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From Domination to Coloring

Stephen Hedetniemi's Graph Theory and Beyond

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Dedicated to

Stephen T. Hedetniemi
On the Occasion of his 80th Birthday
February 7, 2019

Preface

As with every area of mathematics, graph theory has a number of mathematicians who have contributed to its development in a number of ways, namely (1) by proving theorems that are instrumental in its growth, (2) by giving lectures and writing survey papers and books that popularize graph theory, and (3) by creating new concepts and topics that have drawn mathematicians into various areas of graph theory. One mathematician responsible for all of this is Stephen T. Hedetniemi. Steve earned his Ph.D. in mathematics with a specialization in graph theory at the University of Michigan in 1966 under the direction of the well-known graph theorist Frank Harary.

Two major areas of research by Steve Hedetniemi are domination and coloring. In this book, we begin by discussing several topics, results, and problems in domination in which Steve has made a major contribution. From domination, we move on to a number of coloring topics. Along the way *from domination to coloring*, we also discuss other research topics in Stephen Hedetniemi's graph theory, including distance in graphs and two types of traversing walks. In the eight chapters that follow, while the material presented represents only a small sample of Steve's research in graph theory, we believe that beyond what is included lies other avenues for research.

Through studying chessboard problems, Stephen Hedetniemi introduced total domination, which has become one of the major topics of study in domination. Hedetniemi and others showed that there is a chain of inequalities involving the domination number of a graph, the independent domination number, and other domination-related parameters. These are the primary topics of Chap. 1. The independent domination number and total domination number are discussed in more detail in Chap. 2. If every vertex in a dominating set S of a graph G is assigned the value 1 and the vertices not in S are assigned 0, then the sum of the values of each vertex of G and its neighbors is at least 1. This observation by Hedetniemi led to the introduction of a dominating function of a graph. This concept, together with some variations, is the subject of Chap. 3. Two recent domination-related parameters introduced by Hedetniemi, namely Roman domination and alliances in graphs, are the subject of Chap. 4. In the first four chapters then, we discuss some of the primary and most recent results dealing with prominent domination parameters, as well as new and interesting concepts and problems derived from these concepts.

Many areas of graph theory different from domination have also been influenced by the research of Stephen Hedetniemi. One of these is distance in graphs in which Steve has investigated two interpretations of the "middle" of a graph, namely the center and median, which have numerous applications. These and other distance-related subgraphs of graphs are the topics of Chap. 5. In Chap. 6, we discuss two graph traversing concepts studied by Hedetniemi and his coauthors, one in which all edges of a graph are traversed, resulting in Eulerian walks, and a second in which all vertices are traversed, resulting in Hamiltonian walks.

Graph colorings has been a popular area of research for well over a century. This has also been a topic of interest for Hedetniemi for many years. In fact, he wrote his doctoral dissertation on graph homomorphisms, a concept closely tied to proper colorings of graphs. The concept of graph homomorphisms occurs in both Chaps. 7 and 8. Every proper coloring of a graph using the minimum number of colors has the property that for every two distinct colors, there are adjacent vertices with these colors. Any coloring with this property is a complete coloring, which is the primary topic of Chap. 7. The two major methods of evaluating how highly connected a graph involves vertex-cuts and edge-cuts. In Chap. 8, we see relationships of these concepts with graph colorings, resulting in color connection and disconnection in graphs. Recent results involving these connectivity-coloring concepts are presented along with suggestions for new avenues of research.

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Chapter 1
Pioneer of Domination in Graphs

Stephen Hedetniemi is perhaps best known for his pioneering work in domination in graphs. In this chapter, we explore some of his contributions to the direction and advancement of this field of study. We focus on two topics, namely domination of chessboard graphs and the domination chain.

1.1 Introduction

Honest pioneer work in the field of science has always been, and will continue to be, life's pilot. Wilhelm Reich

Pioneering is the work of individuals. Susanne Katherina Langer

Stephen Hedetniemi is at the top of the list of individuals who have most influenced the growth of the popular area of domination in graphs. In this chapter, we first discuss the origin of domination as a chessboard covering problem and consider Steve's contribution to this area of study. Then we turn our attention to the "so-called" domination chain, which was introduced by Hedetniemi along with Cockayne and Miller.

In the subsequent two sections, we will use the following terminology and introduce additional notation as needed. A set S of vertices of a graph G is *independent* if no two vertices in S are adjacent, and the maximum cardinality of an independent set of G is the *independence number* of G, denoted $\alpha(G)$. A *dominating set* S of G is a set of vertices of G such that every vertex in $V \setminus S$ is adjacent to a vertex in S, and the *domination number* $\gamma(G)$ is the minimum cardinality of a dominating set of G. The *independent domination number* of G, denoted $i(G)$, is the minimum cardinality of an independent dominating set of G.

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