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Jorge Luis González-Velázquez

Mechanical Behavior and Fracture of Engineering Materials

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Jorge Luis González-Velázquez

Mechanical Behavior and Fracture of Engineering Materials

Springer

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Preface

Mechanical behavior is a discipline that studies the performance of materials under the action of external forces, such as load, impact, bending, or pressure. Basically, it is a part of physics aimed at the study of the stress-strain phenomena of solid bodies. For the engineer, the knowledge of mechanical behavior is fundamental as it provides the basics to analyze and understand the reaction of physical components under the action of forces. The components specifically intended for bear, transmit, or resist forces are called mechanical or structural, for example: beams, shafts, piping, pressure vessels, gears, and etcetera; therefore, they have to be designed, operated, and maintained to fulfill this requirement, and the Mechanical Behavior discipline provides the basis for these tasks in addition to characterize the mechanical properties of the materials used for the fabrication and construction. Even components that are not intended to support loads as their primary function must be designed to resist the action of eventual o secondary forces, such as dead weight, impacts, wind, earthquakes, and etcetera. For example, a utility cable may have the primary function of conducting electricity, but it has to resist the weight of ice and snow, the wind pressure, and the weight of hanging objects during service.

This book describes the fundamentals of the stress and strain theory introduced by Cauchŷ and others in the nineteenth century, as well as the mechanisms of deformation, strengthening and fracture, applied to materials commonly used in engineering, with emphasis on metals and alloys, but also entails polymers and composite materials. Chapters 1 and 2 have to do with the definition of stress and strain under the assumption that the solid body is continuous, homogeneous, and isotropic; hence, this subject is known as *Continuum Mechanics*. In addition, the method for stress transformation, known as Mohr's Circle, the most yield criteria and the elastic stress-strain relations are described, with emphasis on its practical use in engineering. A brief description of the finite element method to perform the numerical analysis of stress and strain and the experimental methods to measure them are included and at the end of Chap. 2, and finally the hardness test is briefly described.

Chapter 3 describes the plastic strain mechanisms, primarily by dislocation slip, aimed to understand the strengthening mechanisms of metals and alloys discussed in Chap. 4. A section describing the basics of transmission electron microscopy as the most used method for direct observation of dislocations is included in Chap. 3. Since both plastic strain and strengthening mechanisms depend on the microstructure, these chapters introduce basic concepts of metallurgy and heat treatment as the means to control microstructure, and thus the mechanical properties. The mechanical behavior of polymer and composite materials is described in Chap. 5.

The fracture phenomenon is studied in Chap. 6, presenting the basic concepts of fracture along with a description of the mechanisms of brittle and ductile fracture, followed by a comprehensive introduction to fracture mechanics and completed with an introduction to structural integrity, as the in-field application of fracture mechanics. The chapter concludes with a description of the Charpy impact test. Chapter 7 entirely focused on fatigue, due to its importance as fracture mechanism of in-service mechanical components, the interpretation of the S-N curve and the fatigue life prediction methods are explained. The chapter concludes with a brief introduction to fatigue crack growth.

Finally, Chap. 8 deals with high-temperature behavior, specifically the creep phenomenon. The Larson-Miller method to determine creep life is described, as well as the creep mechanisms of dislocation climb and grain boundary sliding. The chapter concludes with the application of the phenomenological creep equations to the design of high-temperature resistant materials.

Mexico City, México Jorge Luis González-Velázquez
April 2019

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About the Author

Jorge Luis González-Velázquez has been Professor of the Metallurgy and Material Department at the Superior School Chemical Engineering and Extractive Industries of the Instituto Politecnico Nacional (National Polytechnic Institute) of Mexico since 1990. He holds a B.S. degree in metallurgical engineering and a M.S. degree in metallurgy conferred by the same institution. He obtained a Ph.D. from The University of Connecticut, U.S.A. in 1989, working on fatigue crack growth under the supervision of Prof. Arthur J. McEvily. Later on, he joined the IPN and started teaching the subject of Physical Metallurgy, which in 2010 was divided into two courses: Mechanical Behavior and Fracture. He also started the courses of Fracture Mechanics and Fractography at graduate level, for the first time in México.

Alongside his teaching activity, Dr. Gonzalez has worked over the last 30 years on research and technological development in the fields of mechanical behavior, fracture mechanics, physical metallurgy, and structural integrity. He is author of over 340 technical publications and has been director of more than 80 theses at undergraduate and graduate levels. He also has conducted over 180 lectures in international conferences and meetings. He is author of the books titled "Fracture Mechanics" and "Mechanical Metallurgy", both of them published in spanish by Editorial Limusa, Mexico, and "Fractography and Failure Analysis", published by Springer Nature.

Chapter 1
Stress

Abstract A brief introduction of the field of Mechanical Behavior and Engineering Materials is given at the beginning of this chapter, where the importance of this field of study is emphasized, followed by the scope of continuum mechanics study and the definition of stress. By using these ideas, the mechanical behavior in uniaxial tension and the design of structural components is described. A comprehensive description of the Cauchŷ's stress tensor is provided along with a simplified procedure to determine the state of stress, illustrated with practical examples, to continue with a full description of the stress transformation methods, both by matrix algebra and the Mohr's Circle methods. The description of the Tresca and Von Mises yield criteria is given, including solved problems and the use of two-dimension yield maps. The chapter finalizes with a brief introduction to the stress concentration phenomenon.

1.1 Mechanical Behavior and Engineering Materials

Mechanical Behavior refers to the study of the relation among the loads that act upon a solid body and the internal forces and strains produced as a result of such action. It is "mechanical" because it deals with the analysis of forces and their reactions on a solid body, regardless of the origin of the loads and without alteration of the material. According to Newton's third law, which is the foundation of mechanics, to every action corresponds a reaction of equal magnitude and opposite direction; so, whenever a force is applied on a solid static body, an internal reaction force is produced in order to balance the external force and maintain the equilibrium. The magnitude of the internal reaction is the stress and the immediate effect of the presence of stresses in a solid is the strain, therefore, the mechanical behavior discipline analyses the stresses and strains within solids, and determines whether the material have enough strength to withstand such forces without excessively deforming, nor fracturing.

The study of stresses and strains in solid bodies that do not have cracks, voids or discontinuities is known as *continuum mechanics*. Commonly, continuum mechanics is referred as "Strength of Materials", but such term is rather imprecise,

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because what is found in Strength of Materials textbooks are methods and formulas meant for the design of structural components, such as beams and columns, among others. Whereas the study of the mechanical behavior and the strengthening mechanisms, is viewed in textbooks with titles such as: Mechanical Metallurgy, Physical Metallurgy, or Materials Science.

Mechanical Behavior is a very important part of engineering, because any solid component of a machine, a structure, a tool or any objet that will bear loads, should be designed and fabricated to withstand stresses and strains produced by its use. For instance, a gas tank must be able to resist the internal pressure without distortion or rupture, a railroad track must resist the pass of trains without excessive deformation and wear, the columns of a building must bear the dead weight plus earthquake and wind loads without excessively bending or breaking. The mechanical behavior is important in nature as well, a tree stem must support the weight of its branches, leaves and fruits, the bones of a vertebrate animal must withstand the forces of walking and jumping and so on. Even when the main function of a solid component is not to withstand or transmit forces, they must have enough mechanical strength to resist the forces that eventually may act over them; for example, the main function of a conductor cable is to transport electricity, but if it does not have enough strength it will not resist the bending, hanging and twisting loads that it experiences in service and will fail. A glass window has the main function of separating the internal environment of a building from the external one, while letting the light to pass, but if it does not have mechanical strength, it will not resist the wind load and small hits caused by tiny stones, bird crashes and so on. In addition to analyze the stresses and strains in loaded bodies, the field of mechanical behavior has the task of assessing the strength of materials through the mechanical tests such as the tension, hardness and impact tests.

Another important aspect of the Mechanical Behavior is to understand and control the strengthening mechanisms of the materials, both engineering and natural. This study is generally at a microscopic level, since the mechanisms of deformation and strengthening are related to the crystalline structure, the crystallographic defects and the microstructure. The field of deformation and strengthening mechanisms provides the basis for the design of new materials and manufacturing processes that allow to obtain specific mechanical properties and the optimal combinations of them.

Considering the above, the field comprising the Mechanical Behavior of materials has been divided into three parts which are:

- Continuum Mechanics: It is the study of the stress-strain relations, from the macroscopic point of view, assuming the solid is a continuum and homogeneous body.
- Deformation and Strengthening mechanisms: This deals with the study on how plastic deformation takes place at an atomic and microstructural level and the origins of the mechanical properties of materials.
- Fracture: Which is divided into two branches: Fracture Mechanics, which is the study of the mechanical behavior of cracked bodies to determine the fracture

resistance and the dynamics of crack propagation, from a macroscopic point of view; and Fractography, which is the study of fracture surfaces and fracture mechanisms, at both macro and microscopic levels.

In Continuum Mechanics, the distribution of stresses and strains of a loaded solid body are determined as a function of geometry, points and directions of applied loads and the mechanical properties determine whether or not the body will withstand such stresses and strains. The main mechanical properties are: resistance to stresses (yield and tensile strength), resistance to penetration (hardness), stiffness (elastic modulus), maximum attainable deformation (ductility), resistance to strain (resilience) and strain energy (toughness).

The study of deformation and strengthening mechanisms is aimed to learn about the specific relations between the crystalline structure and microstructure with the mechanical properties, whereas fracture mechanics is aimed to assess the performance of cracked materials, which are the basis for the damage tolerance design and structural integrity or fitness-for-service assessments.

The materials used for the manufacturing of components for structural and mechanical applications in which the mechanical behavior is the fundamental design criteria are known as *engineering materials* and they are classified according to the Table 1.1.

Table 1.1 Classification of engineering materials

Class	Characteristics	Example
Ferrous alloys	High mechanical strength, ductile, good resistance to heat, good electrical conductivity, high formability, high impact and fracture resistance	Low, medium and high alloy steels, cast irons
Nonferrous alloys	High to medium mechanical strength, ductile, mild resistance to heat, high electrical conductivity, high formability, mild impact and fracture resistance	Cooper, Aluminum, Nickel, Titanium, Magnesium, Zinc and their alloys
Polymers	Low mechanical strength, elastic, poor resistance to heat, electrical insulators, high formability, high corrosion resistance	Plastics (thermoset y thermoplastic), elastomers, foams
Composites	High mechanical strength, poor resistance to heat, low density, high formability	Polymer matrix reinforced with fibers (glass, carbon, Kevlar), reinforced concrete
Technical ceramics	High wear strength, refractory, high corrosion resistance, low impact and fracture resistance	Alumina, magnesia, silica, tungsten carbide
Glass	High corrosion resistance, transparent, poor impact resistance	Borosilicate, silica and soda glasses
Rocks	High compression strength, refractory, low impact and fracture resistance	Clay brick, granite, basalt
Bio-materials	Low density, easy to work, low mechanical strength, low cost	Wood, leather, bone, cotton, silk, vegetable fibers

1.2 Definition of Stress

The concept of stress was introduced by the French scientist and mathematician Augustin Louis Cauchŷ in 1833, based on the Euler's movement laws, Newtons's mechanics and the infinitesimal calculus. Cauchŷ was a remarkable mathematician, catholic and a devoted royalist, that made numerous and important contributions to virtually every branch of mathematics. He was member of the France Academy and professor at Ecolé Polytechnique Parisien and the Paris Sorbone University, where he studied the deformations in loaded solid bodies, introducing the concept of the stress tensor. The next paragraphs describe the Cauchŷ's stress tensor concept, simplified to meet the objective of this book.

In order to define stress, first it is necessary to identify the types of forces that act on a solid body, such forces are:

- Surface forces. They act throughout an external surface, such as loads, tractions and pressure.
- Body Forces. They are exercised on the entirety of particles of a solid body, such as: gravity, magnetism, inertia, thermal forces, etc.

The study of mechanical behavior only considers surface forces, that is to say, load, tractions and pressure, since body forces do not significantly deform the materials, unless they are extremely high.

To analyze the mechanical behavior of a solid body, it is necessary to simplify the system, because materials are complex arrays of atoms, crystalline defects, grains, second phases and heterogeneities. The simplifications consider that the solid body is:

- Continuum. Matter occupies the entire volume and there are no voids or interruptions.
- Homogeneous. The entire volume is made up of the same type of material.
- Isotropic. The properties are the same in any direction.

Figure 1.1 shows an idealized body under the action of an external force. If the body remains static, the external force F is balanced by an internal reaction force Fr, of the same magnitude, but opposite direction. Physically speaking the internal force is the vector resultant of the many forces that act along the links of the particles that

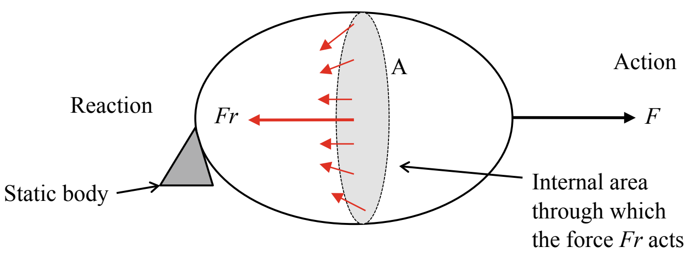


Fig. 1.1 Schematic representation of a static body under an external force

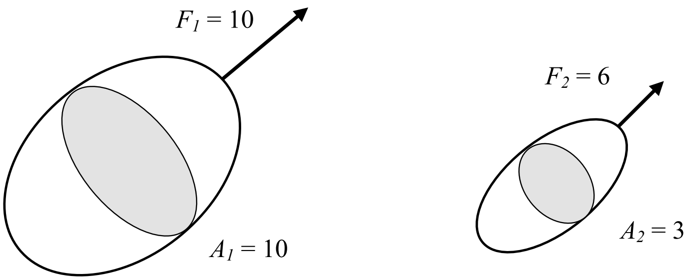


Fig. 1.2 Force/area relation in a static solid under the action of a force

make up the solid (atoms or molecules), thus the effect of the reaction is the stretching, shortening or twisting these links. The magnitude of the internal reaction in the solid, obviously depends on the magnitude of the applied force, but also on the amount of links resisting such force. The number of particle's links resisting these internal forces is proportional to the cross section area A, so the magnitude of the internal effect can be measured by the ratio F/A. If F/A is big, the effect is big and vice versa.

The above concept is illustrated in Fig. 1.2, which shows how the magnitude of the internal effect produced when an external force is applied on a static solid is directly proportional to the applied force and inversely proportional to the cross-section area. The bigger body has a cross area $A_1 = 10$ and an applied force $F_1 = 10$, so it experiences a force unit per area equal to 1.0 force units per unit area. The smaller body has a cross area $A_2 = 3$, with an applied force $F_2 = 6$, so it withstands 2.0 force units per unit area, which is twice of what the bigger body is withstanding, thus it experiences a greater effect.

The magnitude of the internal reaction calculated by the ratio *F/A* is the *stress* and mathematically is defined by the equation:

$$\sigma = \frac{F}{A}$$

Therefore, *stress is the magnitude of the internal reaction produced in a static solid under the action of an external load or force.* According to this definition, stress has derived units, and the typical ones are shown in Table 1.2, along with the conversion factors.

At this point, it is important to differentiate between pressure and stress since they are calculated in a similar way and have the same units, so they are often mistaken. Stress, as it has been stated, is the measure of the effect of an external force a body; hence, the effect is internal and actuates over a cross-section area, while pressure is a force evenly distributed on an external surface. Therefore, stress is internal, and pressure is external. In order to visualize this, consider a hollow cylinder containing a pressurized fluid as depicted in Fig. 1.3. Note that the pressure is acting on an external surface, since the definition of "external" refers to a

Table 1.2 Typical units of stress

System	Units	Common multiple
International	Pascal (Pa = Nw/m^2)	MPa = 10^6 Pa
US customary	psi (psi = lbf/plg^2)	ksi = 1000 psi
Metric	Kgf/mm^2	Kgf/cm^2 = 100 kg/mm^2
Conversion factors:		1 MPa = 10.5 kgf/cm^2 1 kgf/cm^2 = 41.22 psi 1 ksi = 6.895 MPa

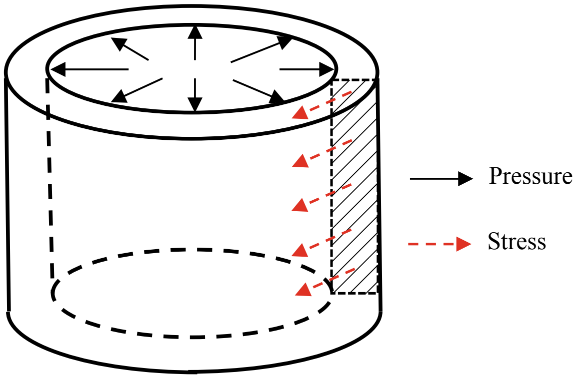


Fig. 1.3 Stress produced in a hollow cylinder with internal pressure

location outside of the body's volume, so even though the pressure is in the internal side of the cylinder, it is applied outside the thickness of the cylinder. Now, it is easy to visualize that the internal pressure will expand the cylinder, and this expansion will generate internal forces in the circumferential direction, represented by the dotted arrows in Fig. 1.3. These circumferential forces act across the shaded area of Fig. 1.3, so it can be foreseen that the stress produced by the circumferential force will have a different value from that of the pressure.

As stresses come from forces, and forces are vectors, they can be decomposed into components; in order to avoid dealing with inclined vectors, Cauchŷ considered one component perpendicular to the cross-section area *Fn* and other component *Ft* parallel to or tangential to it, as shown in Fig. 1.4. Since the effects of these forces on the body are different, so are the stresses resulting from them.

Thus, the stress produced by a force normal to the cross-section area is called *normal* and is designated by:

$$\sigma_N = \frac{F_N}{A}$$

Normal stresses, are divided into two types: when internal forces tend to separate particles or stretch the body they are called *tension stresses* and are of positive sign (+), and when the force brings particles close to each other, or shorten the body,

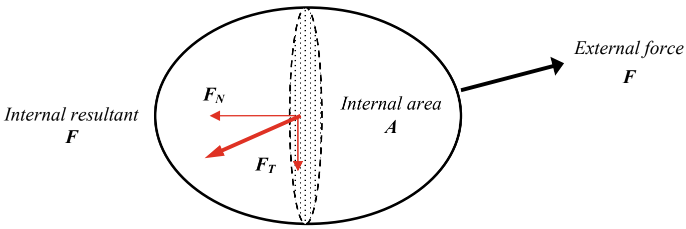


Fig. 1.4 Normal component (F_N) and Tangential component (F_T) of the internal forces

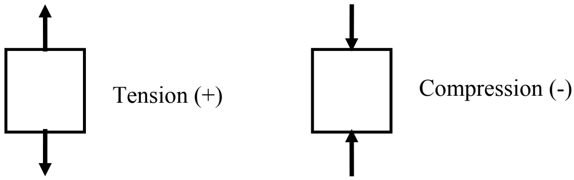


Fig. 1.5 Free body diagrams showing the types of normal stresses

they are *compression stresses* and are of negative sign (−). Such stresses are represented in a free body diagram, as shown in Fig. 1.5.

The stress produced by tangential forces is called *shear stress* and is designated by:

$$\tau = \frac{F_T}{A}$$

The sign of the shear stress is defined in terms of the direction of the bending moment, thus, it is positive if it generates a counter clock moment and vice versa.

1.3 Mechanical Behavior in Uniaxial Tension

The uniaxial tension test is the universally accepted method to characterize the mechanical behavior of engineering materials. Customary, the test is carried out in accordance with the standard ASTM E8, which consists of stretching a test specimen of uniform cross-section (round or rectangular) fixed with a set of grips attached to a load frame. During the test, the specimen's load and elongation are continuously recorded, and the results are plotted in a Load vs. Elongation plot, which for a typical metallic material, looks like the one shown in Fig. 1.6.

The Load-Elongation plot shows that, when a tensile load is applied, the immediate effect is an elongation. Initially, if the load is removed, the test piece recovers its initial shape and dimensions; then, it is said that the deformation is *elastic*. After exceeding a specific load value, a permanent elongation occurs, and then it is said that there is a *plastic* strain. It is important to bear in mind that the

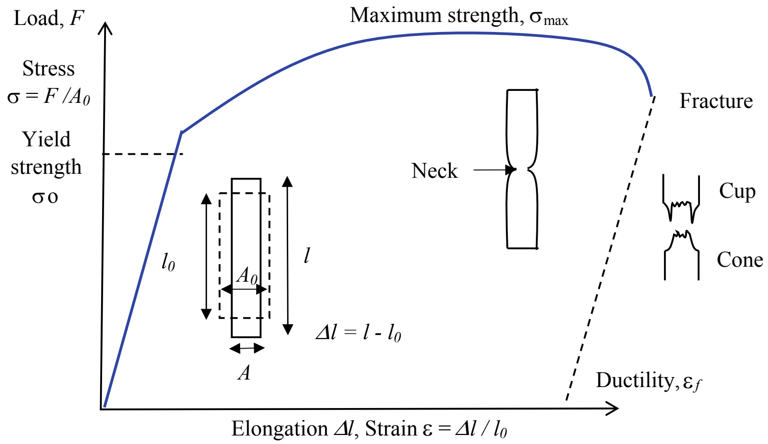


Fig. 1.6 Schematic representation of the Load-Elongation and Stress-strain curves in uniaxial tension of a typical engineering material

elastic deformation does no disappear during plastic deformation, but both are added, therefore the behavior is *elastic-plastic*.

As mentioned before, the result of the uniaxial tension test is a Load vs. Elongation curve, however, the load is divided by the area to give the *engineering stress* (σ), which is defined as:

$$\sigma = \frac{F}{Ao}$$

where F is the applied force and Ao is the initial cross-section area of the test specimen. Similarly, the *engineering deformation* (ε), is defined as:

$$\varepsilon = \frac{\Delta l}{l_0}$$

where Δl is the elongation and lo is the initial length of the test specimen. As Ao and lo are constants, the shape of the Load-Elongation curve does not change when it is transformed into the Engineering Stress-Strain curve, as illustrated in Fig. 1.6. By this way the main mechanical properties of engineering materials are determined from this curve, as described in the next paragraphs.

The stress level at which plastic strain initiates is referred as *yield strength*, represented by the symbol σ_o. The yield strength is a property of the material, and it is of great importance in many engineering applications, since if a component gets plastically deformed, usually it will not perform properly, constituting a failure; for that reason, most engineering designs are done in such a way that the acting stresses do not exceed the yield strength.

The maximum point in the Stress-Strain curve is the *tensile strength*, identified by the symbols σ_{max} or σ_u and it is also a property of the material. It may be noticed that after the tensile strength is reached, the Stress-Strain curve falls down; this is due to the formation of a geometrical contraction of the test specimen, called *neck*. The neck rapidly reduces the cross-section area and induces high local tensile stresses, causing a reduction of the load necessary to continue straining the material. The neck also indicates the initiation of the fracture process, which consists on the formation of an internal crack by the nucleation, growth and coalescence of internal voids. The internal crack reduces the cross-section area to a level where the remaining ligament fails by ductile shear fracture, forming the typical *cup and cone* fracture.

The maximum elongation after failure is called *ductility* and is represented by the symbol ε_f, usually it is a material property, but it is strongly influenced by microstructural and environmental factors, so, often it is a non-mandatory or secondary requirement in material's specifications.

For most engineering materials, the elastic part of the Stress-Strain curve linear, as seen in Fig. 1.7. The ratio between stress and elastic strain is the *Young's modulus*, which is defined as:

$$E = \left(\frac{\sigma}{\varepsilon}\right)_{Elastic}$$

where σ is the stress and ε is the elastic elongation.

Bearing in mind, that $\varepsilon = \Delta l/ lo$ and $\sigma = F/Ao$, it can be stated that:

$$\sigma = E\left(\frac{\Delta l}{l_0}\right) = \frac{F}{A}$$

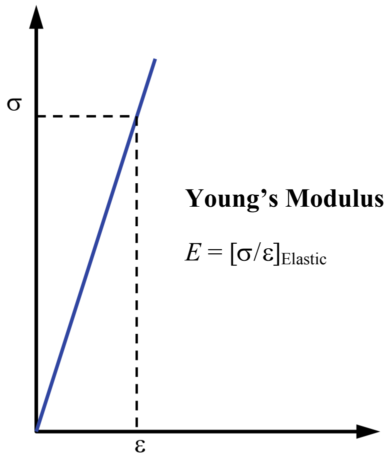


Fig. 1.7 Elastic portion of the stress-strain curve in uniaxial tension

Therefore:

$$F = \left(\frac{AE}{l_0}\right)\Delta l$$

The term (*AE/lo*) is known as the *Elastic Coefficient* and it determines how rigid or flexible a structure is. High values of the elastic coefficient will result in rigid structures, less likely to deform; whereas low values of the elastic coefficient will give flexible structures, easily deformed under load.

The application of the Elastic Coefficient is widespread in the design of, both, mechanical and structural components, where it is desirable to set limits or controls over elastic strain. The most frequent cases of elastic design are:

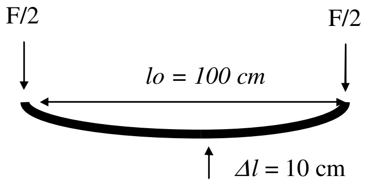
- Flexible or elastic components: those are that should feature fairly large elastic strains or flexions, but not as much as to reach yield, as it is the case of helical and leaf springs. In other words, flexible components are designed to have large controlled elastic strains under the applied loads. According to the elastic coefficient, this can be achieved by long lengths, small cross-section areas and materials with a low Young's modulus. The first two conditions make flexible components long and slender.
- Rigid components: those are where an excessive elastic strain is adverse for their performance, such as building structures, supports, gears and machine parts. In these components, the magnitude of the elastic strain must be limited to a minimum, so the elastic coefficient must be high. This can be attained by widening the cross-section, shortening the length and by selecting high Young's modulus materials. The first two characteristics make rigid components short and thick.

The following examples illustrate the use of the elastic coefficient.

Example 1 A leaf spring of length lo = 100 cm long and width W 10 cm must have an elastic vertical displacement δ l < 10 cm under a load F = 1000 Nw (Approx. 100 kg). Determine the leaf spring thickness B if it is made of steel (E = 211.4 GPa).

Solution A free body diagram of the problem is:

According to the force-elongation elastic formula: $F = (AE\Delta\ l)/lo$, thus:



$$A = (F\,lo)/(E\Delta l)$$

However, $A = W \times B$. Solving for B and substituting values:

$$B = (F\,lo)/(E\Delta l\,W) = (1000\,Nw) \times (1\,m)/\left(211.4 \times 10^9 Nw/m^2\right)(0.1\,m) \times (0.1\,m)$$
$$= 4.73 \times 10^{-3} m = 4.73\,mm$$

Example 2 An engineering design came out with a hollow square column of 10 cm external width, 2.5 mm thickness and 4 m long, to support a load F = 490,500 Nw (50 000 kg). The typical out-of-plumbness tolerance for building columns is 1 mm per meter of height, neglecting curvature effects, it is a tensile strain of $\Delta l/lo$ = 5.73×10^{-3}. A novel engineer suggests to substitute steel by aluminum in order to save weight. Is this a good idea? Assume that the Young's modulus in compression is identical to that in tension. Steel $E = 211.4 \times 10^9$ Nw/m^2; aluminum $E = 70.3 \times 10^9$ Nw/m^2.

Solution According to the force-elongation elastic formula: $F = (AE\Delta\ l)\ /\ lo$, thus:

$$(\Delta l/lo) = F/(AE)$$

A = 9.75 cm^2 = 0.000975 m^2. Substituting values for steel:

$$(\Delta l/lo)_{Steel} = (490\,500\,Nw)/(0.000975\,m^2 \times 211.4 \times 10^9 Nw/m^2) = 2.38 \times 10^{-3}$$

Substituting values for aluminum:

$$\begin{aligned}(\Delta l/lo)_{Aluminum} &= (490\,500\,Nw)/(0.000975\,m^2 \times 70.3 \times 10^9 Nw/m^2)\\ &= 7.16 \times 10^{-3}\end{aligned}$$

Notice that the aluminum column has three times more elastic deformation than steel and it will not meet the specification for out-of-plumbness. This is why aluminum is seldom used in load bearing structures such as buildings and bridges.

1.4 Mechanical Design by the Stress Definition

The main contribution of the stress concept is that the design of mechanical or structural components can be straight forward, once the stress is known. Based on the definition of stress, the design variables can be set as follows:

$$\underset{\text{(Allowable stress)}}{\boldsymbol{\sigma}} \quad = \quad \underset{\text{(Applied load)}}{\boldsymbol{F}} \quad / \quad \underset{\text{(Cross section size)}}{\boldsymbol{A}}$$

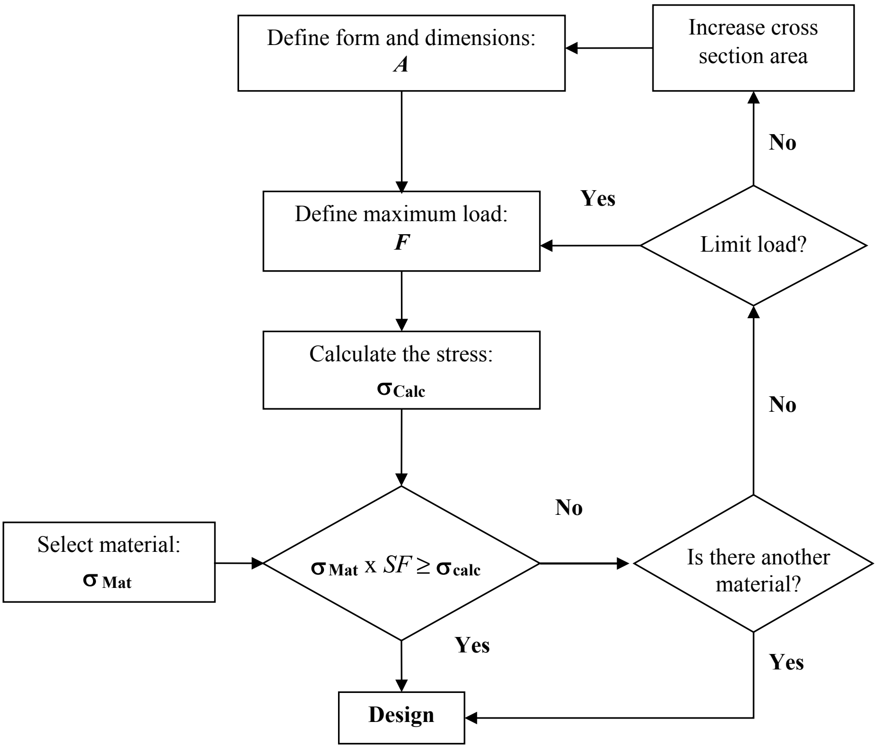


Fig. 1.8 Flow chart of mechanical design based on the definition of stress

According to this equation, any increment or decrement in one variable has to be compensated by adjusting the other two variables. Thus, given a load *F*, which has to be supported by the component and knowing the material's mechanical resistance (for example, the yield strength), the cross-section area *A* that the component needs to have in order to withstand the stress can be calculated. If both *A* and *F* are set, then an appropriate material can be selected, being that which strength is greater than the calculated stress, and finally if the material's strength and *A* are given, then the maximum applied load can be established. As mentioned before, in many designs, the calculated stress should be below the yield strength, so a safety factor less than 1.0 is applied to the material's strength. This process is presented in Fig. 1.8.

The fraction of the yield strength is the maximum allowable stress, called *design stress*. The magnitude of the difference between the design stress and the material's strength is the *safety factor* (*SF*), which value depends on several factors, being the most important: the load uncertainties, the presence of defects that reduce the strength and the severity of failure consequences.

Example A cylindrical bar has to support a load of 1000 kg. Due to design limitations, the diameter cannot exceed 10 mm. It the fabrication material has a yield strength of 3500 kg/cm^2, what is the safety factor?

Solution

$$\sigma_{Calc} = F/A = F/(\pi R^2) = (1000\,\text{kg})/(\pi(0.5)^2\text{cm}^2) = 1273\,\text{kg/cm}^2$$

$$SF = \sigma_{Calc}/\sigma_{Mat} = 0.363$$

Notice that in the previous example *SF* is less than 1.0, because it is applied to the materials strength, however, many design codes use safety factors greater than 1.0, so in such cases it is applied to the applied loads or calculated stresses.

1.5 The Stress Tensor

So far, the stress has been calculated by dividing the total internal reaction force by the cross section area, however, internal force is the resultant of the vectorial sum of many force components acting across the unit area elements that compose the total area. If the body is a continuum, this concept leads to the idea that stress may exist in a point. To demonstrate this, consider an area element a_i of very small size, which is under the action of an internal force component f_i, therefore:

$$A = \Sigma a_i$$

$$F = \Sigma f_i$$

If the equity a_i / fi = F / A, is true, then it is said that the stress is *uniform*. Furthermore, if the body is a continuum, the following limit exists:

$$lim_{a_i \to 0} \frac{f_i}{a_i}$$

And then, the stress can be determined as:

$$\sigma = \frac{dF}{dA}$$

Since dF and dA represent differential quantities (a differential is a value close to zero, but never becomes zero), the previous equation represents the stress in a point, and therefore:

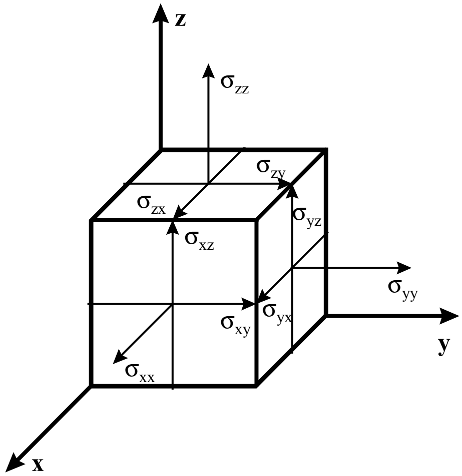


Fig. 1.9 Stress components in a volume element in an orthogonal coordinated system

$$F = \int \sigma dA$$

Cauchŷ applied this concept to a cube shaped element of differential volume placed in a coordinated Cartesian system (*x, y, z*), where the forces are decomposed into three components, each one parallel to the directions *x*, *y* and *z* on each face of the cube, so each face of the cube will have one normal component and two tangential ones. Graphically, the stress components on each cube face would be as shown in Fig. 1.9. The opposite force components that balance the system are not taken into consideration because once the acting components are defined, the opposite are automatically defined, so there is no need to mathematically describe twice as much vectors.

Notice that the following index notation has been introduced, in order to identify the stress components:

$$\sigma_{ij} \quad \textit{is a stress component},$$

where:

i is the cube's face
j is the direction of the force.

As seen in Fig. 1.9 there are nine stress components, three per cube face, and they define the total number of reactions in a static body subject to external loads. Since each component is a vector, by writing the nine components in matrix form, the *stress tensor* is obtained.

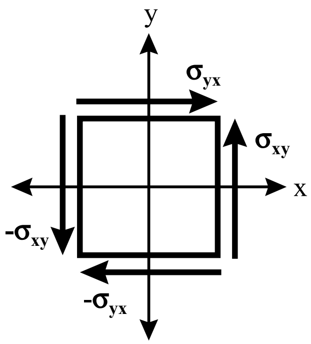


Fig. 1.10 Shear stress components acting on the face of a volume element

$$\sigma = \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix}$$

The stress tensor is made up of three normal components (σ_{xx}, σ_{yy} and σ_{zz}) and six shear components. Now, the shear components also create a momentum that has to be balanced as well in order to keep static balance. To determine the magnitude of the momentum, consider a two dimensional body under the action of shear stresses, as shown in Fig. 1.10.

In static conditions, the in-plane momentum balance is:

$$Myx + Mxy = 0$$

where M is the momentum, defined by:

$$Myx = \sigma_{yx} dxdz(dy)$$

In the latter equation, $dxdx$ is the area where force actuates and dy is the leverage length. Similarly:

$$Mxy = \sigma_{xy} dydz(dx)$$

Which leads to:

$$\sigma_{xy} = \sigma_{yx}$$

In general, $\sigma_{ij} = \sigma_{ji}$, therefore, it is said that the stress tensor is *symmetric*. Symmetry reduces the stress tensor to six independent components.