Solid Mechanics and Its Applications

Sohrob Mottaghi
Rene Gabbai
Haym Benaroya

An Analytical Mechanics Framework for Flow-Oscillator Modeling of Vortex-Induced Bluff-Body Oscillations

Springer

Solid Mechanics and Its Applications

Volume 260

The fundamental questions arising in mechanics are: Why?, How?, and How much? The aim of this series is to provide lucid accounts written by authoritative researchers giving vision and insight in answering these questions on the subject of mechanics as it relates to solids. The scope of the series covers the entire spectrum of solid mechanics. Thus it includes the foundation of mechanics; variational formulations; computational mechanics; statics, kinematics and dynamics of rigid and elastic bodies; vibrations of solids and structures; dynamical systems and chaos; the theories of elasticity, plasticity and viscoelasticity; composite materials; rods, beams, shells and membranes; structural control and stability; soils, rocks and geomechanics; fracture; tribology; experimental mechanics; biomechanics and machine design. The median level of presentation is the first year graduate student. Some texts are monographs defining the current state of the field; others are accessible to final year undergraduates; but essentially the emphasis is on readability and clarity.

Springer and Professors Barber and Klarbring welcome book ideas from authors. Potential authors who wish to submit a book proposal should contact Dr. Mayra Castro, Senior Editor, Springer Heidelberg, Germany, e-mail: mayra.castro@springer.com

Indexed by SCOPUS, Ei Compendex, EBSCO Discovery Service, OCLC, ProQuest Summon, Google Scholar and SpringerLink.

More information about this series at http://www.springer.com/series/6557

Sohrob Mottaghi · Rene Gabbai ·
Haym Benaroya

An Analytical Mechanics Framework for Flow-Oscillator Modeling of Vortex-Induced Bluff-Body Oscillations

Springer

Sohrob Mottaghi
FAA William J. Hughes Technical Center
Atlantic City International Airport
Atlantic City, NJ, USA

Rene Gabbai
Pratt & Whitney
East Hartford, CT, USA

Haym Benaroya
Mechanical and Aerospace Engineering
Rutgers University
Piscataway, NJ, USA

ISSN 0925-0042 ISSN 2214-7764 (electronic)
Solid Mechanics and Its Applications
ISBN 978-3-030-26131-3 ISBN 978-3-030-26133-7 (eBook)
https://doi.org/10.1007/978-3-030-26133-7

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

We dedicate this book as follows:

I would like to dedicate this book to my sister Sarah Mottaghi.
—Sohrob Mottaghi

I would like dedicate this book to my family: to my wife Jessica and children Miriam and Isabelle for keeping me grounded, and to my parents Jacques and Mary for instilling in me a sense of wonder and intellectual curiosity.
—Rene Gabbai

I would like to dedicate this book to my family: my wife Shelley, my mother Esther, my children Adam, Ana, Liz, and Tiffany, my sister Dahlia and her husband Ron, my nephew Max, and to the memory of my father Alfred.
—Haym Benaroya

Preface

It is a pleasure to complete this book that is devoted to the fundamental study of vortex-induced oscillations. This study is borne of the desire to understand such fluid–structure interactions at a fundamental level, and to derive mathematical models within the framework of the flow-oscillator paradigm.

This monograph is a compendium of the efforts of the authors over two decades. We view it as a preliminary effort that has many opportunities for extensions and added insights by others who are so interested.

We find that the variational framework for such modeling efforts provides certain advantages for the derivation of the governing equations, but these equations are not unique. Rather, the derived equations depend on the physical assumptions made initially and throughout the analysis. Our primary goal has been to create a modeling framework within which flow-oscillators can reside, and to show how a number of well-known flow-oscillators, formulated by others, can be viewed as being a part of this framework. An advantage of this framework is that assumptions are explicit and can be removed or changed. Other assumptions can be added. Each of these alterations leads to different governing equations, as one would expect. But the assumptions are explicit, physical, understood, and open to debate.

We appreciate the work of many of our colleagues on this problem of vortex-induced oscillations, and from whom we have learned much. We hope that this effort by us resulting in a new perspective proves to be interesting and useful.

Piscataway, USA — Haym Benaroya
June 2019

Acknowledgements

We are pleased to acknowledge the following people. Dr. Tom Swean, while he was a program manager at ONR, supported our work for many years. We discussed the science and engineering of vortex-induced vibration as colleagues, and we will always be grateful for his insights and his support.

We thank Dr. Yuriy Gulak for our discussions on this subject, as well as many other subjects, over the past decades. His deep insights and knowledge, and friendship, are gratefully and warmly appreciated.

We thank Dr. Joey Sanchez for his valuable assistance with the integration of the various documents that represented our work of over two decades, over multiple computer systems and TEX operating systems, that led to this monograph.

SM wishes to express his gratitude to Prof. Haym Benaroya for his continuous mentorship in the field of vortex-induced vibration. He is also thankful to RDG since SM's studies were, in part, inspired by RDG's earlier work in this field.

RDG would like to express his sincere gratitude to his coauthors. In particular, he would like to thank Prof. Haym Benaroya for introducing him to the fascinating field that is vortex-induced vibration.

HB would like to acknowledge Dr. Timothy Wei for our early collaborations in this research area. HB also sincerely acknowledges the great pleasure of working with his two coauthors, who are very talented researchers as well as being kind and generous persons, from whom much has been learned.

Contents

Chapter 1
Introduction

Abstract This chapter introduces the focus problem of this monograph, vortex-induced oscillations, which is within the fluid–structure interaction class of problems. The organization of the monograph is provided.

1.1 Background and Overview

The problem of fluid–structure interaction (FSI) has long been one of the great challenges in engineering. It is a crucial consideration in the design of many bluff-body engineering structures, such as offshore structures, skyscrapers, aircraft, and bridges. It is also a serious design consideration for aerodynamic bodies, such as wings, but this is beyond our scope. While the importance of the subject has been understood for well over a century, it has been only in the past few decades that efforts have been made to analytically model the general behavior of such systems. Parallel to analytical attempts, many experiments have been devoted to gathering data and interpreting such interactions. Consequently, analytical dynamics-based modeling of such problems has evolved with coupling to experimental data resulting in various semi-analytical representations. Generally, attempts have been made to model vortex-induced vibration (VIV) problems as few degrees-of-freedom (DOF) oscillatory models; therefore, they are referred to as *reduced-order* models.

Due to the complexity of the interactions between fluid and structure, in particular for vortex-induced vibration, a variety of efforts have been undertaken to explain the physics of this coupling. Initially, the efforts were experimental so that "reality" could be visualized, and then explained. Tremendous efforts have led to impressive results by numerous experimentalists along with an extensive phenomenological understanding of this behavior. The practical needs of industry required more than just understanding; it required designs of structures and machines that could operate safely for long periods of time in fluid environments where complex interactions occur. For vortex-induced oscillations, this led to the need for design equations that were representative of the experimental data, as well as technologies to minimize the effects of shedding vortices. Physical theory lagged experimental data, of course, but the need for governing design equations was there, resulting in the formulation

S. Mottaghi et al., *An Analytical Mechanics Framework for Flow-Oscillator Modeling of Vortex-Induced Bluff-Body Oscillations*, Solid Mechanics and Its Applications 260,
https://doi.org/10.1007/978-3-030-26133-7_1

of governing equations that qualitatively mimicked the data and could be made to fit the data in specific instances by the use of nonphysical "arbitrary" parameters. Such semi-empirical equations have formed the backbone of reduced-order modeling for VIV.

Our monograph represents a line of work with the goal of laying a fundamental foundation for such reduced-order modeling. This effort is based on the variational principles of mechanics. Before we go to that work in Chap. 4 and subsequent chapters, we review the efforts of the community. In Chap. 2, we provide a representative review of the literature for bluff bodies. In Chap. 3, we summarize variational mechanics. In Chaps. 4–7, we provide detailed derivations of a sequence of our analytical dynamics modeling efforts of VIV.

1.2 Introduction to the Model Problem

For experimental studies of VIV, certain types of structural configurations have been preferred in the literature, where a rigid solid body with one or two degree(s)-of-freedom is immersed in a flow. While the experiments have been conducted on a variety of solid shapes (and occasionally on flexible bodies), *reduced-order semi-analytical models* have been generally developed for single DOF rigid bluff bodies, specifically for circular cylinders. The most commonly used model, called the *model problem* [5], is a type of inverted solid pendulum that is immersed in a flow, rests on elastic supports and can only move transversely to the flow direction. A second model is the translating cylinder. Schematic diagrams of elements of two representative configurations of the *model problem* are shown in Figs. 1.1 and 1.2. The *model problem* has been widely used since it possesses a simple geometric configuration, and yet, it exhibits the majority of the nonlinear behaviors observed in VIV systems. Consequently, the majority of VIV experiments have been conducted based on the *model problem*. Both in experimental and analytical studies, the flow is controlled or considered to be two dimensional for all time, as are the shedding vortices.

The purpose of this work is to present our theoretical studies that derive reduced-order models from first principles, where assumptions are explicitly stated. Therefore, experimental observations are not the main focus of this research work. However, a few key features observed in the experimental studies are summarized for those who are not familiar with the subject. An in-depth review of experimental studies of VIV can be found in [4].

Starting with the stagnant fluid, if the speed of the flow past a bluff-body cylinder is increased, three different behavioral regimes are identified: *pre-synchronization*, *resonant synchronization*, and *classical lock-in*. Pre-synchronization is the first regime where the structure starts oscillating and vortices are first observed. The amplitudes of the structural oscillations are low and the vortices' strength are weak to moderate. Observed in this region is a beating behavior, that is, the peak amplitudes of structural response increase and decrease gradually as the structure oscillates. Moreover, the flow drives the structure in this region.

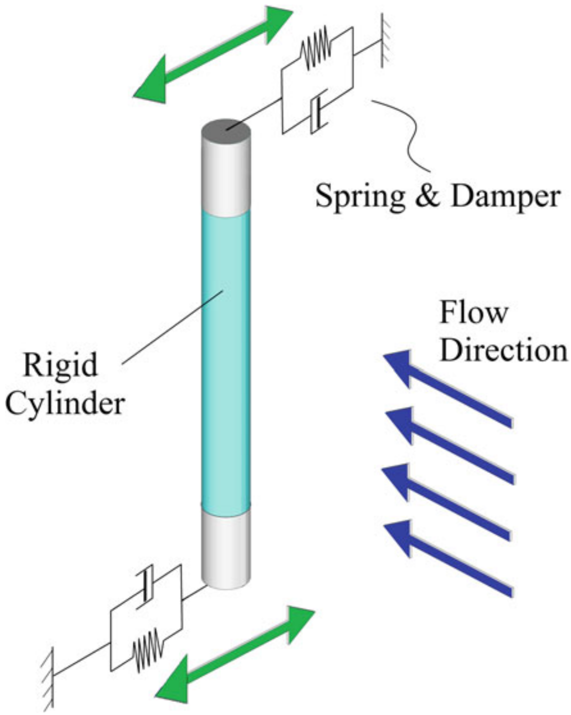

Fig. 1.1 A representative configuration of the model problem: translating cylinder

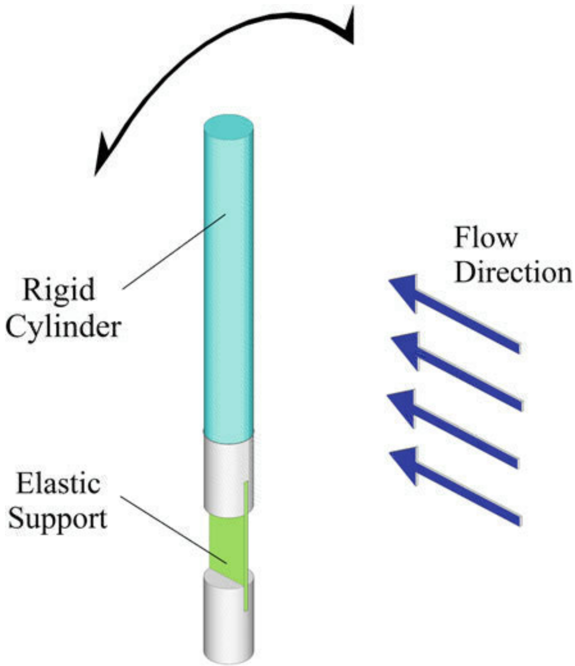

Fig. 1.2 A representative configuration of the model problem: inverted pendulum

As the average velocity of the flow is increased, vortices become stronger until the frequency of the vortex shedding reaches the natural frequency of the structure, where near-resonant behavior is observed. Thus, the structural response reaches a maximum and this is called the *resonant synchronization* region. Similar to the *pre-synchronization* region, beating behavior is noticeable but weaker, and the structure remains driven by the flow.

If the flow velocity is increased further, constant structural oscillation amplitude and frequency are observed for a range of flow velocities. This phenomenon is called *classical lock-in*. Unlike the other two regions, the flow is modulated by the structure and the observed vortices are the least organized. The existence of three distinct regimes in the frequency–amplitude response curves of an inverted pendulum is shown in Fig. 1.3. As in Fig. 2.1, many experiments show the existence of hysteretic behavior, where the maximum amplitude of the oscillations are larger as the velocity is increased than when it is decreased. VIV is a complicated phenomenon. The structural response depends on many factors, such as shedding frequency, Reynolds number, material damping, structural stiffness, surface roughness, cylinder length, density of the fluid, and mass of the cylinder, [4, 8]. Therefore,

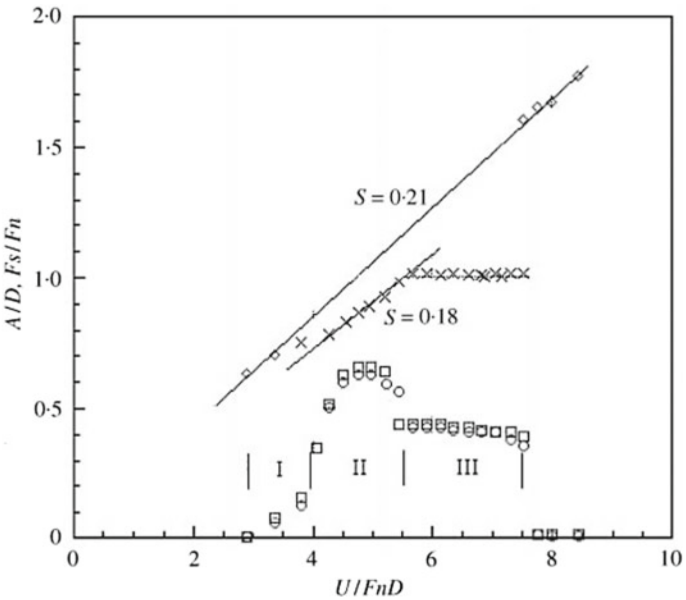

Fig. 1.3 The frequency–amplitude response curves of an inverted pendulum, where A is the amplitude of oscillation, D is the diameter of the cylinder, F_s is the frequency of oscillation, F_n is the natural frequency of the cylinder, U represents the fluid velocity; □, ◯ amplitude of oscillation for two independent but identical experimental runs; × frequency of oscillation and vortex shedding frequency in which VIV was observed; ◇ frequency of vortex shedding where the cylinder was stationary; I pre-synchronization; II resonant synchronization; III classic lock-in [2]. Reprinted with permission

reduced-order modeling of VIV has evolved in parallel to experiments in order to quantify our understanding of this phenomenon.

Efforts to model VIV as reduced-order systems can be divided into two categories: empirical models and first-principles models. Moreover, the empirical models can be divided into two subcategories: wake-oscillator (wake-body) models and experimental force-coefficient models. The wake-oscillator models are based on the assumption that an immersed structure in a flow experiences nonlinear oscillator-like hydrodynamic forces. Therefore, the aim is to obtain nonlinear fluid force equations from the experimentally acquired data that can be coupled with the structural equation of motion. One of the early models is the one proposed by Hartlen and Currie [6]. They used a van der Pol-type fluid oscillator to model the fluid–structure system,

$$\ddot{x} + 2\zeta\dot{x} + x = a\omega_0^2 C_L \tag{1.1}$$

$$\ddot{C}_L - \alpha\omega_0\dot{C}_L + \frac{\gamma}{\omega_0}\dot{C}_L^3 + \omega_0^2 C_L = \beta\dot{x}, \tag{1.2}$$

where a, ω_0, and ζ are the known structural parameters, and the fluid parameters α, β, and γ are found experimentally.

The experimental force-coefficient models are single degree-of-freedom models. They only include a single forcing function obtained experimentally. Generally, the empirical models have relative success in capturing the features of VIV. However, these models neglect the dynamic coupling between the flow and the structure by only considering the forces as they are seen by the structure. Therefore, they do not provide much understanding of the physics of the problem, as the fluid and structure exchange energy. These *ad hoc* methods are outside the scope of this work that is focused on first-principles models, specifically, using variational principles. Useful reviews of the empirical models can be found in [1, 3, 4]. While variational principles have been known for well over a century, it was not until 1973 that McIver was among the first researchers to propose the use of variational methods in modeling fluid–structure interaction problems [7]. Also, the work by Benaroya and Wei in 2000 is one of the earliest attempts to use such methods for VIV problems [2]. Consequently, the literature on the subject is very limited.

In the next chapter, we provide an overview of the literature on VIV. The field is vast and our review should be considered to be representative rather than comprehensive. This review is intended to provide the reader a feel for the physics of the dynamic behavior, and a summary of relevant modeling efforts.

References

1. Benaroya H, Gabbai RD (2008) Modelling vortex-induced fluid structure interaction. Philos Trans Math Phys Eng Sci 366(1868):1231–1274
2. Benaroya H, Wei T (2000) Hamilton's principle for external viscous fluid structure interactions. J Sound Vib 238(1):113–145

3. Gabbai RD (2006) Hamilton's principle for fluid-structure interaction and applications to the free-vibration of an elastically-mounted cylinder. PhD dissertation, Rutgers, the State University of New Jersey, New Brunswick, NJ
4. Gabbai RD, Benaroya H (2005) An overview of modeling and experiments of vortex-induced vibration of circular cylinders. J Sound Vib 282:575–616
5. Gabbai RD, Benaroya H (2008) A first-principles derivation procedure for wake-body models in vortex-induced vibration: proof-of-concept. J Sound Vib 312:19–38
6. Hartlen RT, Currie IG (1970) Lift-oscillator model of vortex induced vibration. J Eng Mech 96(5):577–591
7. McIver DB (1973) Hamilton's principle for systems of changing mass. J Eng Mech 7(3):249–261
8. Sarpkaya T (2004) A critical review of the intrinsic nature of vortex-induced vibrations. J Fluids Struct 19(4):389–447

Chapter 2
Literature in Vortex-Induced Oscillations

Abstract A literature review is provided in this chapter of vortex-induced oscillations. While the literature is vast, our review is selective but representative of the field. Reviewed are: (i) experimental studies on: fluid forces, three-dimensionality and free-surface effects, vortex-shedding modes and synchronization regions, frequency dependence of the added mass, the dynamics of cylinders with low mass-damping; (ii) semi-empirical models: wake-oscillator, single degree-of-freedom, force decomposition; (iii) variational approaches; and (iv) numerical approaches.

2.1 Introduction

Vortex-induced vibration (VIV) occurs when shedding vortices (a von Kármán vortex street) exert oscillatory forces on a cylinder in the direction perpendicular to both the flow and the structure. The structure starts to oscillate due to these forces if it is not fixed. For fixed cylinders, the frequency of shedding is related to the nondimensional Strouhal number, defined as $S = f_v D/U$, where f_v is the frequency of vortex shedding, U is the steady velocity of the flow, and D is the diameter of the circular cylinder. The Strouhal number is found to be nearly constant with a value of 0.2 for a large range of Reynolds numbers. This range is often called the subcritical range and spans the Reynolds number range from 300 to 2×10^5 [19].

For flow past cylinders that are free to vibrate, the phenomenon of synchronization or lock-in is observed. For low flow speeds, the cylinder will initially respond at the frequency f_v. This frequency is fixed by the Strouhal number. As the flow speed is increased, the shedding frequency approaches the fundamental natural frequency of the cylinder, f_n. In this regime of flow speeds, the vortex-shedding frequency no longer follows the Strouhal relationship. Rather, the shedding frequency becomes "locked-in" to the natural frequency of the cylinder. Within the lock-in regime large body motions are observed (the structure undergoes near-resonance vibration).

It is also well known that a hysteresis behavior may exist in the amplitude variation and frequency capture depending on the approach to the resonance range—whether from a low velocity or from a high velocity [88]. As will be discussed later, the two branches of this hysteresis loop are associated with different vortex-shedding

S. Mottaghi et al., *An Analytical Mechanics Framework for Flow-Oscillator Modeling of Vortex-Induced Bluff-Body Oscillations*, Solid Mechanics and Its Applications 260,
https://doi.org/10.1007/978-3-030-26133-7_2

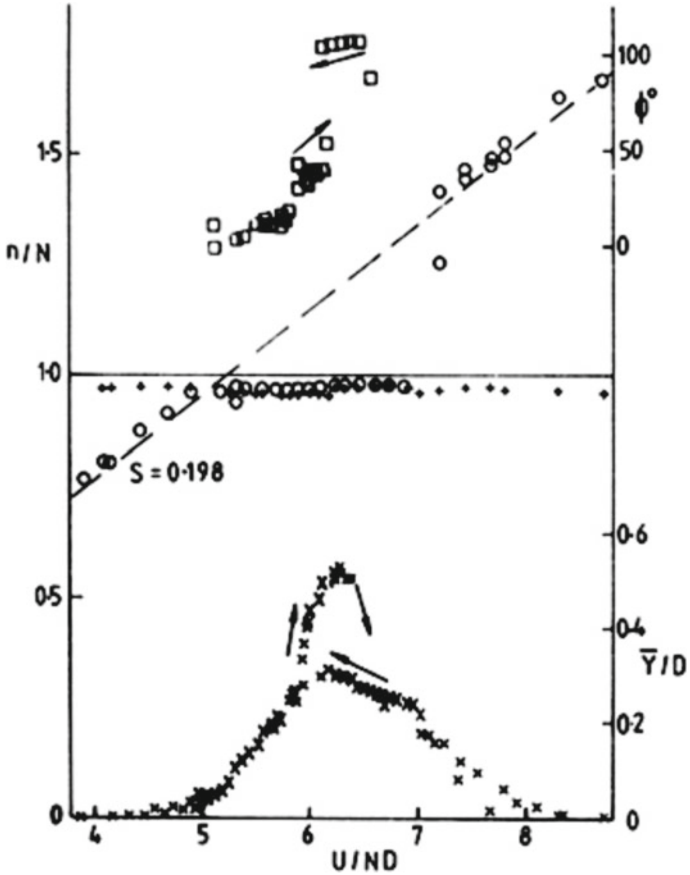

Fig. 2.1 Oscillation characteristics for a freely vibrating circular cylinder with light damping. N is the body oscillation frequency, n is the vortex-shedding frequency, $\overline{V} = D$ is the normalized maximum amplitude of oscillation measured at a particular value of the reduced velocity, and ϕ^o is the phase angle between the fluid force and the cylinder displacement. O, vortex-shedding frequency; +, cylinder frequency; □, phase angle; x, oscillation amplitude [4]. Reprinted with permission of the author

modes and transition between these branches is associated with a phase jump of ~180° [65]. Shown in Fig. 2.1 is a typical response in the lock-in region of a freely vibrating circular cylinder with light damping. The hysteresis effect is clearly seen, with higher amplitudes achieved when the reduced velocity is increased over a certain range. Here, N is the body oscillation frequency, n is the vortex-shedding frequency, $\bar{Y}$ is the maximum amplitude of oscillation measured at a particular value of the reduced velocity, and $\phi°$ is the phase angle between the fluid force and the cylinder displacement. The straight line $S = 0.198$ is the line of constant Strouhal number.

The amplitude of the structural response during lock-in and the band of fluid velocities over which the lock-in phenomenon exists is strongly dependent on a reduced damping parameter expressing the ratio of the damping force to the excitation force. The Scruton number, $Sc = 4\pi m\zeta/\rho D^2$, is but one of many representations for this reduced damping parameter found in the literature. As the reduced damping parameter increases, lock-in becomes characterized by a decreasing peak structural amplitude and occurs over a decreasing band of velocities. It is also worth noting that different phenomena are seen in structures with high and low structure–fluid density ratios $M^* = m/\rho D^2$, where m is the cylinder mass per unit length and ρ is the fluid density. For systems with high M^*, the vortex-shedding frequency is entrained by the structural frequency. For systems with low M^*, it is the fluid oscillation which sets the frequency, and the entrainment frequency instead tends toward the shedding frequency f_v.

The engineering implications of VIV have been well documented in the literature. Structures such as tall buildings, chimneys, stacks, and long-span bridges develop pronounced vibrations when exposed to fluid flow. For example, studies focusing on the VIV of these structures are found in references [18, 22, 59, 74]. The length and higher flexibility of some of these structures further aggravates the problem. In offshore applications, VIV of long slender structures such as pipelines, risers, tendons, and spar platforms challenge engineering designers [17]. Some examples of fundamental studies on the nature of the VIV of marine structures are included in references [25, 30, 50, 100, 105]. Extensive research has also been done in the area of VIV assessment [21, 69, 72] and suppression [3, 49] .

In this review, both experimental and theoretical investigations of the fundamental aspects of vortex-induced vibration of circular cylinders are discussed in some detail. The goal has been to be thorough without being exhaustive. The main focus is on the semi-empirical models used to predict the response of the cylinder to the forces from the flow. These models are not rigorous and generally provide minimal insights into the flow field. To understand the flow effect on a structure, it is important that the actual flow field be described. Consequently, a secondary focus of this review is to discuss the flow characteristics around the cylinder. The flow field generated by flow separation around a body is a very complex fluid dynamics problem. However, much progress has been made toward the understanding of flow around bluff bodies. This is especially true in the field of computational fluid dynamics (CFD), and in keeping with the primary focus of this review, only selected papers highlighting this progress have been included.

Many reviews of the subject have been written that primarily focused on the experimental data [4, 7, 8, 64, 73, 88]. A recent one is by Sarpkaya [92]. While there continues to be extensive work on VIV, this work is still an excellent representation of our understanding. At about the same time, a review paper by two of us [34] focused on semi-empirical, reduced-order, modeling efforts. Since that time, additional review papers have appeared: Williamson and Govardhan [109], Bearman [5], and Wu et al. [111].

While VIV continues to be the subject of intensive research efforts and is quickly evolving, the need for reduced-order models continues to this day. Among their attractions is the fact that they can be used in higher Reynolds number flows than CFD models and they have been solved in both the time and frequency domains. In addition, an alternative new method for the modeling of VIV is discussed, an approach that is the basis for this monograph. The method is based on the variational principles of mechanics and leads to a more fundamental (without ad hoc assumptions) derivation of the reduced-order equations of motion, yet remains inexorably linked to physical data. Experimental data helps to verify the model predictions, thus leading to the most advantageous model framework.

2.2 Experimental Studies

There are innumerable experimental studies on the vortex-induced vibration of bluff bodies, especially circular cylinders. These studies have examined a multitude of phenomena, from vortex shedding from a stationary bluff body to vortex shedding from an elastic body. The vibration caused by vortices generated by the flow past a structure depends on several factors. The correlation of the force components, the Reynolds number, the shedding frequencies, and the added mass effects are just a few of these. The literature is rich with experiments in which many of these factors have been considered, usually by varying one or two factors and holding the rest fixed. Here, key papers highlighting the influences of some of these factors on the structural response are discussed. Attention is focused mostly on results pertaining to the structural response. However, since VIV is indeed a coupled phenomena, some mention must be made of the hydrodynamics.

Before proceeding, it is worthwhile to define those variables that consistently appear in the equations developed in this section of the review. The outer diameter of a circular cylinder is designated by D, the length of the cylinder by L, the free-stream velocity of the flow by U, and the fluid density by ρ. The Strouhal number, S, is defined as $S = f_v D/U$, where f_v is taken to be the natural vortex-shedding frequency of a fixed cylinder. The reduced velocity is defined as $V_r = U/f_n D$, where f_n is the natural frequency of the structure. The normalized damping is defined as $\zeta = c_{sys}/c_{crit}$, where c_{sys} is the system damping, and c_{crit} is the critical damping.

Bearman [4] presents a comprehensive review of experimental studies related to vortex shedding from bluff bodies. He addresses the important question of the role of afterbody shape in vortex-induced vibration and results pertaining to a variety of afterbody shapes are included. Bearman first examines the mechanism of vortex shedding from a fixed bluff body. The presence of two shear layers is primarily responsible for vortex shedding. The presence of the body does not directly cause the vortex shedding, but it instead modifies the vortex- shedding process by allowing feedback between the wake and the shedding of circulation at the separation points.

Another important point discussed is the absence of two-dimensionality in the vortices shed from a two-dimensional bluff body in uniform flow. The spanwise coupling between the two shear layers that lead to the generation of vortex shedding is generally weak. This implies that unsteady quantities related to vortex shedding (e.g., surface pressure) are not constant along the span of the body. However, continuous regions of similar properties are characterized in terms of correlation lengths. Small departures from two-dimensionality, in the form of a taper along the axis of the bluff body or the presence of shear flow, leads to significant reductions in the vortex-shedding correlation length.

Bearman also examines vortex shedding from oscillating bluff bodies. The fundamental difference between fixed and oscillating bluff bodies is that the motion of the cylinder can take control of the instability mechanism that leads to vortex shedding. This is manifested in the capture of the vortex-shedding frequency by the body natural frequency over a range of reduced velocities. The vortex-shedding correlation length is significantly increased when the vortex-shedding frequency coincides with the body oscillation frequency. The range of reduced velocities over which the vortex-shedding frequency coincides with the natural frequency of the body depends on the oscillation amplitude. Larger ranges of frequency capture result from larger oscillation amplitudes.

It is worth pointing out that the capture range will always include the reduced velocity value corresponding to the inverse Strouhal number, and that maximum amplitude is attained near to (but not exactly) this value. In other words, the reduced velocity for maximum amplitude is close to $1/S$. The location of this resonant point within the capture range depends on the shape of the afterbody.

In the capture range, flow conditions around a bluff body change rapidly. The fluctuating lift coefficient increases due to the improved two dimensionality of the flow. This improved two dimensionality (increased correlation length) increases the strength of the shed vortices. The increase in the lift coefficient can also be attributed to the influence of the body motion, which manifests itself through the reduction of the length of the vortex-formation region and the formation of stronger vortices near the base of the body. The mechanism governing the phase of the vortex-induced force relative to the body motion has also been explored by Bearman. The changes in phase angle through the capture range occur in a progressive and not discontinuous fashion. In the lower end of the lock-in range, a vortex formed on one side of the cylinder is shed when the cylinder is near to attaining its maximum amplitude on the opposite side (Mode 1). As the reduced velocity is increased, the timing of vortex shedding suddenly changes, and the same vortex is now shed when the cylinder reaches its maximum amplitude on the same side (Mode 2). Clearly, the point in an oscillation cycle at which the cylinder receives its maximum transverse thrust changes drastically over a narrow range of reduced velocities. Zdravkovich [113] discusses in detail the modification of vortex shedding in the synchronization range. The existence of the two modes, Mode 1 and Mode 2, is used to explain the existence of the hysteresis effect.

Bearman [4] discusses free versus forced vibrations in experiments. Forced vibration experiments offer the advantage that the reduced velocity and amplitude ratios can be independently varied. In free vibration experiments, these two parameters are inseparable, since varying the reduced velocity leads to changes in the amplitude ratio. The major disadvantage of forced vibration experiments is that only a very limited range of reduced velocities and amplitude ratios studied will actually correspond to those encountered in a free vibration. Bearman states that free and forced vibration flows are the same, provided that one assumes that the exact history of motion is inconsequential.

2.2.1 Fluid Forces on an Oscillating Cylinder

Vortex shedding from a circular cylinder produces alternating forces on the cylinder and it is these forces that cause the cylinder to vibrate if it is free to do so. Experiments by Sarpkaya [87] determine the in-phase and out-of-phase components of the time-dependent force acting on a rigid circular cylinder undergoing forced transverse oscillations in a uniform stream. These force components are used in the prediction of the dynamic response of an elastically mounted cylinder in the synchronization range. The details of this aspect of the investigation are relegated to the section of this review describing semi-empirical models. Preliminary experimental work measures the mean fluid-induced force on the cylinder in the direction of flow for various amplitudes and frequencies of cylinder oscillation in the transverse direction. The in-line force is found to increase as A/D increases, where A is the transverse oscillation amplitude. For a given value of A/D, the in-line force reaches a maximum for $D = \overline{V}T$ (mathematically similar to a Strouhal number) in the range 0.18–0.20, where T is the oscillation period and $\overline{V}$ has the same meaning as U. Furthermore, synchronization is found to occur at a frequency slightly lower than the Strouhal frequency for a stationary cylinder, 0.21, corresponding to the range of Reynolds numbers considered by Sarpkaya, 5000–25,000.

In considering the transverse force on the cylinder, the lift coefficient C_L is expressed in terms of an in-phase inertia force and an out-of-phase drag force. The inertia coefficient C_{ml} characterizes the in-phase force, while the out-of-phase force is characterized by the drag coefficient C_{dl}. The drag and inertia coefficients are assumed independent of the Reynolds number in the range considered, 5000–25,000. Synchronization is manifested by a rapid decrease in the inertia coefficient and a rapid increase in the absolute value of the drag coefficient. The experiments also confirm that the net effect of the cylinder–flow interaction near synchronization, for $A/D < 1$, is the same as for periodic flow over a cylinder at rest. This suggests that the fluid becomes the oscillator under these conditions.

The major implication is then that use of the maximum inertia coefficient obtained by oscillating the cylinder in a fluid otherwise at rest, $C_{ml} = 1$, does not give the correct results since C_{ml} has been shown to reach a value of about 2 near synchronization. There is a range of $V_r = \overline{V}T/D$ near perfect synchronization, $V_r \sim 5$, where the drag coefficient is found to be in-phase (negative) with the direction of motion of the cylinder. In this range, the drag coefficient actually helps to magnify the oscillations, and for this reason the range is often referred to as the negative damping region.

Gopalkrishnan [36] measures the vortex-induced lift and drag forces on a smooth circular cylinder undergoing forced sinusoidal oscillations transverse to the free stream. The measurements are conducted in water. The lift force phase angle (defined in the same way as ϕ^o in Fig. 2.1) is found to be very different for large oscillation amplitudes than for small oscillation amplitudes. This is partially responsible for the amplitude-limited nature of VIV. The range of reduced velocities where the cylinder is excited into oscillations by the flow (the lift coefficient excitation region) is found to not coincide with the lock-in region. Furthermore, the excitation region is found to be dependent on the phase, while lock-in is found to be a frequency-dependent effect. The author also measures the lift and drag forces on a cylinder subjected to an amplitude-modulated force causing beating motions. The presence of beating is found to cause a reduction in the mean drag coefficient, an increase in the rms oscillating drag coefficient, and increased extent of the primary excitation regions (vs. sinusoidal excitation). The overall magnitude of the lift coefficient was comparable to that corresponding to sinusoidal forcing.

2.2.2 Three-Dimensionality and Free-Surface Effects

Three-dimensional features naturally arise in the VIV problem, where elastic structures are characterized by their eigenmodes and wake flows show secondary instabilities [30]. The transition to three dimensionality in the near wake of a circular cylinder is discussed by Williamson [106]. Three-dimensional structures in the wake were found to occur for Reynolds numbers greater than about 178. These three-dimensional structures are attributed directly to the deformation of the primary wake vortices, and were not the result of any secondary (Kelvin–Helmholtz) vortices caused by high-frequency oscillations within the separating shear layers. The transition to three dimensionality is found to involve two successive transitions, each characterized by a discontinuity in the Strouhal–Reynolds number relationship. These discontinuities can be seen in Fig. 2.2. The first discontinuity (Re: 170–180) is associated with the transition from periodic and laminar vortex shedding to shedding involving the formation of vortex loops. The second discontinuity (Re: 225–270) is related to the transition from the vortex loops to finer scale streamwise vortices. The first discontinuity is found to be hysteretic, while the second discontinuity is not. A more comprehensive discussion on these discontinuities (so-called Mode A and Mode B secondary $3D$ instabilities), and vortex dynamics in bluff body wakes in general, can be found in two review papers by Williamson [107, 108]. Specifically, comparisons of measurements and theoretical predictions of spanwise instabilities for modes “A” and “B” are given in Fig. 10 of Williamson [108].

The question of three dimensionality in the wake of a surface-piercing rigid cylinder mounted as an inverted pendulum is examined in detail by Voorhees and Wei [102]. The cylinder is characterized by a low mass ratio, $m^* = 1.90$, and high mass-damping, $m^*\zeta = 0.103$. The mass ratio is defined as the mass of the cylinder assembly divided by the mass of water displaced by the cylinder, $m^* = m/\rho\pi r^2 L$. The ratio of mechanical to critical damping is represented by ζ. This study, for Re: 2300–6800, found that the response characteristics of the cylinder are similar to those seen in elastically mounted cylinders of similar m^* and $m^*\zeta$. Strong axial flows

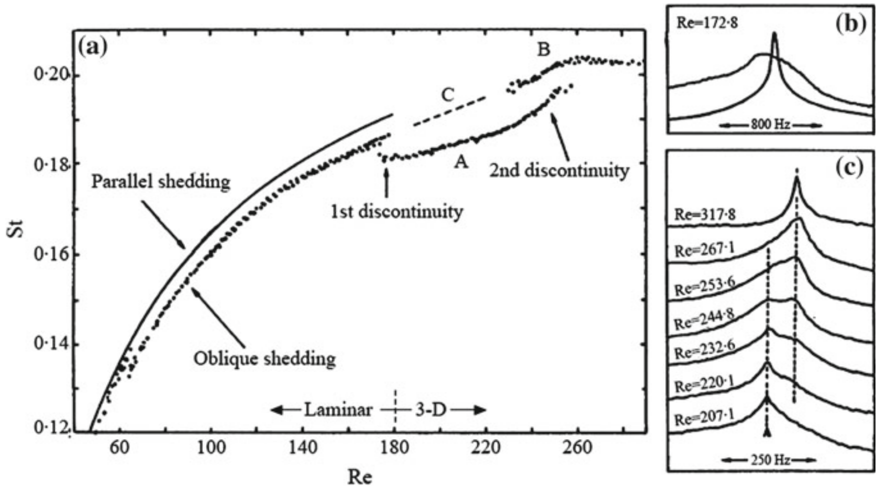

Fig. 2.2 **a** Variation in Strouhal number as a function of Reynolds number; **b** frequency spectra at first discontinuity; **c** frequency spectra at second discontinuity [114]. Reprinted with permission

associated with the Kármán vortices are observed, and these flows are generally directed upwards toward the free surface. Below the free surface, these axial flows can be predominantly attributed to the linearly increasing oscillation amplitude along the span. Near the free surface, however, there is an equal probability of upflow and downflow. These upflows and downflows are shown to be well correlated to the quasi-periodic beating of the cylinder amplitude at the reference reduced velocity $U^* = 4.9$ (Re: 3400) in the synchronization range. In essence, the effect of the free surface is to disrupt the primary upflow mechanism and also to induce lateral spreading of the top portions of the Kármán vortices.

Regarding free surfaces, several fundamental aspects of vortex-formation are found to depend on the gap between the cylinder and the free surface, as discussed by Lin and Rockwell [71] for the case of a fully submerged cylinder oriented parallel to the free surface. The influence of a free surface on the wake structure has also been investigated by Sheridan et al. [94, 95].

2.2.3 Vortex-Shedding Modes and Synchronization Regions

The character of the vortex shedding is important in that it influences lift force phase and, consequently, the energy transfer between the fluid and the body. Williamson and Roshko [110] explore the existence of regions of vortex synchronization in the wavelength–amplitude plane. From the outset, the Reynolds number is not treated as an independent parameter in this study. The Reynolds number is kept within a certain range, $30 < \text{Re} < 1000$, but is never held fixed. The amplitude ratio equals