FOURTH EDITION Medical Statistics from Scratch

An Introduction for Health Professionals



DAVID BOWERS

WILEY Blackwell

Medical Statistics from Scratch

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An Introduction for Health Professionals

Fourth Edition

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Preface to the 4th Edition

I noticed when looking at the prefaces to each of the first three editions, that there seems to be a gap of five years between each one. And this fourth edition is no exception – it's five years since I wrote the preface to the third edition, in 2013. Spooky or what! But anyway, this new edition has given me an opportunity to completely refresh the book. I have added three shiny new chapters covering important areas in medical statistics:

- On incidence, prevalence and standardisation.
- On Poisson regression.
- And on missing data.

I have also made numerous additions in most of the other chapters. These include, for example:

- In the chapter on measures of spread, I have added material on testing for Normality using the Shapiro–Wilk and Kolmogorov–Smirnov tests, as well as the graphical QQ plot.
- In the chapter on describing data with charts, I have added more material on the scatterplot, and introduced the bubble plot.
- In the second of the chapters on research design I have added brief notes on *per protocol* analysis.
- In the confounding chapter I have introduced the idea of confounding by indication, residual confounding, and propensity score matching.
- In the chapter on comparing population parameters using confidence intervals, I have revisited the interpretation of confidence intervals and the size of corresponding samples to try and make it easier to understand. I wanted to emphasise that we should take a flexible approach to the interpretation of confidence intervals and p-values.
- In the hypothesis chapter I have introduced the notion of correcting for multiple comparisons.

- In the logistic regression chapter I have added new material on goodness-of-fit to include the Cox and Snell, and Nagelkerke's *pseudo* goodness-of-fit measures.
- In the chapter on systematic review I have added a brief discussion of two statistical tests of funnel plot asymmetry: Begg's test and Egger's test. I have also added a little something on the trim and fill method for dealing with funnel plot asymmetry.

Apart from the above additions, I have added many new examples and exercises, and have rewritten much of the existing text to (hopefully) make it easier and more accessible.

Finally, I want to thank all of those who helped me in the writing of this book, and Barbara Noble in particular for bravely (and very competently) copy-editing and proofreading the material. Any remaining errors are of course mine.

As always I am happy to receive comments and criticism of my book, and I welcome any suggestions for improvement.

"Reader, I married him." Whoops! Wrong book.

David Bowers, 2019

Preface to the 3rd Edition

The 1st edition of this book was published in 2002 and the 2nd edition in 2008. I was surprised when I discovered it was quite such a long time ago. Where did the time go! Anyway, over the course of the last five years, I have received many favourable comments from readers of my book, which of course is immensely gratifying. I must be doing something right then.

This edition contains a completely new chapter (on diagnostic tests), there is a quite a lot of new material and most of the chapters have received an extensive re-write. I have also updated virtually all of the examples drawn from the journals and added many new exercises. I hope that this gives the book a fresh feel – as well as a new lease of life.

The book should appeal, as before, to everybody in health care (students and professionals alike) including nurses, doctors, health visitors, physiotherapists, midwives, radiographers, dieticians, speech therapists, health educators and promoters, chiropodists and all those other allied and auxiliary professionals. It might possibly also be of interest to veterinary surgeons, one of whom reviewed my proposal fairly enthusiastically.

My thanks to Jon Peacock and all the others at Wiley who have shepherded me along in the past and no doubt will do so in the future. I must also thank Barbara Noble, who patiently acted as my first-line copyeditor. She read through my manuscript, discovered quite a few errors of various sorts and made many valuable suggestions to improve readability. Any remaining mistakes are of course mine.

I also want to acknowledge my great debt to Susanne, who always encourages me, enthusiastically, in everything I attempt.

Finally, I would like to mention another book which might be of interest to any readers who are thinking of embarking on research for the first time – *Getting Started in Health Research*, Bowers et al., Wiley, 2012. This book covers both quantitative and qualitative research. It will guide you through the research process, from the very first idea to the interpretation of your results and your conclusions.

David Bowers, 2013

Preface to the 2nd Edition

This book is a "not-too-mathematical" introduction to medical statistics. It should appeal to anyone training or working in the health care arena – whatever his or her particular discipline is – who wants either a simple introduction to the subject or a gentle reminder of stuff that they might have forgotten. I have aimed the book at:

- students doing either a first degree or a diploma in clinical and health care courses
- students doing post-graduate clinical and health care studies
- health care professionals doing professional and membership examinations
- health care professionals who want to brush up on some medical statistics generally or who want a simple reminder of a particular topic
- anybody else who wants to know a bit of what medical statistics is about.

The most significant change in this edition is the addition of two new chapters, one on measuring survival and the other on systematic review and meta-analysis. The ability to understand the principles of survival analysis is important, not least because of its popularity in clinical research and consequently in the clinical literature. Similarly, the increasing importance of evidence-based clinical practice means that systematic review and meta-analysis also demand a place. In addition, I have taken the opportunity to correct and freshen the text in a few places, as well as adding a small number of new examples. My thanks to Lucy Sayer, my editor at John Wiley & Sons, for her enthusiastic support, to Liz Renwick and Robert Hambrook and all the other people in Wiley for their invaluable help and my special thanks to my copyeditor Barbara Noble for her truly excellent work and enthusiasm (of course, any remaining errors are mine).

I am happy to get any comments from you. You can e-mail me at: d.bowers@leeds.ac.uk.

Preface to the 1st Edition

This book is intended to be an introduction to medical statistics but one which is not too mathematical – in fact, it has the absolute minimum of maths. The exceptions however are Chapters 17 and 18, which have maths on linear and logistic regressions. It is really impossible to provide material on these procedures without some maths, and I hesitated about including them at all. However, they are such useful and widely used techniques, particularly logistic regression and its production of odds ratios, which I felt they must go in. Of course, you do not *have* to read them. It should appeal to anyone training or working in the health care arena – whatever his or her particular discipline is – who wants a simple, not-too-technical introduction to the subject. I have aimed the book at:

- students doing either a first degree or a diploma in health care-related courses
- students doing post-graduate health care studies
- health care professionals doing professional and membership examinations
- health care professionals who want to brush up on some medical statistics generally or who want a simple reminder of a particular topic
- anybody else who wants to know a bit of what medical statistics is about.

I intended originally to make this book as an amalgam of two previous books of mine, *Statistics from Scratch for Health Care Professionals* and *Statistics Further from Scratch*. However, although it covers a lot of the same material as in those two books, this is in reality a completely new book, with a lot of extra stuff, particularly on linear and logistic regressions. I am happy to get any comments and criticisms from you. You can e-mail me at: slothist@hotmail.com.

Introduction

My purpose in writing this book was to provide a guide to all those health care students and professionals out there who either want to get started in medical statistics or who would like (or need) to refresh their understanding of one or more of the topics which are common in medical statistics. I have tried to keep the mathematics to a minimum, although this is a bit more difficult with the somewhat challenging material on modelling in later chapters.

I have used lots of appropriate examples drawn from clinical journals to illustrate the ideas, as well as many relevant exercises which the readers may wish to work through to consolidate their understanding of the material covered (the solutions are at the end of this book).

I have included some outputs from SPSS and Minitab which I hope will help the readers interpret the results from these statistical programs.

Finally, for any tutors who are using this book to introduce their students to medical statistics, I am always very pleased to receive any comments or criticisms they may have which will help me improve the book for any possible future editions. My e-mail address is: d.bowers@ leeds.ac.uk.

Enjoy!

David Bowers