Earth Pressure

Achim Hettler
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For Marie-Hélène

Achim Hettler

and Claudia

Karl-Eugen Kurrer

Preface

"You only have a future if you understand the past"
Wilhelm von Humboldt, 1767–1835

Decades have passed since the publication of an entire book on the subject of earth pressure. For this purpose, refer for example to "Erddrucktheorien" by Árpád Kézdi from 1962 or to part II of the series on excavations by Anton Weißenbach from 1975, which essentially includes earth pressure issues although in this case mainly concerned with its application for excavation walls. In the meantime, the topic has been treated repeatedly as part of works in the fields of soil mechanics and foundation engineering, see for example "Bodenmechanik" by Gerd Gudehus from 1981 or the contributions to the "Grundbau-Taschenbuch". Despite the importance of earth pressure theories in structural engineering, the current view has not been written yet. Many analytical applications have proved useful for decades. In recent years, the Finite-Element Method has been added as a new tool, and in practice, the displacement dependency of earth pressure has to be considered in more detail.

Essentially, this book has three major themes. Firstly, to make a set of working instructions available to civil and structural engineers in construction companies, engineering firms and design departments as well as students. This is supplemented with comments on the current earth pressure standard of 2017 and the collection of samples from 2018. Then current methods for determining earth pressure are presented in detail. However, a basic understanding of today's common theories and rules is hardly conceivable without a thorough study of history. The first empirical design rules were already known to the Romans; hints can be found in the publications by Vitruvius. Today's theories began in France more than three centuries ago and are closely associated with French military engineers. The third major theme is therefore dedicated to historical development, complemented by the biographies of selected researchers who have made significant contributions to the subject of earth pressure.

Without the support of assistants, it is hardly possible to complete a book. Jan Deutschmann has provided untiring, quick and competent support, as well as

Marcel Deckert, Ingmar Zehn and Annette Richter. Furthermore, the publisher Ernst & Sohn supported the idea for the present book and its implementation from the very beginning.

Achim Hettler, Karl-Eugen Kurrer
Dortmund and Berlin, 2019

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Contents

1 Introduction

The topic of earth pressure is considered one of the oldest and most extensive chapters in soil mechanics and foundation engineering. It is also one of the three pillars of structural engineering together with arch theory and beam theory. The first written sources, dating back to Vitruvius, are more than 2000 years old and therefore much older than the well-known theories of Coulomb (1773/1776) or Rankine (1857). In the first and sixth volume of his ten books, Vitruvius deals with the mode of action of earth pressure on retaining walls and proposes buttresses. Vauban, one of the greatest engineers in history, already published design tables for retaining walls with heights of up to 15 m in 1684, which cannot be bettered even today. The development of the earth pressure theory is described in detail in chapter 2 which is based on the extended edition of "The History of the Theory of Structures. Searching for Equilibrium" by Kurrer (2018). The present book can only include a limited selection of current design methods. The aim of the book is to provide a set of work instructions for foundation engineers and structural engineers in construction companies, engineering consultancies and in design departments, but also for students. In order to further theoretical understanding, the essential principles for determining earth pressure are initially presented in chapter 3. Chapters 4 to 12 contain the most important methods of determining active and passive earth pressure as well as at-rest earth pressure. In chapters 7 and 8, the spatial effects of earth pressure are taken into account. One concern of this book is to give a short overview of non-everyday questions and to refer to further literature (see chapter 14). In recent years, the displacement dependency of earth pressure has increasingly come into view. This applies not only to passive but also to active cases (see chapter 15). The book offers also instructions for practical application in chapter 16 and is supplemented by earth pressure tables for the most important basic cases.

Many questions were submitted to the DIN Committee "calculation methods", and a selection of these is discussed in the commentary to DIN 4085 in chapter 17. In the last section of this chapter, references are provided to the examples in the supplement to DIN 4085, which was published in December 2018.

The history of earth pressure theory in chapter 2 includes a few selected short biographies of scientists and engineers working in the field who have taken up and developed the subject over the centuries, see chapter 18. The book is supplemented by two appendices with terms, symbols and indices (Appendix A) and

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earth pressure tables in Appendix B. For historical reasons, the current terms and formulas in chapters 3 to 17 and in the Appendices may differ from the original terms in chapter 2.

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2

The history of earth pressure theory

Digging, piling, tipping, stretching, arching, placing and laying are the archetypal forms of building which, in terms of their historical manifestation, appeared in this sequence and formed and still form the foundation for all great architecture. Even today, the archetypal forms are the basic ways of building (v. Halász, 1988, p. 257). Whereas digging reaches back into the depths of the animal-human transition, the *teocalli* of the Aztecs were magnificent pyramids built by piling and tipping. In fact, *teocalli* means "covered by stones" (v. Halász, 1988, p. 257), the core of the pyramid consisting of a pile of earth. Building with earth – earthworks – is, even today, based on three elementary forms of activity: digging, piling and tipping. Moving great bodies of soil to form the embankments, cuttings and cuts required during the building of roads, railways and waterways has changed and still changes not only the relief of the natural landscape, but also the urban landscape (Guillerme, 1995).

The evolution of geotechnical engineering up to 1700 has been summarised in an extensive congress paper by Jean Kérisel, who from 1951 to 1969 was honorary professor of soil mechanics at the École Nationale des Ponts et Chaussées in Paris (Kérisel, 1985). In contrast to that work, this chapter will try to trace the theory of earth pressure from its beginnings shortly before the turn of the 18th century right up to the present day from the perspective of the history of theory of structures. Besides original sources, the following historical studies have been consulted: (Corradi, 1995 & 2002), (Chrimes, 2008), (Feld, 1928 & 1948), (Golder, 1948 & 1953), (Guillerme, 1995, pp. 85-145), (Habib, 1991), (Herries & Orme, 1989), (Heyman, 1972), (Jáky, 1937/1938), (Kalle & Zentgraf, 1992), (Kérisel, 1953), (Kötter, 1893), (Llorente, 2015), (Marr, 2003), (Martony de Köszegh, 1828), (Mayniel, 1808), (Mehrtens, 1912, pp. 55-73), (Ohde, 1948-1952), (Peck, 1985), (Reissner, 1910), (Skempton, 1981 & 1985), Verdeyen (1959) and (Winkler, 1872).

Around the middle of the 19th century, Alexandre Collin (1808-1890) started to shape the theory of earthworks through his theory of embankments made from cohesive soils backed up by experiments (Collin, 1846). Ten years later, Culmann published his article *Ueber die Gleichgewichtsbedingungen von Erdmassen* (on the equilibrium conditions of bodies of soil) (Culmann, 1856), which was followed in 1872 by his paper on earthworks (Culmann, 1872). In 1888

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Karl von Ott, professor of soil mechanics at the German Technical University in Prague, divided his lectures into

- the theory of earthworks (or embankments),
- the theory of retaining walls,
- the theory of the masonry arch, and
- elastic theory and its application to timber and iron structures paying particular attention to roofs and bridges.

What he understood by earthworks was the creation of certain soil forms "known by the names of dams, ramparts, cuttings, cuts, etc., the creation of which requires working the material supplied by the natural soil" (v. Ott, 1888, p. 2). Laws governing the equilibrium of such bodies of earth (Fig. 2.1) were postulated in his book *Theorie des Erdbaues oder der Böschungen* (theory of earthworks or embankments) (v. Ott, 1888, p. 2).

It was August von Kaven who provided a classical summary of the theory of earthworks in the middle of the classical phase of theory of structures (1875-1900) (v. Kaven, 1885). But the theory of earthworks did not gain new momentum (e.g. (Hultin, 1916), (Fellenius, 1927)) until the investigations into the collapse of the quayside at Gothenburg on 5 March 1916 (Petterson, 1916). Earth pressure theory backed up by experimentation started to assert itself as soil mechanics evolved in the 1920s, with Terzaghi pointing the way forward with his seminal work *Erdbaumechanik auf bodenphysikalischer Grundlage* (mechanics of soil in construction) (Terzaghi, 1925). Today, the theory of embankments and earth pressure theory are part of soil mechanics (Fig. 2.2), which in turn is a subdiscipline of geotechnical engineering.

Earth pressure theory can look back on 300 years of history. The first half of that was dominated by French engineering officers, a list of names stretching from Vauban to Bélidor to Coulomb to Poncelet, who were involved with the planning, design, construction and upkeep of fortifications. In the following sections, the thesis postulated is that the Corps du Génie Militaire of the early 18th century not only played a decisive role in the development of modern civil engineering, but

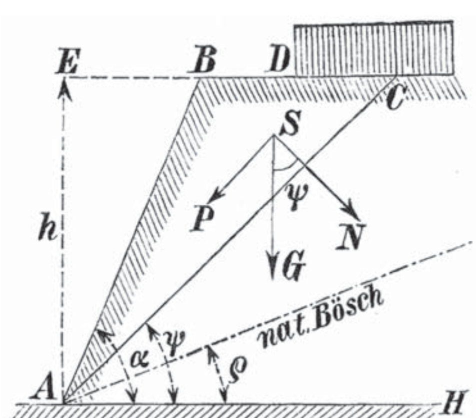

Fig. 2.1 Investigating the stability of an embankment loaded through excavation; ψ = angle of slip plane, ρ = angle of internal friction (v. Ott, 1888, p. 20).

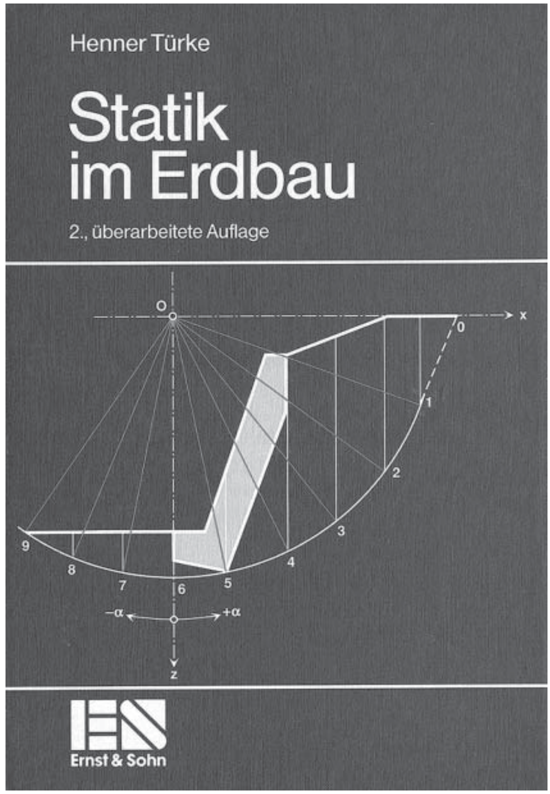

Fig. 2.2 The illustration on this book cover shows a schematic view of the investigation of a slip circle in the subsoil behind a retaining wall (Türke, 1990).

also that the engineering officers of that corps created the first genuine engineering science theory in the form of earth pressure theory, providing civil engineers with a scientific conception for their work. Not until the establishment phase of theory of structures (1850-1875) (Kurrer, 2018, pp. 20-21) would the supremacy of the engineering officer in the field of earth pressure be overtaken by that of the railway engineer. So the building of fortifications, with earth pressure theory providing a scientific tool, marks the birth of modern civil engineering.

2.1 Retaining walls for fortifications

The building of fortifications in Europe from the early years of the modern era right up to the completion of the Industrial Revolution in the countries of continental Europe was based on earthworks, which together with masonry resulted in large-format structures that were to leave a mark on towns and cities. One example is Luxembourg, where building works between 1543 and 1867 turned it into one of the strongest fortresses in Europe (Fig. 2.3).

One of those who worked on extending Luxembourg's fortifications was Vauban (Fig. 2.4), who in 1678 had been appointed Commissary General of all French fortifications by Louis XIV and who was in charge of the conquest of the city in 1684. However, Luxembourg is only one small part in the output of

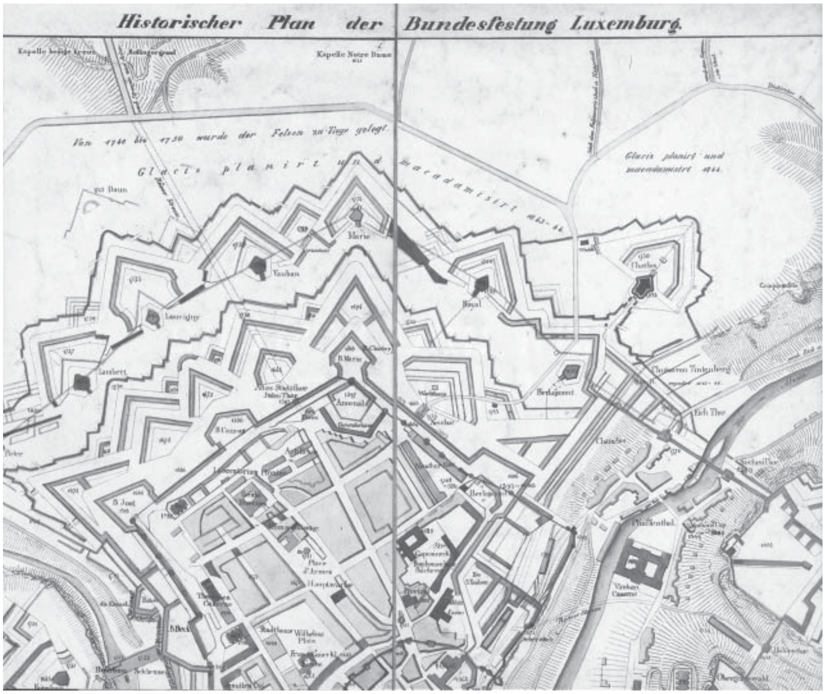

Fig. 2.3 Historical map of Luxembourg by First Lieutenant Cederstolpe showing the city's fortifications, *c*. 1845 (Reinert & Bruns, 2013, p. 48).

Fig. 2.4 Sebastien le Prestre de Vauban (1633-1707); copy by Antoine Coysevox of the marble bust (since lost) produced by Pietro Marchetti by order of Napoleon I (Neumann, Hartwig, 1984, p. 379).

this "Ingénieur de France", as he was called during his lifetime. As is written in the *Larousse universel* of 1923: "Towards the end of his life, Vauban – who Saint-Simon (1760-1825) described as one of the most virtuous men of his century – published his *Projet d'une dixme royale* [project for a royal tithe] in which, driven by a genuine philanthropic feeling, he called for fair taxes, which resulted in him falling out of favour with Louis XIV" (cited after (Göggel, 2011, p. 136)). In just a few decades, Vauban built 33 new fortifications and rebuilt about 300; so far, 411 construction measures at 160 locations have been proved to be his work (Neumann, Hartwig, 1984, p. 381). Vauban's fortification and civil works involved about nine million cubic metres of masonry (Petzsch, 2011, p. 191). According to his own figures, Vauban used more than 3.7 million cubic metres of masonry for retaining walls supporting the ramparts with their bastions at the corners of the star-shaped fortifications and the intermediate masonry walls, the curtain walls, (see (Poncelet, 1844, p. 67)), which corresponds to 41% of the total amount of masonry built.

As early as 1684, Vauban published design tables for retaining walls with heights of 3 m $< H <$ 25 m (Kérisel, 1985, p. 55). Three years later, Vauban, in his role as newly appointed Commissary General of all French fortifications, sent his engineers in the Corps du Génie Militaire his *Profil général pour les murs de soutènement* in which he presented his retaining wall profiles that were later adopted by engineering offers such as Bélidor (1729), Poncelet (1840) and Wheeler (1870) (see (Feld, 1928, p. 64ff.)). This "universal Vauban profile" (Poncelet, 1844, p. 4) was investigated by Poncelet, who compared this "main principle of Vauban's rules" (Poncelet, 1844, p. 68ff.) with the results of his earth pressure theory. Fig. 2.5, which shows the retaining walls for the fortifications at Ypres, conveys an impression of the Vauban profile, which Vauban drew in an entry in his diary for 1698 (see (Kérisel, 1985, p. 86)). The trapezoidal form of the retaining wall on the right of bastion 63 for the Ypres fortifications has the following dimensions: height $H = 11.38$ m, width at base $b = 3.52$ m, width at top

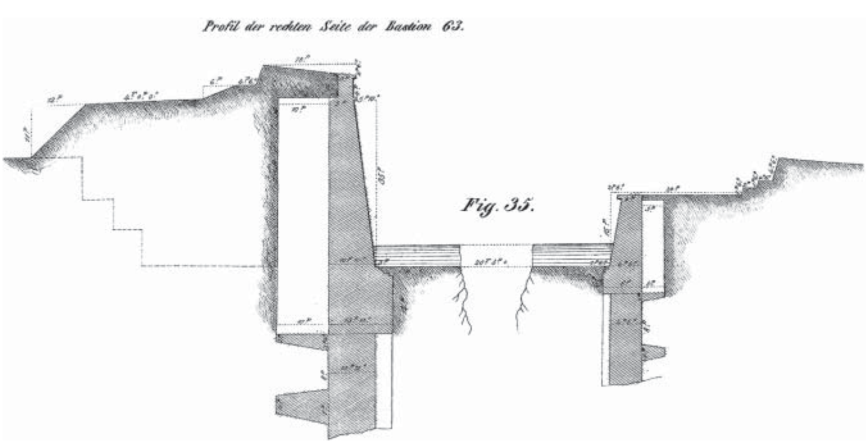

Fig. 2.5 Retaining wall with buttresses for the fortifications at Ypres designed by Vauban in 1699, after a drawing by A. de Caligny (Poncelet, 1844, plate IV, Fig. 35).

k = 1.62 m, batter of wall on air side m = (3.52-1.62)/11.38 = 1:6, average depth of soil covering to top of masonry h' = 0.5·(2.11+1.35) = 1.75 m. The retaining wall is stiffened by buttresses 16.90 m high every 4.87 m, which themselves have a trapezoidal cross-section with depth h = 3.25 m, width at base b_u = 2.60 m and width at top b_o = 1.30 m. The buttresses increase the stability enormously.

In a 1953 essay on the history of soil mechanics in France, Kérisel mentions a paper by M. Chauvelot which was presented to the Paris-based Académie des Sciences by Gaspard Monge (1746-1818) and Alexandre-Théophile Vandermonde (1735-1796) in 1783 and contains examples (with figures) for Vauban's design principles. For a retaining wall with buttresses at a spacing of 5.75 m and a batter m = 1:5 on the air side, he gives the following formula for the width of the base of the retaining wall:

$$b_{Vauban,1:5} = m \cdot H + k_{Vauban,1:5} = \frac{1}{5} \cdot H + 1.48 \tag{2.1}$$

Of course, Vauban based his formula of 1684 on the units of length used at that time, the *toise* (1 T = 1.95 m) and the *pied* (1 p = 0.325 m), which, when converted to the metric system, results in the Vauban formula of eq. (2.1) (see (Kérisel, 1985, p. 55)). In eq. (2.1), H is the depth of the earth backfill and $k_{Vauban,1:5}$ the width of the top of the retaining wall. Kérisel also published the table specified by Chauvelot (Kérisel, 1953, p. 153). Eq. (2.1) can be easily fitted to the retaining wall shown in Fig. 2.5:

$$b_{Vauban,1:6} = m \cdot H + k_{Vauban,1:6} = \frac{1}{6} \cdot H + 1.625 \tag{2.2}$$

which with H = 11.38 m results, according to eq. (2.2), in a base width of

$$b_{Vauban,1:6} = \frac{1}{6} \cdot 11.38 + 1.625 = 1.90 + 1.625 = 3.52 \text{ m.}$$

In the case of retaining walls with soil surcharge (see Fig. 2.6) and small buttresses, Vauban apparently proposed this formula:

$$b_{Vauban,surcharge} = \frac{1}{5} \cdot H + 1.625 \tag{2.3}$$

(see (Feld, 1928, p. 64)); again, this equation (like equations 2.1 and 2.2) has been converted to metric.

In Fig. 2.6, h' = C-G stands for the averaged depth of soil surcharge, H = C-B the height of the retaining wall, or soil backfill, $b_{Vauban,surcharge}$ = A-B the width at the base and A-C = 1.625 m the width at the top.

According to Audoy, Vauban based his retaining wall profiles on a factor of safety against overturning $\nu_{K,Vauban}$ = 3.8 and a factor of safety against sliding $\nu_{G,Vauban}$ = 4.7 (see (Feld, 1928, p. 65)). However, estimating the stability of Vauban's retaining wall (Fig. 2.5) according to the calculations in (Kurrer, 2018, pp. 55-58) and using the same soil mechanics parameters results in much lower factors of safety than those given by Audoy: at the base of the wall there is an overturning safety factor ν_K = 2.3, which is > $\nu_{permiss}$ = 1.5, and the sliding safety factor ν_G is nearly 1.6, again > $\nu_{permiss}$ = 1.5. If the buttresses are left out of the equation, the stability of the retaining wall against overturning is ν_K = 1.2 and sliding ν_G = 1.07, which are both just on the safe side.

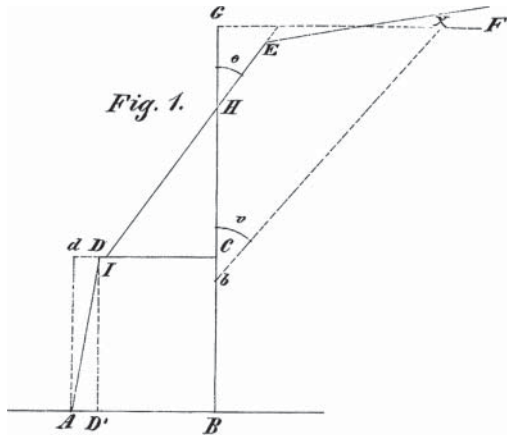

Fig. 2.6 Designations for retaining walls with soil surcharge after Poncelet (Poncelet, 1844, plate I, Fig. 1).

According to that, Vauban's retaining walls with their trapezoidal profile and buttresses cannot be further optimised structurally. Even Poncelet therefore assumed that Vauban's dimensioning rules – e.g. eqs. (2.1) to (2.3) – handed down to us do not represent empirical rules, instead can be attributed to "an exact geometric theory" (Poncelet, 1844, p. 4). Therefore, the Vauban profiles provided the structural/constructional reference for more than 150 years. And it was against this that earth pressure theories had to measure their modelling quality and practicability.

2.2 Earth pressure theory as an object of military engineering

More than 2,000 years ago, Vitruvius – for many years responsible for the building of military engines in the armies of Caesar and Augustus – investigated the phenomenon of earth pressure and how to deal with it in structural and constructional terms. In chapter V, "The City Walls", in Book I of his Ten Books on Architecture, Vitruvius writes about the walls between the towers, which require a "comb-like arrangement" of buttresses between them which are filled with earth (Fig. 2.7): "With this form of construction, the enormous burden of earth will be

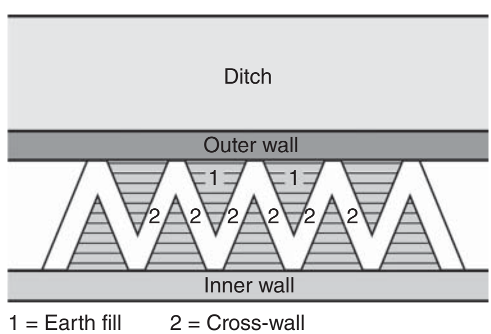

Fig. 2.7 Horizontal section through fortifications after Vitruvius (Vitruvius, 1981, Fig. 6).

distributed into small bodies, and will not lie with all its weight in one crushing mass so as to thrust out the substructures" (Vitruvius, 1981, p. 59).

In chapter VIII "On Foundations and Substructures" in Book VI, Vitruvius describes earth pressure not only in qualitative terms, but also tells us how to calculate the earth pressure for the retaining walls of Fig. 2.7: "Particular pains, too, must be taken with substructures, for here an endless amount of harm is usually done by the earth used as filling. This [earth fill] cannot always remain of the same weight that it usually has in summer, but in winter time it increases in weight and bulk by taking up a great deal of rain water, and then it bursts its enclosing walls and thrusts them out ... The following means must be taken to provide against such a defect. First, let the walls be given a thickness proportionate to the amount of filling" (Vitruvius, 1981, p. 297). Vitruvius then proposes rules for dimensioning the system of retaining walls and explains that "to meet the mass of earth, there should be saw-shaped constructions attached to the wall" and "with this arrangement, the teeth and diagonal structures will not allow the filling to thrust with all its force against the wall, but will check and distribute the pressure" (Vitruvius, 1981, p. 299). These quotes are the oldest known references to the nature and effect of earth pressure.

Like those involved with building had condensed the nature of masonry arch thrust into structural and constructional knowledge in the form of a structural theory in a lengthy historical process through their observations, own experiences during construction and many years of checking structures in use, so the knowledge of the phenomenon of earth pressure at the end of the 17th century culminated in Vauban's design theory for retaining walls. The beginnings of the changeover from empiricism to theory took place in masonry arches (see section 4.3.1) as it did in earthworks under the auspices of the Académie Royale d'Architecture de Paris (Kurrer, 2018, p. 212). Whereas La Hire proposed that the *règles de l'art* for the masonry arch problem be based on classical mechanics, Pierre Bullet (1639-1716) was the first (in 1691) to attempt to model physically and quantify earth pressure on retaining walls (Bullet, 1691, pp. 159-177). Both La Hire and Bullet were committed to the rationalism of René Descartes. It is therefore the classical rationalism of Descartes and Leibniz that formed their scientific theory and epistemological sounding board at the transition from the orientation phase (1575-1700) to the application phase (1700-1775) of theory of structures (Kurrer, 2018, pp. 15-16). The inductive structural theory ideas of Leonardo da Vinci and other engineers of the Renaissance was to be replaced by the deductive method (Polónyi, 1982), which to date shapes the way that this fundamental engineering science discipline sees itself. The difference between masonry arch theory and earth pressure theory in the application phase right up to the end of the constitution phase (1825-1850) of theory of structures (Kurrer, 2018, pp. 19-20) is that earth pressure theory is not the work of civil engineers, but essentially military engineers.

2.2.1 In the beginning there was the inclined plane

The first earth pressure theories were based on the model of the inclined plane (Fig. 2.8), which Stevin had cleverly used as long as go as 1586 for his equilibrium

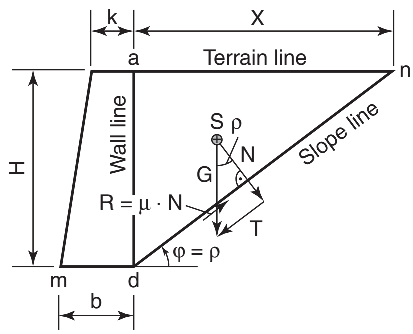

Fig. 2.8 Determining the earth pressure according to the fundamental model of the inclined plane.

observations (Kurrer, 2018, pp. 29-30). The starting point for these studies was the observation that when loose cohesionless materials are tipped out, they form a conical pile, the slant line of which forms a natural slope and the angle of the slope line with respect to the horizontal φ corresponds to the angle of internal friction ρ of this soil type. If further material is tipped out on top of this, it rolls downwards and in this case a retaining wall must be built upwards from point *d* to resist the descending material. This resistance was interpreted as earth pressure.

In the standard model of the first earth pressure theories, the wedge of soil bounded by slope line *d-n*, wall line *d-a* and terrain line *a-n* was considered as a rigid body with weight *G* which slides without friction parallel with the slope line. The components of *G* acting perpendicular *N* and parallel *T* to the slope line can be determined from the similarity between triangle *d-a-n* and the triangle of forces (Fig. 2.8):

$$\frac{N}{G} = \frac{x}{dn} \Rightarrow N = G \cdot \frac{x}{dn} = G \cdot \cos\varphi \tag{2.4}$$

$$\frac{T}{G} = \frac{H}{dn} \Rightarrow T = G \cdot \frac{H}{dn} = G \cdot \sin\varphi \tag{2.5}$$

The force *T* acting parallel with the slope according to eq. (2.5) functions as earth pressure *E* on the retaining wall. If, however, the slope line is affected by friction, then the earth pressure is reduced to

$$E = T - R. \tag{2.6}$$

In his *Mémoire de l'Académie Royale* of 19 December 1699, which described the design of waterwheels, Guillaume Amontons (1663-1705) realised that the friction force *R* is proportional to the normal force *N* and independent of the contact area. He assumed a value 1/3 for the proportionality factor μ (Amontons, 1699/1718). The fundamental model of the inclined plane modified to include friction force *R*

$$E = T - R = G \cdot \sin\varphi - \mu \cdot N = G \cdot (\sin\varphi - \mu \cdot \cos\varphi) \tag{2.7}$$

for earth pressure was already in use for finding dimensions for retaining walls in the first half of the 18th century by way of diverse simplifications. These earth pressure theory approaches differ in the first place in the figures assumed for the slope angle $\varphi = \rho$, the magnitude of the friction force and the definition of the point of application of *E*.

Bullet

Bullet modelled the cohesionless soil material, e.g. sand, as a regular pile with small, spherical particles with a theoretical slope angle φ = 60° (Fig. 2.9). For reasons of safety, his further studies were based on a slope angle φ = 45° (Fig. 2.10).

In the next step, Bullet determined the force at the inclined plane that prevents a particle of weight G' from rolling downwards:

$$E' = \frac{\sqrt{2}}{2} \cdot G' \approx \frac{5}{7} \cdot G'. \tag{2.8}$$

Of course, this relationship also applies to the entire earth pressure wedge with weight G (see Fig. 2.8):

$$E_{Bullet} = \frac{\sqrt{2}}{2} \cdot G \approx \frac{5}{7} \cdot G = \frac{5}{7} \cdot 0.5 \cdot \gamma_E \cdot H^2 = 0.35 \cdot \gamma_E \cdot H^2 \tag{2.9}$$

Eq. (2.9) can also be found from eq. (2.5) with $\varphi = \rho = 45°$. As an example, Bullet now calculated the area of the earth pressure wedge with leg lengths $x = 6$ *toisen* as $A_G = 0.5{\cdot}6{\cdot}6 = 18$ *square toisen*. As G is proportional to E_{Bullet}, then according to eq. (2.9), $A_E = (5/7){\cdot}18 = 13$ *square toisen* is valid for the "area of earth pressure". Where the earth and the masonry of the retaining wall have the same unit weight $\gamma_E = \gamma_{MW}$, Bullet can determine the wall's dimensions from the area A_E assumed by him to be equal to the cross-sectional area of the retaining wall A_S. Consequently, the width of the base of the retaining wall can be calculated from

$$b_{Bullet} = \frac{5}{7} \cdot H - k \tag{2.10}$$

where H is the height and k the width of the top of the retaining wall. Here, for $H = 6$ *toisen* (= 6·1.95 = 11.7 m) and $k = 10/6$ *toisen* (= 3.25 m), b_{Bullet} takes on a value of about 110/42 ≈ 16/6 = 2.66 *toisen* (= 5.20 m) (Fig. 2.11).

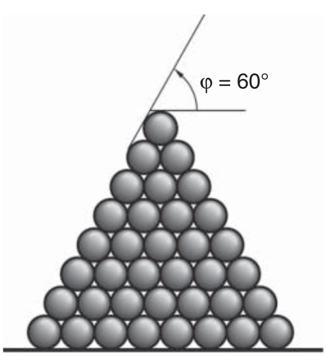

Fig. 2.9 Natural slope of small spherical grains of sand after Bullet (redrawn and modified after (Bullet, 1691, p. 171)).

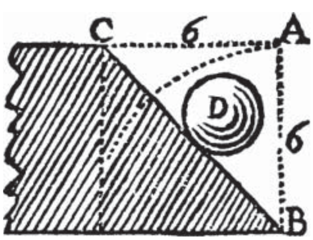

Fig. 2.10 Earth pressure determination after Bullet (Bullet, 1691, p. 172).

Fig. 2.11 Retaining wall design according to Bullet (Bullet, 1691, p. 173).

When determining the width of the base of a retaining wall, according to Emil Winkler (1835-1888), Bullet divided the "area of the earth pressure" A_E by the height H (Winkler, 1872, p. 59):

$$b_{\mathrm{Bullet,Winkler}} = \frac{1}{H} \cdot A_E = \frac{1}{H} \cdot \frac{5}{7} \cdot A_S = \frac{1}{H} \cdot \frac{5}{7} \cdot \frac{H^2}{2} = \frac{5}{14} \cdot H \approx 0.35 \cdot H \tag{2.11}$$

If $H = 6$ *toisen* is entered into eq. (2.11), then, according to Winkler, Bullet would have obtained a value of 2.14 *toisen* for the width of the base. Feld, too, specifies the same formula as Winkler (Feld, 1928, p. 65). From this it follows that both Winkler and Feld have either misunderstood these parts of Bullet's work or their misunderstanding is down to having adopted secondary sources without criticism.

Obtaining the dimensions of retaining walls using Bullet's method owes more to geometry than it does to statics, because he is only interested in the magnitude of the vectors of the earth pressure with the weight of the retaining wall and does not consider their point of application or direction at all.

Gautier

Hubert Gautier (1660-1737) worked with Vauban and was set to make his mark on French engineering in the early days of the Corps des Ingénieurs des Ponts et Chaussées, which was founded in 1716. Gautier became known for his monographs on roadbuilding (1693) and bridge-building (1716), which progressed to become the number one textbooks for modern civil engineering and remained so for a number of decades. He was an inspector of roads and bridges from 1713 to 1731 and therefore was also involved in solving earthworks problems such as those that occur when laying out routes for roads. We have Gautier to thank for the first figures regarding the most important soil parameters. He measured a unit weight of 18.1 kN/m^3 and a slope angle of 31° for dry, clean sand; the corresponding values for customary, loosened earth fill were, according to Gautier, 13.4 kN/m^3 and 45° (Gautier, 1717, pp. 37-51). Although Gautier based his dimensions for retaining walls on geometric rules or rules of proportion, his measurement of these two soil parameters laid the foundation for the evolution of a theory of earth pressure.

Couplet

In his first *Mémoire de l'Académie Royale* on earth pressure, Couplet criticised Bullet's assumptions (Couplet, 1726/1728):

- The assumed slope angle of 60° is incorrect (see Fig. 2.9).
- The pile of spherical particles is not two-dimensional (see Fig. 2.9), but three-dimensional (see Fig. 2.12).
- The slope line *d-n* cannot be understood as an inclined plane down which the wedge of soil *a-d-n* slides (see Fig. 2.8).
- The factor 5/7 in Bullet's earth pressure equation (2.9) is incorrect because earth pressure *E* does not act horizontally.

Couplet assumed a configuration of frictionless spherical particles in the shape of a tetrahedron (Fig. 2.12), with every sphere making contact with three others and transferring to those three the compressive forces acting perpendicular to the areas of contact. From this, Couplet initially derived a fictitious slope line *L-K* (Fig. 2.13). So, the sphere on the outside does not roll down *C-B*, but down *L-K*. Couplet showed further that his frictionless theory requires a constant horizontal pressure acting on the smooth wall line which is independent of the slope line angle and proportional to $0.5 \cdot H^2$. Taking the elementary tetrahedron with side length $2 \cdot \sqrt{3}$ and triangle *A-I-D* (Fig. 2.12), Couplet found that the ratio of earth pressure *E* to the weight of the sliding wedge of soil *G* was $2 : \sqrt{8}$, i.e. the triangle

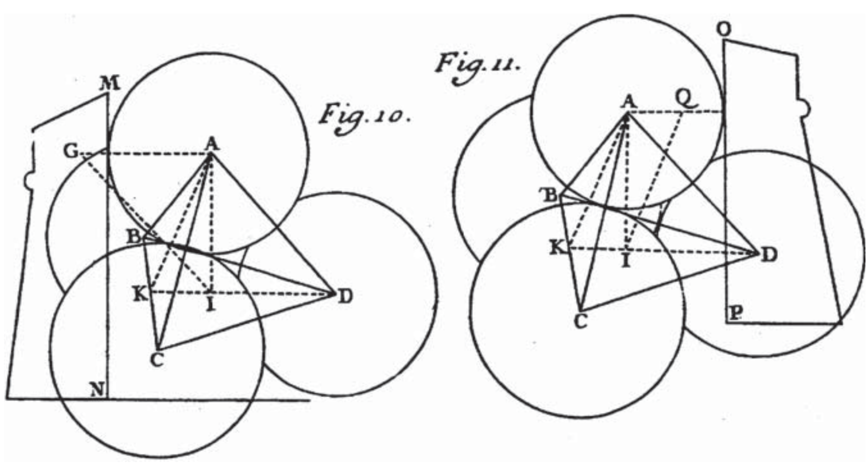

Fig. 2.12 Pile of spherical particles in the form of a tetrahedron after Couplet (Couplet, 1726/1728, plate 4, Figs. 10 & 11).