive schools for several years, in which almost all children learn together after the 4th grade. In addition, the special needs school system has been reduced for many areas, so that the heterogeneity of learning groups and thus the promotion of special talents (albeit in a reduced manner) has increasingly come to the fore in research and practice.

Fostering projects for mathematically gifted students were continued or newly developed at various university locations. The Mathematics Olympiad was established as a major nationwide competition. A large number of dissertations on mathematical giftedness were written in several university working groups. Currently, there is a nationwide project "Leistung macht Schule" (LEMAS), which is strongly funded by the state and aims to promote high-performing and potentially high-performing students in various school subjects, including mathematics. The project will support school development and networking in particular, and aims in strengthening teacher abilities in diagnosing and promoting giftedness amongst others in the STEM fields.

GERMAN RESEARCH IN THE DOMAIN OF MATHEMATICAL GIFTEDNESS

Due to reasons of space the following description of important research approaches and results offers (only) a selection of mathematical didactic work or psychological work with very strong mathematical didactic references, not taking into account research about more sociological aspects of giftedness like gender specific (e.g. Benölken, 2013) or classroom related research (Nolte & Pamperien, 2017).

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[1] In 1962, the GDR government passed a resolution for systematic development of extra-curricular offers for students in the field of mathematics ("mathematics resolution", in German: "Mathematikbeschluss").

[2] Cf. https://mathematikalpha.de/alpha and https://www.wurzel.org.

Nature of Mathematical Giftedness

In all models of mathematical giftedness we know, the core element is seen in a high level of the cognitive domain. Nevertheless, they have in common to take into account influencing factors like intrapersonal and environmental aspects and their interconnectedness. Thus, the developmental process is essential for shaping the potential. Furthermore, used characteristics of the cognitive domain can be related to different levels, which are illustrated in the following figure. While the level of thought and action patterns is purely descriptive, the levels of abilities and the underlying elementary mental processes and structures are explanatory.

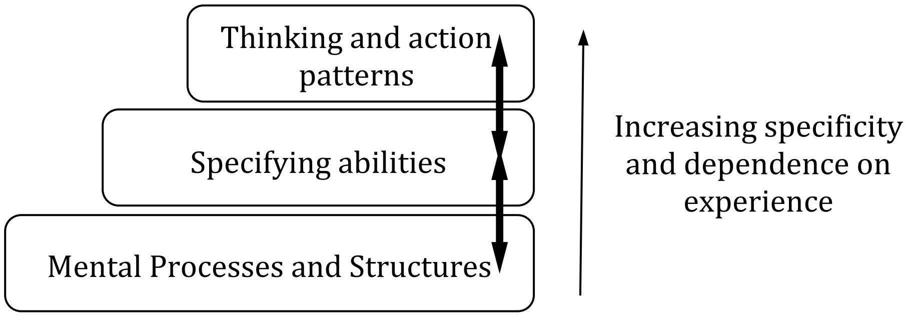

Fig. 1: Description levels for the cognitive core of mathematical giftedness (cf. Fritzlar, 2013)

We assume that specific or specified abilities develop over time based on more general mental processes and structures through mathematical activities. On appropriate occasions, these can then manifest themselves in characteristic patterns of thought and action.

In the GDR, psychological studies of mathematical activities and abilities were conducted relatively early (e.g. Gullasch, 1971). Building on this, the Academy of Pedagogical Sciences developed several qualification (PhD-) projects on mathematical giftedness and its promotion – later also from a more pedagogical and didactic perspective.

The dissertation by Dassow (1983) is an early empirical work on mathematical giftedness in primary school. He investigated a method for early identification of mathematically gifted students at the age of 9 to 10 years. For this purpose, he developed a theory-based model of the structure of mathematical giftedness and empirically examined it in a group of 60 students. The model includes the following abilities:

- Ability for mathematical abstraction, for rapid and comprehensive generalization of mathematical objects, relationships and operations
- Ability to think elastically
- Ability to curtail the mathematical thinking process
- Ability for logical and systematic sequences of thoughts
- Ability to quickly and easily change to reverse thinking

The first one is of particular importance; but overall, these abilities can only be found in elementary form in such young students.

Additionally, the model includes personality traits in the sense of action dispositions, which are regarded as necessary components of mathematical giftedness:

- Strong interest in mathematical activities
- High mental activity
- Strong self-confidence
- High degree of independence in mathematical activities
- Low fatigue in mathematical activities

The above listed abilities are partly based on elementary mental processes (lower level in Fig. 1). For example, switching to reverse thinking is understood as bidirectionally use of network connection, mental bonds sensu Krutetskii (1976). Therefore, Dassow was unable to develop test items that mainly test the extent of this ability (due to the occurance of these connections in almost all tasks); thus, it remains the only component of the developed structural model of mathematical giftedness that could not be empirically supported in the study. In addition, the theoretically derived special importance of interest in mathematical activities could not be confirmed.

At that time, there was no comparable scientific interest in the field of mathematical giftedness in the FRG. Only Kießwetter attempted to characterize activities of mathematically gifted secondary school students. While Dassow approached mathematical giftedness from a rather psychological point of view, Kießwetter followed a mathematical didactic perspective.

According to Kießwetter, theory-building processes are characteristic for the research work of mathematicians (Kießwetter, 2006, Fritzlar, 2008). In analogy, his approach for fostering gifted students is based on offering opportunities for age appropriate similar processes. At primary grade level, problems fields open an access for typical thinking processes like argumentation and generalization, at higher secondary level, the students even develop small mathematical theories.

Based on indicated characteristics of doing mathematics and the results of Krutetskii, at the beginning of the 1980s Kießwetter constructed a “catalog of categories of mathematical thinking” or “patterns of action”, which are useful in working on mathematical problems. Using them in mathematically rich situations gives hints for a high mathematical potential. On that basis, Kießwetter developed the “Hamburg Mathematical Talent Test” (HTMB) whose problems can be solved by using these patterns: organizing material; recognizing patterns or rules; recognizing problems, finding (constructing) related problems; changing the representation of a problem and recognizing patterns and rules in this new area; comprehending very complex structures and working within these structures; reversing processes (cf. Wagner & Zimmermann, 1986). However, this catalogue is explicitly not intended to provide a comprehensive description mathematical giftedness (e.g. Kießwetter, 1985). It seems important, that it includes relatively complex patterns of mathematical operations, which, even more than Krutetskii’s catalog of mathematical abilities, are formed by specific experiences and solidify and expand with them (see Fritzlar, 2010).

In Germany, the description of mathematical talents in primary school age developed by *Käpnick* had a great impact on the scientific discussion. Käpnick worked out theoretical and empirical mathematics-specific features of a potential mathematical talent, which he combined with general behavioral personality traits (Käpnick, 1998). This description was later incorporated into a talent and performance development model that is strongly oriented on the “Differentiated Model of Giftedness and Talent” by Gagné (2004).

Selected Characteristics of Mathematical Giftedness

If one examines more closely, what is meant with characteristics used for describing mathematical giftedness in the literature and how the authors try to capture their manifestations, then different accentuations become clear in some cases. In recent years, there have been increased efforts in Germany to elaborate such characteristics and possible relationships between them in more detail.

An outstanding example for these efforts is the dissertation of Aßmus (2017), which examines the cognitive characteristics of mathematical giftedness not only empirically for second grade students (8 years olds), but also profoundly theoretically.

One important result among others is the detailed theoretical analysis of the component “reversing thought processes”. On this basis, she succeeds, unlike Dassow, in proving that the ability in reverse thinking is better developed in mathematically gifted second graders than in students of the same age who are not mathematically gifted. The advance is based above all on the ability to recognize reverse questions and situations. In contrast, the correct revising of a longer train of thought is a great challenge for all students of this age group.

For some characteristics of mathematical giftedness, a psychological perspective can be very fruitful. *Klix* describes four “basic components” of thinking which decisively determine the cognitive capacity of an individual: analogy, complexity reduction, multiple classification and multimodality or double representation. In this context, multimodality is understood as simultaneous use of several modality-specific representations and double representation is understood as simultaneous use of visual and symbolic representations in coping with cognitive demands (Klix, 1992, 1993).
The psychologists Krause, Seidel et al. (1999) investigated the double representation hypothesis for mathematical problems. As a result, they found that mathematically gifted students show better performance. They were able to solve more problems than comparison groups and solve them faster. The better performance, however, could not be explained by the traditional measures of experimental psychology; in terms of IQ, visualization or memory capacity; corresponding differences between the two groups of subjects were not significant. By means of EEG analyses, however, it could be shown that in mathematically gifted students, already within fractions of a second of understanding the instruction, those brain regions are activated which are responsible for the conceptual and pictorial-vivid modality. In the comparison group, however, such a double activation was not detectable. (Fritzlar & Heinrich, 2010).

These neuroscientific findings at the lower level in Fig. 1 support the indicators for mathematical giftedness at higher levels formulated by Kießwetter or Käpnick. Psychological investigations are also available, for example, for analogizing by mathematically gifted students (Foth & van der Meer, 2013). Detailed mathematics didactic studies in this area are currently carried out e. g. by Aßmus and Förster (2013).

Relations between giftedness and other theoretical constructs

Giftedness and IQ

It is often argued that the validity of the IQ for subject-specific requirements – for example problem solving in mathematics – is controversial and limited. Among other things, Nolte investigated this problem as part of a research and fostering project for elementary school children at the University of Hamburg, which celebrates its 20th anniversary this year.

Since there must be a selection in the city of Hamburg among the third-graders interested in participating in the fostering project, the children are initially invited to trial lessons – the “math club for math fans” (Nolte, 2004). Afterwards, the students work on a specially developed mathematics test and an intelligence test, which correlates particularly strongly with performance in mathematics. Important results from the study of more than 1660 students from nine years, of which complete data are available, can be summarized as follows (see Nolte, 2013a): The results of both tests correlated with -0.34. However, this correlation was significantly weaker for children who achieved particularly good results in the math test; for example, the correlation coefficient for positions 1-15 dropped to -0.14.

The not very strong statistical relationship and its further reduction are partly due to the (increasing) selectivity and the (decreasing) sample size. However, the results show that a special mathematical giftedness cannot be deduced from the IQ (Nolte, 2013). The doubts expressed by mathematical didactics on a simple connection between IQ and giftedness (e.g. Bardy, 2007, Bauersfeld, 2003, Käpnick, 1998) are thus further supported.

Mathematical giftedness and developing mathematical expertise

Originally, ‘giftedness’ and ‘expertise’ were understood as two fundamentally different constructs that have their roots in different research traditions. While ‘giftedness’ refers to an area-specific potential of high performance, ‘expertise’ is characterised by continuously outstanding performance levels in a certain domain. However, nowadays it is undisputed that giftedness does not ‘evolve’ on its own. Rather, it is based on long term learning processes and practical experience which themselves are reliant on support and stimulating encouragement functioning as inter- and intrapersonal catalysts.

Therefore, there are currently increased efforts to synthesise both approaches, resulting in integrated models, which explain the development of giftedness, talent or expertise. For the domain of mathematics, Fritzlar (2015) described a model of developing mathematical expertise. For reasons of space, it is not possible to go into this model in detail in this article. However, some of the consequences, which we believe to be essential, are listed below:

- Individual factors, such as the cognitive apparatus or its genetically predetermined part, will lose their anticipatory nature in the model of developing expertise. This is true also for the current (measured) level of expertise, which does not allow any reliable statements on which level will or can potentially be reached (Sternberg, 1998).

- Rather, the model emphasises that any individual must continuously progress in his or her development through a suitable interplay of all participating factors. This way, he or she has the chance to reach the level of expertise that enables access to fostering programmes etc. and/or lead him to be identified as 'gifted' (Sternberg, 2000).
- Developing extraordinary performance is generally not understood as an autocatalytic process. The environment, in this context, does not merely serve in a defensive function by preventing potential 'disturbances'. Rather, experience in mathematics and thus opportunities to learn are necessary preconditions to develop one's expertise. This also points to the responsibilities of schools and society in supporting necessary long-term and specific learning activities.
- From a pedagogical perspective, inclusive fostering activities should at first be given priority over exclusive ones.
- A (amongst others) cognitive 'basic configuration' is differentiated from mathematical abilities and special achievements. This points to and considers their context-dependence and therewith, for instance, the problem of identifying gifted students or initiating positive feedbacks.
- The model brings to the fore the systemic character of expertise (cf. Ziegler & Phillipson, 2012): the current level of expertise appears simultaneously as an emergent feature and as an element of a system that is characterised by cross-linkages, dynamics and equifinality, amongst others.

Looking Ahead

In the past decades, there was a large number of research projects in the field of mathematical giftedness in Germany. Nevertheless, from our point of view, there are still many open questions, the answers to which could be worthwhile. On the one hand, it seems important to us to continue theoretical work in the form of a more detailed elaboration of possible characteristics of mathematical giftedness, relationships between these and relationships between giftedness and other constructs. On the other hand, the practice-oriented further development of promotion approaches is important, which, for example, also include children with special support needs. In this respect, we consider the concretization of a systemic view of mathematical giftedness and its promotion to be very promising.

Theoretical developments

The construction and use of structures (or abstraction and generalization) seem to be of central importance for mathematical giftedness. So far, however, this characteristic has been investigated in German-language studies predominantly in connection with geometric or number patterns. Already in Krutetskii's classical book, the *ability for formalized perception of mathematical material, for grasping the formal structure of a problem* is seen as an essential component of mathematical giftedness. However, this refers to a more general concept of "structure", in which a structure is understood as a set of elements that are in (different) relationships to each other; the set with its internal relationships then appears as a whole, underlying the corresponding mathematical situation. On this basis, it seems promising to us to take a closer empirical look at the ability to grasp underlying mathematical structures, structural similarities and differences between math-related situations and to investigate its development across different age groups.

An important related question refers to change which arise with focusing on investigations in regular math lessons. A first study with primary school students suggests that more students see mathematical patterns than 15 years ago. Therefore, it seems questionable to us whether seeing patterns can still be regarded as a hint for giftedness (Nolte & Richter 2019).

Those who work with mathematically gifted children and adolescents have certainly often experienced a problem solver suddenly seeing the solution. Already Krutetskii (1969) describes these observations impressively and, above all, attributes them to a very rapid succession of solution steps: "... that in several cases one gets the impression that, in essence, there is no process, but rather that there is an analytico-synthetic "vision" of the mathematical material in a single act, a single step"(Krutetskii, 1969, p. 108). This, in our view, can be related to the somewhat broader construct of *intuition* introduced by Käpnick (2012) into German-language research on mathematic giftedness. This construct also points to the importance of (possibly implicit) knowledge. In terms of research methodology, the phenomenon of intuition (with possible references to knowledge and speed of information processing) is certainly very difficult to access, yet it seems highly interesting and worth further empirical efforts.

Creativity and giftedness are often seen in close connection. However, there is no agreement on the exact relationship between the two constructs. Thus, creativity is sometimes seen as a prerequisite, as a possible component or a possible consequence of giftedness, or both constructs are seen as independently of each other. A clarification of the relationship is difficult also because both constructs are partly fuzzy. Assmus and Fritzlar (2018) make a proposal that combines a modern concept of giftedness with a relativistic understanding of creativity. Here, further scientific efforts seem important to us, with which in particular the question could be investigated how creative mathematical activity at (primary) school age can be expressed itself and how this can be promoted.

Practice-oriented developments

Discussions about the realization of inclusion also underline the necessity to foster students with a high potential. Nolte’s study about twice exceptional students (Nolte, 2013b, 2017) underline the importance of interdisciplinary collaboration. The special needs of these students do not belong to the usual professional knowledge of mathematics teachers. Yet, not enough is known about the special needs of mathematical gifted and promising students who must overcome or handle barriers in their learning processes.

Ziegler (2005) used the Actiotope Model to develop a systemic view of giftedness that comprises four components: the individual's action repertoire, its subjective action space, its goals, and the environment surrounding the individual. In our view, the next step should be to attempt to specify this model for the domain of mathematics and to derive from it possible conditions of success for the promotion of mathematical giftedness.

Going back to Kießwetter's patterns of action we question how it is possible to use the results of research in the field of giftedness for working with all students. Nolte (2006) distinguishes between patterns of action which can be observed and "action" from cognitive components of problem solving which may lead to an action but are not observable. Thus, for instance. students can be taught how to organize material as an observable action. But, we cannot lead a student to recognize pattern and structures. Thus, inclusive fostering activities in regular classroom may be based on explicitly discussing patterns of action and cognitive components. Even though there has been more or less research on giftedness in Germany for more than 100 years, much remains to be done, at least with regard to the domain of mathematics.

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TEACHING HIGHLY ABLE LEARNERS IN DIVERSE CLASSROOMS: PEDAGOGICAL POSSIBILITIES THROUGH COLLABORATION

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We argue in this paper that, with appropriate support, teaching of highly able learner can occur in diverse classrooms. We draw on a constructivist theory of learning and a differentiation paradigm (Dai & Chen, 2013). The claim that teachers can orchestrate teaching for highly able students in diverse classrooms is considered with evidence of our own and other data, warrant, backing, qualifier and rebuttal. Results from many studies have given knowledge of learning needs of mathematically highly able learners as well as of successful teaching to meet a diversity of learners. Drawing on our research, and work with school development, we share ideas about possibilities for teachers to support learning for all students, that is, including the highly able, within a diverse classroom. In particular, we advocate the possibilities from professional collaboration and our practice examples illustrate this claim.

Keywords: Gifted education, Differentiated instruction, collaboration

INTRODUCTION

The importance of improving student results and raising the status of the teaching profession are often highlighted in government policy and media. Many countries across the globe have looked for ways to improve their education systems and international rankings of student achievement. However, there are several studies reporting that highly able learners are not given the educational support they need (e.g., Leikin & Stanger, 2011; Pettersson; 2011). The first author of this publication personally asked Françoys Gagné and Linda Silverman, both leading researchers in the field of gifted education, if they believe it is possible to teach highly able learners in diverse classrooms. Both quickly responded: No. This is a contradiction to our experience, research and beliefs as educational researchers and teachers and in this paper we propose an alternative pedagogical possibility.

It is reasonable to assume that differences in perceptions of what is possible or not, partly has its roots in which definition, or paradigm, of gifted education is used. As researchers in pedagogical work we acknowledge that students have differing inner qualities; some have higher cognitive abilities, others have lower cognitive abilities. There are also differences in terms of learning dispositions, motivation, volition and wider individual experience. Consequently, we acknowledge that some students have possibility to excel faster and deeper into a subject, for example mathematics, than most other students. Aiming to support students to develop as far as possible in knowledge, the process of how to recognize and support students' learning should be in focus. Moreover, since most highly able students are placed in diverse classrooms (Shayshon, Gal, Tesler, & Ko, 2014) it is important that developmental research includes focus on teachers' practice.

Through our research and experiences from school development, Erasmus and other projects, this paper presents an argument that it is possible to teach highly able learners in diverse classrooms.

THEORETICAL CONSIDERATIONS

The aim of this paper is to give an argument on that highly able students can be taught according to their learning needs in diverse classrooms. The theoretical perspective of learning is constructivist, and the chain of argument is inspired by Toulmins' (1958) model of argumentations as described by Brunström (2015, p. 19[3]). The model is described by six parts that are connected. It was developed to analyze argumentations made in every-day life.

Claim: The statement that is going to be defended or the hypothesis or conclusion that has been drawn,

Data: Facts that the statement/hypothesis/conclusion is based on,

Warrant: Explanation to why the step from facts to statement/hypothesis/conclusion is reasonable,

Backing: Sometimes the *warrant* need support. Support giving the *warrant* validity is called "backing",

Qualifier: Marking the validity of the reasoning, that is how valid the step from facts to statement/hypothesis/conclusion is,

Rebuttal: In some circumstances there may be exceptions from where the conclusions are not valid. Such circumstances are called "rebuttal".

Constructivism is a theory of learning which advocates the acquisition of knowledge as an individually tailored process of construction. To design effective teaching environments, constructivism promotes understanding of what children already know when they come into the classroom. Curriculum should be designed to build on students' experience and knowledge and allowed to develop with them. The teacher acts as a facilitator who encourages students to discover principles for themselves, to construct knowledge through open-ended discovery and problem-solving. To do this, a teacher should encourage curiosity and discussion amongst students as well as promoting their autonomy (Ernest,1995; Steffe & Gale, 1995).

LITERATURE REVIEW

In this section we elaborate on different views of gifted education based on the review by Dai and Chen (2013). Thereafter we continue with a brief overview of highly able students in school from a pedagogical perspective, focusing on their learning needs as characteristics to recognize them. Further we briefly elaborate on teacher professional development, collaborative work between teachers and principals' support to teachers to enable teaching inclusive of highly able students.

Three views of gifted education

Dai and Chen (2013) have done a thorough overview of different ways of addressing gifted education, as a result they divide the views into three paradigms: The gifted child paradigm, The talent development paradigm and The differentiation paradigm. Different

[3] Brunström (2015) is here translated to English.

answers will be given to the four questions; *What is high ability?*, *Why should school practice bother?*, *Who are the highly able?* and *How should teaching be adapted for the highly able students?* Responses to these questions depending on which paradigm is used.

The gifted education paradigms proposed by Dai and Chen (2013) are here very shortly summarized. In *The gifted child paradigm*, giftedness is something individuals are born with, school should adapt teaching for these students since they are the leaders of tomorrow, high IQ is a frequent example of how a student may be defined as highly able or not, and these students need special schools with a designed curriculum. In *The talent development paradigm*, high ability is something that can develop - although some individuals have potential to develop faster and deeper than others - the highly able students are seen to be the future leaders and the innovators that can solve the big problems of tomorrow. IQ measurements are not enough to find who is highly able or not – other standardized and also non-standardized measurements are of importance, and these students need special school or special classes where acceleration and enrichment are possible ways to adapt teaching. In *The differentiation paradigm* high ability is something that can be developed - and all individuals are seen to have capacity to develop - the school is important to support all students to develop as far as individually possible - the highly able students can develop far more than expected according to age, teaching should be adapted so that all students are given possibilities for learning.

This paper addresses the diverse classroom, therefore *The differentiation paradigm* is the only possible view. We do however acknowledge that students have differences and that these depends on both genetics and environment. That is, we acknowledge the two other paradigms answers given to *Who is highly able?*. But our focus is on the question *How should teaching be adapted for the highly able students?*

Highly able students in school

The reason to identify students learning needs, from a pedagogical perspective, is to be able to adapt teaching towards their learning needs. That is, to offer teaching that enables learning so students can develop as far as possible in knowledge.

In this paper highly able students in mathematics are seen as a sub-domain to highly able students in general; they are assumed to share the same learning needs. That is, they are quick learners, prefer complex and abstract tasks (Leikin, Leikin, Paz-Baruch Waisman, &, Lev, 2017; Rogers, 2007; Tomlinson, 2016), can get bored and even drop-out of school when teaching goes too slow (Mohokare & Mholo, 2017). This paper address high ability in general, with students with high ability in mathematics as a specific cohort example.

To successfully teach in diverse classrooms, Le Fevre, Timperley and Ell (2016) suggest that teachers need to have knowledge of their students' learning needs and to be able to adapt teaching towards these needs. This means that to be able to teach highly able students in diverse classrooms teachers need to be able to first recognize the students' learning needs and second orchestrate teaching to include those students (Mattsson, 2013).

Most highly able students do not need repetitions (e.g., Persson, 2010), their learning may even be hindered if they are forced to it (Rogers, 2007). They are fast learners, in mathematics for example they need less time than other students to solve complex tasks (Nolte & Pamperien, 2017). Many studies report that acceleration is suitable for highly able students (e.g., Colangelo & Assouline, 2009). However, Sheffield (2015) means that acceleration may be contra productive if the aim is to increase the number of individuals in mathematics intensive occupations.

In general, students with high ability in mathematics easily grasp the formal structure of a problem and quickly generalize (Krutetskii, 1976; Sheffield, 2003). They may show great interest on why and how a solution to a problem gets right or wrong (Mohokare & Mhlolo, 2017; Sheffield, 2003). It does not necessary mean that highly able students become successful in achievement, they may become bored and act rebellious for having to work on a level they perceive too low for them (Mohokare & Mhlolo, 2017).

To be able to meet highly able students' learning needs, several studies have concluded that teachers need professional development on gifted education (e.g., Shayshon et al., 2014). According to Desimone (2009) collaboration is one out of five core features making teachers' professional development successful, that is to improve practice and increase teacher knowledge and skills. Collaboration between colleagues is by teachers perceived to be necessary to orchestrate teaching meeting the learning needs of highly able students in diverse classrooms (Mellroth, 2019). Collaboration with students is also an integral element of pedagogy and recommended between teachers and parents of highly able students (Porter, 2008). However, this paper focuses on the collaboration between teaching colleagues.

In Sweden, the principal is the pedagogical leader on her or his school. S/he is responsible that each student receives an education that develops their learning. For principals to support such education, Forssten Seiser (2017) found that they should focus on teachers' prerequisites to develop students' learning environments. They should encourage teachers to try different ways to adjust their teaching, they should also strive to create a common understanding of what is needed, from the school, to give students learning support. Related to this paper, the findings of Forssten Seiser (2017) is interpreted as meaning the principal has an important role in supporting and encouraging teachers to learn about highly able students' learning needs and gifted education. In the study of Johnsen, Haensly, Ryser and Ford (2002) principals showed interest and were involved in a professional development project on gifted education, and encouraged the participating teachers. In the end of that project the majority of participating teachers had positively changed their classroom practices and adapted teaching to include highly able students in teaching.

CLAIM: THROUGH COLLABORATION, TEACHERS CAN ORCHESTRATE TEACHING HIGHLY ABLE STUDENTS IN DIVERSE CLASSROOMS

One claim on teaching highly able students in diverse classrooms will be given here. In this paper it is that, through collaboration, teachers can orchestrate teaching for highly able students in diverse classrooms. The example follows Toulmins' (1958) model of argumentation presented earlier. The claim is considered with evidence of our own and other data, warrant, backing, qualifier and rebuttal – following the chain of argument is inspired by Toulmins' (1958) model of argumentations noted earlier in this paper.

Data

Mellroth (2019) showed that teachers who participated in a two-year long professional development program on gifted education perceived they had competence in recognizing highly able students in diverse classrooms. Those teachers connected their teaching practice with theories of gifted education, for example they showed knowledge, in line with research, of highly able student characteristics, in general, as well as in mathematics.

They were aware of that their own mathematical knowledge may be to low to meet the learning needs of mathematically highly able students. Low mathematical knowledge among teachers is shown to be a problem when teaching mathematically highly able students (e.g., Hoth et al., 2017). However, the teachers in the study of Mellroth (2019) gave a solution on to how to solve the problem. They acknowledged that teachers and other school staff create a team of colleagues. Therefore, they suggested that teachers with deeper mathematical knowledge can be found through collaboration with colleagues. Those teachers can give support to teachers with less mathematical knowledge on how to orchestrate teaching for mathematically highly able students.

Earlier work by Margrain (2005, 2010, 2011) explored the experience of highly able young children in early childhood and with transition to starting school. Through classroom observations, teacher and parent interview, and document analysis, evidence was gathered of how teachers worked in various ways to understand diversity, differentiate curriculum, and document learning. The teachers worked particularly closely with parents and colleagues to gain understanding of the competencies children displayed in varying contexts, and to share responsibility for supporting development. A way of documenting such collaboration is through the Individual Education Plan (IEP), advocated by Mazza-Davies (2008) and used in Margrain's own school teaching experience.

Warrant

By training teachers (n=74) in differentiating instructions, over a period of two years, to meet the learning needs of highly able students, Johnsen et al. (2002) showed that, by the end of the second year of the program, none of the participating teachers continued to ask their students to wait while other students finished, 57% of the teachers used pre-assessments to compact the curriculum, provide acceleration or enrichment, 77% of the mathematics teachers chose to accelerate instructions and 71% of the teachers began offering a variety of learning activities, compared to 13% before training. The project, seen as a professional development program, supported 99% of the participating teachers to change their classroom practices to adapt for highly able students. Collaboration between teachers and other project members is highlighted as important for the developed internal and an external support structure in the project. Therefore, the study of Johnsen et al. gives a warrant to the subsequent data collected by Mellroth (2019) and Margrain (2005, 2010, 2011).

Backing

Nolte and Pamperien (2017) showed that challenging tasks, developed to support mathematically highly able students, were successfully applied in diverse classrooms, but that highly able students needed less time and achieved better regarding generalization and proving. To be able to use such challenging tasks in teaching, Nolte and Pamperien highlight that the teachers needed training on how to work with highly able students. In line with Mason and Johnston-Wilder (2006), they mean that teachers should ask questions to students which supports them become more independent in their solution process rather than giving them an answer on a problem. The study of Nolte and Pamperien gives back-up to the warrant based on the study of Johnsen et al. (2002). For example, they show that same kind of tasks can be used successfully for all students in diverse classrooms, but that teachers who orchestrate teaching need extra training to be able to implement them. That is, collaboration between teachers with deeper mathematical knowledge and knowledge of gifted education and teachers less skilled can be a solution.

A survey conducted by Margrain, Lee and Farquhar (2013) confirmed – unsurprisingly - that teachers who had experience working with high ability students were stronger advocates of these students and felt more confident in their practice. This study supports the argument that working with high ability students in diverse classrooms disseminates awareness of this group and facilitates skill in differentiation across the teaching profession.

Qualifier

Examples from three countries are used as qualifiers. First we share three systems-level examples of collaborative pedagogical support structures, followed by brief classroom-level examples of collaborative pedagogy which supports mathematically high ability students. Our experiences – including from the first author's work in an Erasmus Plus project, undertaken between 2017 and 2019 (European Comission, 2019) have illustrated examples of how teachers can be supported to recognize and orchestrate teaching inclusive of highly able students.

In The Netherlands we were told that each school ought to have a gifted coordinator. Several solutions are used in the country to give teachers in regular schools support on how to recognize and orchestrate teaching for highly able students. During the Erasmus project (European Comission, 2019) we visited and learned from an organization, DeDNKRS[4], where special education teachers, also trained in gifted education, gave support to and trained teachers in regular classrooms. In practice, the pedagogues from DeDNKRS were called in to counsel regular teachers at schools who needed guidance in how to support highly able students learning. During the time of counseling, regular teachers and pedagogues at DeDNKRS acted as colleagues, sharing knowledge of students and expertise in gifted education to improve practice.

In the federal state Hamburg, Germany, a political decision from 2014 (Landesinstitut für Lehrerbildung und Schulentwicklung, 2014) declared that each school should have a gifted coordinator. Through the Erasmus project (European Comission, 2019) we learned that this person aims to give support to teachers at the school on recognizing and orchestrating teaching for highly able students. To organize and guarantee the quality of these coordinators a central organization, Beratungstelle besondere Begabungen[5], (BbB), gives professional development for teachers and they administer networks between the gifted coordinators at schools in the regions of Hamburg. In addition, BbB works closely with the University of Hamburg which validates their scientific connection.

Thirdly, Gifted Aotearoa[6] is a New Zealand national network of expertise, established to improve quality of education provided to learners, and provides support through sharing professional expertise, growing local networks, nurturing local leadership and developing professional pathways. The five culturally-framed collaborative initiatives are: keeping connected, keeping afloat, learning together, leading together, and raising

[4] Link to the website of DeDNKRS; https://dednkrs.nl/
[5] Link to the website of BbB; https://li.hamburg.de/bbb/
[6] Link to the website of Gifted Aotearoa; https://www.giftedaotearoa.nz/

new leaders. Gifted Aotearoa as an organization is in itself a professional collaboration between professional learning providers and a parent advocacy organization, and the work is funded by the New Zealand Ministry of Education.

These examples from show that high ability is acknowledged in differing school systems. They can also be compared to the professional development program on which Johnsen et al., (2002) made their study on, but in contrast the examples from The Netherlands, Hamburg and New Zealand are not time-limited, they are part of the countries' school practices. Further, the examples show that examples of organizational structures give teachers support to recognize students who are highly able. Support by principals is an aspect shown to be of important to change practice by Johnsen et al. (2002) and Forssten Seiser (2017). We now give some brief examples of classroom-level practice from our own research to illustrate pedagogical possibilities.

In Mellroth, van Bommel and Liljekvist (2019) an example is given on how collaboration among teacher colleagues in diverse classrooms is perceived as a possibility to orchestrate teaching inclusive of highly able students in mathematics. The example comes from a teacher who on regular basis had another teacher in her mathematics lessons, a common way to arrange teaching in Sweden and in all subjects, called "the two-teaching system". The teacher declared that both were happy for this collaboration and recognized that through this arrangement they could work together to meet the learning needs of the highly able students. The main teacher contributed deeper knowledge of the individual class members which the other teacher needed to learn, and the other teacher had deeper mathematical knowledge and he therefore orchestrated challenges for the highly able. Thus, both teachers contributed essential elements to the collaboration for the benefit of students, and the two teachers were also able to learn from each other.

Margain's (2005) doctoral thesis documented several ways in which teachers could collaborate within a school to support curriculum differentiation. In one example, a five-year old student who was highly able in mathematics and reading had a 'home' school class with age peers, but attended classes with students several years older in the areas of strength. In another example an early childhood teacher met with teachers from a local primary school to borrow resources and share teaching ideas so that a four-year old could be given extension in the preschool setting. In a third example, the principal of a five-year old met with parents and teachers and an IEP was developed which attended to social, emotional and academic goals. In this final example no members of the school staff felt they had particular expertise in gifted education, but they recognized the high ability of the given student and were committed to student well-being and learning more as a professional, collaborative team.

For the claim given, these qualifiers validate that collegial work can make it possible for teachers to recognize and orchestrate teaching of highly able students in diverse classrooms. Important colleagues to teachers are in these cases gifted coordinators. These collaborations work in a similar way as special education teachers give support to teachers in meeting the learning needs of students with learning disability.

Rebuttal

The claim is based on two important conditions; Firstly, teachers who can orchestrate teaching inclusive for highly able students have professional development in gifted education. Secondly, the organization has a structure for teachers to collaborate with other colleagues. If these two conditions are not fulfilled there is a rebuttal to the claim.