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Thorsten Hens
Marc Oliver Rieger

Solutions to *Financial Economics*

Exercises on Classical and Behavioral Finance

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Preface

Writing a textbook takes time and effort. Everybody knows that. But designing exercises is by no means an easier task as we have noticed when composing this book. Good exercises are challenging and enlightening and even fun to solve! It's not easy to find them. We hope that we were able to find such exercises for this book, if not always, at least sometimes. A few times, however, others have found so wonderful exercises for a topic that it is difficult to find something better and we have therefore—with permission of the authors—reprinted classical exercises from other books.

The structure of the book is self-explanatory: in the first part you find exercises and in the second part solutions. The chapter numbers follow the chapters of our textbook *Financial Economics* to make it easy to navigate. We hope you will learn a lot from these exercises, we also hope that you like them and we appreciate the effort you take studying them!

We thank Marie Hardelauf for the typing and the layout of the book, Artem Dyachenko for his input to some of the exercises, and Anastasiia Sokko, Nilüfer Schindler, Urs Schweri and Sabine Elmiger for their general help with composing this book. We thank Biljana Meiske for recomputing and improving many exercises. Finally, we thank Philipp Baun and his team from the Springer publishing house for their immense patience with us and their steady support.

Zürich, Switzerland — Thorsten Hens
Trier, Germany — Marc Oliver Rieger
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Part I

Exercises

1 Introduction

In this part, we present a large number of exercises that can accompany our book *Financial Economics*. They are sorted by the chapters of the book. Within each chapter, the exercises are roughly sorted by topics such that topics covered earlier in the chapter come first. A second criterion is by the difficulty (the easier exercises first). Many exercises are far from being "routine". We think that exercises that just plug in numbers into formulas being learned by heart do not help students much to comprehend a topic. They also don't help the lecturer when designing exams or homework assignments: such simple "plug-and-play" exercises are easier designed from scratch then copied from a textbook. Instead, we tried to design exercises that inspire thinking and encourage deeper understanding of the subject. Often there is not only one solution and sometimes students who do not find the optimal solution can at least try to get partial or approximative results. Teasing out the creativity of students in solving problems is very helpful for guiding them into making their own research and this is what we aim to achieve with our exercises.

That does not mean that our exercises are all very difficult and only solvable for top students. To the contrary, we hope that most students who put enough thought and effort into them will be able to solve them—at least partially.

Finally, we hope that the exercises do not only encourage thinking and help to understand the topics of our book, but also are interesting—and sometimes even fun—to solve!

Complete solutions to all exercises are given in the second part of this book.

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2 Decision Theory

2.1. Consider the following game: you roll a dice, if you roll a 6, you win 6 million € otherwise you win nothing. You can play only once. Let us assume your expected utility function is given by $u(x) = \log_{10} x$ (base 10 logarithm, i.e., $\log_{10}(10^n) = n$) and your initial wealth is 10,000 €.

(a) How big is your expected utility after playing this game? Imagine instead that you get 1 million € for sure, how big is your utility afterwards? Which of the two variants would you therefore prefer? How could you have seen this without doing any computation?
(b) Now, the prize of the game is only 61 €. What would be the certainty equivalent of the game, given the same expected utility function as above? Should you participate for a fee of 10 €? Why is the result surprising?

2.2. Prove that the Expected Utility Theory (EUT) satisfies the Continuity Axiom.

2.3. In a city center, parking space is rare. Hence, legal parking costs an amount of $t > 0$. Some people decide to park illegally. There is a probability $p > 0$ of being caught which leads to a fine $f > t$. In order to decrease the number of illegal parkers, there are two possible concepts: doubling the fine f or doubling the controls (i.e., the probability p). Assuming that the illegal parkers are risk-averse, which is the better concept?

2.4. Consider two assets: a stock and a bond. There are two states of the world (each with probability $1/2$): boom and recession. The stock's returns are $+8\%$ in a boom and -2% in a recession, the bond yields $+2\%$ each.

(a) Compute their mean and variance.
(b) Find the value of α such that an investor with the mean-variance utility function $U(\mu, \sigma) = \mu - \alpha\sigma^2$ is indifferent between both assets.

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(c) If this investor buys some stocks (say a proportion $\lambda \in [0, 1]$ of his total investment) and some bonds (a proportion of $1 - \lambda$), how will his returns be distributed? Which $\lambda \in [0, 1]$ is optimal for him?

2.5. Daniel Bernoulli, one of the founders of expected utility theory, and Daniel Kahneman, one of the founders of Prospect Theory, go on vacation. They each have two credit cards and two wallets. With a certain probability a wallet could be stolen. The probability that a particular wallet is stolen is independent from the probability that another wallet is stolen.

Assume that both act according to their theories. Would they put both credit cards into the same wallet or each in a different wallet? [Hint: Assume an identity probability weighing function, i.e., $w(p) = p$.]

2.6. Can the standard form of PT with the standard PT-parameters explain that people play a lottery if the winning probability is 1:1,000,000, the prize is one million € and a lottery ticket costs 2 €?

2.7. Show that Cumulative Prospect Theory explains Allais' Paradox. To this aim, compute the CPT values of the four lotteries and compare!

Lottery A	State	1–33	34–99	100
	Outcome	2500	2400	0
Lottery B	State	1–100		
	Outcome	2400		
Lottery C	State	1–33	34–100	
	Outcome	2500	0	
Lottery D	State	1–33	34–99	100
	Outcome	2400	0	2400

2.8. Can the certainty equivalent of a lottery in Prospect Theory (PT) be larger than the largest outcome of the lottery? How is it in Cumulative Prospect Theory (CPT)? How is it in the normalized version of Prospect Theory (by Karmarkar)? Give an example or proof! (This property is called "violation of internality".)

2.9. We say that a person is loss averse if he does not like to participate in a lottery with 50% chance of winning X and 50% chance of losing X. Let us assume a person's decisions are described by classical prospect theory with parameters $\alpha < \beta$. For simplicity, assume $X = 100$, $\alpha = 0.8$, $\beta = 1$, i.e. risk neutrality in losses, and $\gamma = 1$, i.e. no probability weighting.

Compute the values of λ for which the person is loss averse! Show that for any $\alpha < \beta < 1$ the person can be loss averse for some $\lambda < 1$!

2.10. Let us assume that a value function v is given by $v(x) := x$ and a weighting function w is $w(F) := \sqrt{F}$. A lottery is described by the probability measure

$P := p(x)\,\mathrm{d}x$, where the probability density p is given as

$$p(x) := \begin{cases} x, & \text{if } 0 \le x < 1, \\ 2 - x, & \text{if } 1 \le x < 2, \\ 0, & \text{otherwise.} \end{cases}$$

Compute the CPT-value of this lottery. Use this to compute the certainty equivalent (CE). Explain the difference between the CE and the expected value!

2.11. Jerome is a student. If you ask him whether he prefers 100 € now or 110 € next week, he prefers to get the money now. If you ask him, however, whether he prefers 100 € now or 200 € in 4 months, he prefers to wait. Can you explain these preferences with classical time discounting? Can you explain it with hyperbolic discounting? Assume linear utility e.g. $U(w) = w$.

Angelika is a student. If you ask her whether she prefers 100 € now or 120 € next year, she prefers to wait. If you ask her, however, whether she prefers 100 € now or 1000 € in 10 years she prefers the money now. Can you explain these preferences with classical time discounting? Can you explain it with hyperbolic discounting? What if you consider an increase in her wealth level in 10 years? [Hint: assume a non-linear utility function e.g. $U(w) = \min(w, c)$ where c is a constant.]

2.12 (Samuelson Paradox). We all know that we can take more risk in our investment decisions when we have a longer investment horizon—do we? Consider the following counter argument by Paul Samuelson: let us suppose you are not willing to play a certain gamble only once, but you are willing to accept the offer to play it ten times. Now, after playing it nine times, why don't you want to stop here? After all, past is past, and at this point you just have to decide to play this gamble *once* (more) or not and you preferred in this case not to play it, didn't you? So, you would rather only play nine times. But then of course the same argument could be iterated and you would finally not play the gamble at all. Now replace "gamble" by "investing in the stock market for one year" and you have just disproved that you should be willing to take more risk on the long run.

On the other hand, if you choose a utility function, say, $u(x) = x^{\alpha}$, you can construct a lottery L such that the utility of this lottery is lower than the utility of not playing, but the utility of playing the lottery twice (or ten times) is larger than not playing. (Construct such a lottery as an exercise!) So this tells you that, yes, indeed a rational person might want to take more risk on the long run.

Now we have two nice proofs that contradict each other, a situation we tend to call a paradox.

> How wonderful that we have met with a paradox. Now we have some hope of making progress

we could say in the words of Niels Bohr. But how do you solve this paradox?

2.13. There are three assets. Their payoffs are as follows:

Probability	Asset 1	Asset 2	Asset 3
50%	5%	5%	– 5%
50%	10%	−5%	−10%

Assume $R_f = 0\%$.

(a) Calculate the certainty equivalent of each asset, if the investor has the utility function

$$v_{KT}(R) = \begin{cases} (R - RP)^{\alpha^+}, & \text{if } R > RP, \\ -\beta\,(RP - R)^{\alpha^-}, & \text{if } R \le RP, \end{cases}$$

with $\alpha^+ = \alpha^- = 0.88$, $\beta = 2.25$ and $RP = 0\%$. The values for the alphas and beta are the classical results of Kahneman and Twersky.

(b) Find the parameters of

$$v_{DHM}(R) = \begin{cases} (R - RP) - \frac{\alpha^+}{2}\,(R - RP)^2, & \text{if } R > RP, \\ \beta\left((R - RP) - \frac{\alpha^-}{2}\,(R - RP)^2\right), & \text{if } R \le RP, \end{cases}$$

such that the three assets have the same certainty equivalent as in the v_{KT}-case (assume $RP = 0\%$ also in v_{DHM}).

2.14. In a market we have two states, a risky and a risk-free asset. State 1 occurs with a probability $p = 75\%$. The risky asset pays 7% in state 1 and −10% in state 2. The risk-free asset pays 1% for sure.

(a) Determine the optimal portfolio of a mean-variance maximizer with $\gamma = 2$.

(b) Determine the optimal portfolio of a prospect theory maximizer with

$$v(R) = \begin{cases} (R - RP) - \frac{\alpha^+}{2}\,(R - RP)^2, & \text{if } R > RP, \\ \beta\left((R - RP) - \frac{\alpha^-}{2}\,(R - RP)^2\right), & \text{if } R \le RP, \end{cases}$$

with $\alpha^+ = -\alpha^- = 1.5$, $\beta = 2$ and $RP = R_f$. Assume that the agent is not doing probability weighting. Assume there are short selling constraints i.e. $\lambda \in [0, 1]$.

(c) Assume the stock has now a return of −2% in state 1 and 16% in state 2. Determine the optimal portfolio of the same investor but he weights the probabilities as follows: $w(0.25) = 0.3$ and $w(0.75) = 0.7$.

2.15.

(a) Suppose you have a 50% chance to double your yearly income. Which percentage of your yearly income are you willing to risk in the other 50% of the cases?

(b) Assume you have an expected utility function with constant relative risk aversion. Compute the size of your CRRA.

(c) Compute your risk aversion in the mean-variance approach. The utility function is $U(R) = \mu(R) - \gamma/2\,\sigma(R)^2$.

(d) Which percentage of income would the following representative prospect theory agent of Kahneman and Tversky with

$$v(R) = \begin{cases} (R - RP)^{\alpha^+}, & \text{if } R > RP, \\ -\beta\,(RP - R)^{\alpha^-}, & \text{if } R \leq RP, \end{cases}$$

$\alpha^+ = \alpha^- = 0.88$, $\beta = 2.25$, $RP = 0\%$ and without probability weighting risk in that situation?

(e) Find the risk aversion of a mean-variance investor such that he would split his wealth equally between stocks and bonds. To do so recall that the equity premium is 6.4% and the standard deviation of stocks is 21%.

(f) Now suppose you have some background wealth which is 50% of your yearly income. Take the percentage of answer (a) and find your CRRA for this case.

3 Two-Period Model: Mean-Variance Approach

3.1. There are two risky assets, $k = 1, 2$ and one risk-free asset with return of 2%. Risky assets cannot be short sold. The expected returns of the risky assets are $\mu_1 := 5\%$ and $\mu_2 := 7.5\%$. The covariance matrix is:

$$COV := \begin{pmatrix} 2\% & -1\% \\ -1\% & 4\% \end{pmatrix}.$$

(a) Calculate the Minimum-Variance Portfolio and the Tangent Portfolio.
(b) Some mean-variance investor assuming the Covariance Matrix given above chooses the portfolio $\lambda := (0.2, 0.5, 0.3)$. Assume that investor's risk aversion is $\rho := 1$. Which implicit expected returns does he hold?
(c) Suppose the market portfolio is $\lambda^M := (0.4, 0.6)$. Compute the beta-factors. Assume the excess return of the market portfolio is 3%. Determine the expected returns of the two risky assets.

3.2. Download the data from the homepage of this book (you find it at http://www.financial-economics.de). The datafile contains the price levels of equity indices of several countries as well as the world risk-free interest rate, which represents the monthly returns of the risk-free asset. Prepare the country index data by calculating the monthly net returns first.

(a) Compute the mean and standard-deviation of the monthly net-returns of the indices given. Determine the corresponding annual returns. Is there differences in the risk-return tradeoff across different country equity markets? Interpret briefly.
(b) Plot the histograms of the monthly returns. Explain the distributional properties of the country returns such as possible fat tails or skewness.

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(c) Compute the covariance/correlation matrix. What country indices are highly correlated? Are there any negative correlations?
(d) Draw the efficient frontier for the asset universe given in the data set. Show both the unconstrained and the constrained case, where no short sales are allowed.
(e) Compute the tangent-portfolio with and without short-sales. Are they different? Explain why.
(f) Illustrate the sensitivity of the tangent portfolio with respect to changes in means by drawing different frontiers (unconstrained).
(g) Compute the SML based on the tangent-portfolio with short-sales. Show that all the assets are on the SML graphically.

3.3. Construct a simple example with three risky assets $k = 1, 2, 3$ such that none of them have a pairwise correlation of $+1$ or -1, but a combination of asset 1 and 2 has a correlation of $+1$ with asset 3.

Use this example to prove that the problem described in Sect. 3.1.4 of the book cannot be solved by controlling for pairwise correlation between assets.

3.4. An investor with mean-variance utility $U(\mu, \sigma) := \mu - \sigma^2$ can invest in three risky assets, $k = 1, 2, 3$ and one risk-free asset. The risk-free return is 2%. Risky assets cannot be short sold. The expected returns of the risky assets are $\mu_1 := 5\%$, $\mu_2 := 7.5\%$ and $\mu_3 := 10\%$ The covariance matrix is:

$$COV := \begin{pmatrix} 2\% & -1\% & -2\% \\ -1\% & 4\% & 6\% \\ -2\% & 6\% & 8\% \end{pmatrix}.$$

(a) Calculate the optimal portfolio λ^{opt} of the investor if he can only invest in the first two assets. Calculate the mean and the variance and also the investor's utility for that portfolio.
(b) Now consider the third asset and show that it has positive Alpha with respect to the optimal portfolio λ^{opt}. Suggest a new portfolio mix consisting of the tangential portfolio and the third asset so that the investor improves upon the tangential portfolio.
(c) Now suppose the investor had initially chosen the portfolio consisting of asset 2 only. Show that adding asset three to this portfolio worsens his situation!

3.5. Suppose that financial markets consist of two risky assets and one riskless asset. Let $R_f = 1\%$ and there be four investors each of whom has different beliefs for the expected returns of the two risky assets as follows:

$$\mu^1 = \begin{pmatrix} 6\% \\ 1\% \end{pmatrix}, \quad \mu^2 = \begin{pmatrix} 3\% \\ 2\% \end{pmatrix}, \quad \mu^3 = \begin{pmatrix} 2\% \\ 3\% \end{pmatrix}, \text{ and } \mu^4 = \begin{pmatrix} 1\% \\ 5\% \end{pmatrix}.$$

All investors have the same degree of risk aversion in the mean-variance preferences, $\rho^1 = \rho^2 = \rho^3 = \rho^4 = 2$, and their invested wealth levels are all the same as well $w_0^1 = w_0^2 = w_0^3 = w_0^4 = 10$. The variance-covariance matrix and the true expected returns of the risky assets are given by

$$COV = \begin{pmatrix} 2\% & 0\% \\ 0\% & 2\% \end{pmatrix} \text{ and } \hat{\mu} = \begin{pmatrix} 2\% \\ 2\% \end{pmatrix}.$$

(a) Find which of the investors should be actively investing and which should rather not.
(b) Show that investor 2 has a negative alpha portfolio but should rather be active.
(c) Show that investor 4 has a positive alpha portfolio but should rather be passive.

3.6. "Equities, bonds and other traditional asset classes have an economic rationale for giving positive mean returns. Hedge funds have no economic theory underlying their positive performance. There is no risk premium in the classic economic sense. The returns are achieved by the managers' ability to exploit inefficiencies left behind by other (less informed, less intelligent, less savvy, ignorant, or uneconomically motivated) investors in what is largely considered a zero or negative sum game."
Alexander M. Ineichen (UBS Investment Research, March 2005, page 31.)
In the following we analyze this statement critically:
Consider a two-period financial market model with $k = 0, 1, \ldots, K$ assets and heterogeneous beliefs. Let $k = 0$ be the risk-free asset.

(a) For the CAPM, define the ex-post alpha of an asset k, $\widehat{\alpha}_{k,M}$, and of an investment strategy $\lambda^i = \left(\lambda^{i1}, \ldots, \lambda^{iK}\right)$, denoted by $\widehat{\alpha}^i$.
(b) Let r^i be the relative wealth of investment strategy λ^i. Argue that $\sum_i \widehat{\alpha}^i r^i = 0$, i.e., with respect to the alphas financial markets are a zero sum game.
(c) In the last 10 years Hedge Funds have generated positive returns of about 10% p.a. Is this finding compatible with the CAPM?
(d) Comment on the quotation from Ineichen given above. Are his statements supported by financial economics as it has been taught in this class?

3.7. Let $R_f := 1\%$ and let there be two risky assets and three investors with the following characteristics:

$$\mu^1 := \begin{pmatrix} 3\% \\ 1\% \end{pmatrix}, \quad \mu^2 := \begin{pmatrix} 2\% \\ 1\% \end{pmatrix}, \quad \mu^3 := \begin{pmatrix} 1\% \\ 2\% \end{pmatrix},$$

$$\gamma^1 := \gamma^2 := \gamma^3 := 2,$$

$$w_0^1 := w_0^2 := w_0^3 := 5.$$

Let

$$\mathrm{cov} := \begin{pmatrix} 2\% & 0\% \\ 0\% & 2\% \end{pmatrix} \text{ and } \hat{\mu} = \begin{pmatrix} 2\% \\ 1\% \end{pmatrix}.$$

(a) Calculate the (ex-ante) market expectation $\overline{\mu}$.
(b) Calculate the optimal portfolio for all investors (if they are active).
(c) Calculate the market portfolio λ_M assuming that all investors are active.
(d) Which investors should invest active, which passive?
(e) Calculate the ex-post alphas of the investors.
(f) Show that investor 1 has a positive ex-post alpha, if he is active, but should better be passive.

3.8. Consider two risky assets with

$$(\mu_1, \sigma_1) := (5\%, 5\%) \qquad \text{and}$$
$$(\mu_2, \sigma_2) := (10\%, 10\%).$$

The correlation between the two assets is $\rho = 0.5$.

(a) Compute the tangent portfolio for $R_f = 0\%$ with and without short-selling.
(b) How does the tangent portfolio change when R_f increases?

4 Two-Period Model: State-Preference Approach

4.1. Consider a two-period economy with uncertainty in the second period. Consumption is in terms of a single consumer good. In the second period there are **S** many possible states and every consumer aims to maximize the consumption across states. There are **I** many consumers with utility functions $\mathbf{U}^i$ (strictly increasing, concave and continuous). The consumption good has a price π_s in each state. Consider the following market structures:

(A) Arrow Debreu Equilibrium: pure exchange, consumption c_s^i, endowment ω_s^i with prices π_s.

(B) Financial Markets Equilibrium: consumption c_s^i, endowment ω_s^i with an economy where exchange is done solely via financial markets. There are $\mathbf{K}+1$ many financial assets in the economy with period 1 prices q^k and each asset has **S** many possible payoff scenarios A_s^k in the next period and agents transfer their consumption via asset allocations $\theta^{i,k}$.

(a) State the budget constraints of the consumers in both economies A and B.
(b) State the market equilibrium for both economies A and B.
(c) Assume that the financial markets are complete and show that the equilibrium of the economies A and B will yield the same consumption allocations.

4.2. Suppose we have financial markets with the following payoff matrix and market equilibrium prices

$$A = \begin{pmatrix} 1 & 4 \\ 1 & 2 \\ 1 & 0 \end{pmatrix} \qquad q = (1, 2)$$

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Introduce a new asset with a payoff (2 0 0). This can be a call option written on the second asset with a strike price of 2. Find the price or pricing bounds using the no-arbitrage condition.

4.3. Suppose we have a financial market with the following payoff matrix of two assets in two states:

$$A = \begin{pmatrix} 1 & 3 \\ 1 & 1 \end{pmatrix}$$

(a) Find which of the following set of prices are arbitrage-free or not by trying to form arbitrage strategies:
  (i) $q' = [1,\ 2]$
  (ii) $q' = [1,\ 1/2]$
  (iii) $q' = [1,\ 1]$

(b) Find which of the following set of prices are arbitrage free or not by calculating the state prices implied by *FTAP*, conclude which set of state prices are positive for which set of asset prices.
  (i) $q' = [1,\ 2]$
  (ii) $q' = [1,\ 1/2]$
  (iii) $q' = [1,\ 1]$

(c) Consider the asset prices that are arbitrage-free and introduce a call option which has a strike price of 1.5 on the second asset in the economy.
  (i) State the payoff of the call option.
  (ii) Is it possible to replicate the payoff of the call option?
  (iii) Calculate the price of the call option by considering the arbitrage free prices given in the previous parts.

4.4. Consider an economy with two agents, two equally likely states $\Rightarrow S = 2$, each agent has the expected utility of the form:

$$U^1(c) = U^2(c) = \ln(c_0) + \frac{1}{2}(\ln(c_1) + \ln(c_2))$$

and endowments

$$w^1 = (1, 1, 2),$$
$$w^2 = (1, 2, 1).$$

In financial markets, suppose we have a single bond paying one in both states.

Then the payoff matrix is $A' = [1\ 1]$

(a) Is the market of this economy complete?

(b) Derive the financial markets equilibrium asset and consumption allocations and asset prices. Show whether there is trade among consumers.

(c) Is there a way to complete this market? What kind of payoff vector of an asset would complete the market? Give an example.
(d) Consider a new asset with the payoff vector $[x, \quad y] \in \mathbb{R}^2$ where $x \neq y$, where $x = 2$ and $y = 1/2$. Can you find the price of this asset? Otherwise, find the valuation bounds for this asset.
(e) Introduce a new asset to the market with the payoff vector $[x, \quad y] \in \mathbb{R}^2$ where $x \neq y$. Now that the market is complete, derive the equilibrium allocations and prices.
(f) Find the first asset's equilibrium price in the new financial market economy and compare whether the introduction of the new asset changes the existing asset's equilibrium price.
(g) Suppose the payoff vector of the second asset as it given in part (d). $[x, \quad y] \in \mathbb{R}^2$ where $x \neq y$, where $x = 2$ and $y = 1/2$. Find the equilibrium price of the asset in the new financial markets equilibrium. Compare if the new asset belong to the valuation bounds you found in part (d).
(h) With the introduction of the second asset, did the asset allocations of the consumers change?

4.5. Suppose we have financial markets with a payoff matrix

$$\begin{array}{cc} \text{Bond} & \text{Stock} \end{array}$$
$$\begin{pmatrix} 1 & 3 \\ 1 & 1 \\ 1 & 1 \end{pmatrix}$$

(a) Introduce a call option to this market with $K = 1$

$$\text{Payoff} = \max(0, S - K) \Rightarrow \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

What happens to the market? Can we price the call option with bond and stock? Is it a redundant asset?
(b) Assume now with the new asset that we have three assets in the markets as in part (a). Introduce a second call option with a different strike price $K = 2$.

$$\text{Payoff} = \max(0, S - K) \Rightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Show whether the second option becomes redundant and if we can replicate the option payoffs.

(c) Introduce a put option (as a second option) $K = 1$ in the markets with 3 assets, 1 bond, 1 stock and 1 call option.

$$\text{Payoff} = \max(0, K - S) \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

How can we price this asset? Derive the put-call parity by using the replicating strategy method.

4.6. Consider an economy with two consumers, two time periods, two equally likely future states and financial markets with 2 assets. Each agent has the following expected utility function form

$$U^i(\mathbf{c}) = \ln(c_0^i) + \frac{1}{2}\ln(c_1^i) + \frac{1}{2}\ln(c_2^i)$$

and initial endowments $\omega^1 = (1,\ 1,\ 2)$ and $\omega^2 = (1,\ 2,\ 1)$. The financial markets have two assets in zero net supply with the following payoff matrix:

$$\begin{pmatrix} 1 & 1/2 \\ 1 & 2 \end{pmatrix}$$

(a) Check whether the markets are complete.
(b) Define the financial market equilibrium for this economy.
(c) Define the Arrow Debreu equilibrium for this economy.
(d) Find the equilibrium state prices.
(e) Find the financial assets' prices by using the state prices.
(f) Find the optimal consumption allocations for both agents.
(g) To derive the optimal asset allocations, use the budget constraints of the financial markets equilibrium as both equilibrium solution would yield the same consumption allocation. We could use the asset allocations that would achieve the desired optimal consumption allocation we derived in the Arrow Debreu equilibrium.
(h) Is there any wealth transfer across time? Explain why.
(i) Is there wealth transfer across states? Explain why for each agent in detail.
(j) In the economy, we introduce a new asset called a derivative. It is a call option written on the risky asset in the economy with the second period exercise price $K = 1.1$. What would be the price of this option. . .
   (i) by using the hedge portfolio method, by deriving the amount to invest in both assets riskless and risky?
   (ii) by using the equilibrium state prices?
   (iii) by using the risk neutral pricing formula?