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Optimization and Inventory Management

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Chapter 1
Economic Production Quantity (EPQ) Inventory Model for a Deteriorating Item with a Two-Level Trade Credit Policy and Allowable Shortages

Ali Akbar Shaikh, Leopoldo Eduardo Cárdenas-Barrón and Sunil Tiwari

Abstract This research work derives an economic production quantity (EPQ) model, and in order to make it a bit close to reality, the stockout is allowed, and this is completely backordered. In addition to this feature, it is incorporated a two-level credit scheme when both supplier and retailer are giving a delay in payment to their respective customers with the aim of enhancing the sales. The inventory model is modeled as a constrained nonlinear optimization problem, and this is resolved by the generalized reduced gradient method (GRG). Moreover, to exemplify and certify the inventory model, five instances are given and solved. Finally, a sensitivity analysis is made for studying the influence of variations of input parameters, modifying one parameter and maintaining the others at their initial input values.

Keywords Production-inventory model · Deteriorating · Constant demand · Full backlogging · Full credit policy

1.1 Introduction

The management of inventories is one of the critical responsibilities that the managers of manufacturing firms need to do carefully. Recently, businesses are highly competitive due to globalization. So, all manufacturing firms are highly engaged in

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how to promote their business in order to have a successful career with the aim of surviving in current volatile markets. For this reason, the researchers and academician are very interested in deriving inventory models that are useful in an inventory decision-making process.

In a supply chain system, there exist several stages and, in each stage, there is a customer that one needs to satisfy. A supply chain system typically involves suppliers, manufacturers, transport system, warehouses capacities, retailers, and the end consumers [8]. The core aim of a supply chain system is to deliver the right products at the right time and right location with a minimum total cost [18].

Trade credit policy is one of the essential issues that are required to promote the business in highly competitive markets. In this context, suppliers/manufacturers give a certain (fixed) time to settle the payment without any interest charge to the customers. This trade credit policy motivates to consumers to procure more goods. With this credit policy, the buyer reduces his/her inventory holding cost because this decision decreases the amount of capital investment during the permissible delay period. It is important to mention that during the interval of the permissible delay in payment, the retailer earns revenue due to the sales of the product as well as obtains interests on that revenue via a banking facility or any other investment alternative. Usually, the trade credit policy enables to increase the demand and capture more and more clientele.

In inventory management, there exist two well-known inventory models: Harris [12] and Taft [19]. The first one is identified as the economic order quantity (EOQ). The second one is called the economic production quantity (EPQ). It is relevant to remark that the EOQ inventory model is a particular case of the EPQ inventory model. Notice that the well-known EOQ inventory model is developed by putting the assumption that when a retailer buys a product, he or she must give the payment to his/her supplier when the items are delivered. For the first time, Goyal [11] introduced the concept of credit policy in an inventory model. Goyal [11] formulated a single-product inventory model with a permissible delay in payment. After that, many EOQ inventory models with trade credit policy have been appearing. In this direction, the reader can see the two comprehensive reviews related to trade credit in Chang et al. [5] and Seifert et al. [15].

The credit strategy examined in Goyal [11] consists in that supplier gives a credit period to his/her retailer. However, the retailer cannot give a credit period to his/her client. This type of problem is identified as a single-level trade credit scheme. The problem becomes more interesting if the retailer gives credit to its client too. Thus, this kind of problem is recognized as a two-level trade credit scheme for a supply chain comprised of supplier–retailer–customer. Perhaps, Huang [13] introduced an EOQ inventory model with two-level trade credit scheme by taking into consideration that the supplier gives to the retailer a delay period (M), and the latter correspondingly gives a delay period (N) to its customer. After that, this type of inventory model has also discussed by Teng [20]. Now, some dissimilarities between Teng [20] and Huang [13] inventory models are mentioned below:

(i) In Huang's [13] inventory model, if the client buys goods from the retailer at time t then he or she must pay the goods at time N. Consequently, the retailer must permit the maximum credit time period to his/her retailer up to time N.
(ii) In Teng's [20] inventory model, if the customer procures products to the retailer at time t which is within in the interval $[0, T]$, then the customer must give payment to its retailer at time $N + t$. Here, the retailer always permits the customer a credit period N. The perspective of Teng [20] is generally applied in the business operations.

Chung and Huang [9] extended Goyal's [11] inventory model. Principally, they developed an EPQ inventory model under a single-level credit policy approach.

Generally, in the real world, there exist so many kinds of goods which deteriorate thru time. So, in inventory analysis, the deterioration cannot be ignored. Ghare and Schrader [10] formulated an EOQ inventory model by considering that the deterioration rate is known and constant. Later, a lot of research works have been done under the trade credit policy.

On the one hand, Liao [14] presented an EPQ inventory model for goods that suffer an exponential deterioration rate involving two-level trade credit scheme based on the concepts Huang's [13] inventory model. On the other hand, Chang et al. [6] discussed an EPQ inventory model with products that suffer an exponential deterioration rate considering two-level trade credit scheme by applying the concepts of Teng [20]. Table 1.1 presents some recent works related to the trade credit scheme. The acronyms IFS and SFI correspond to inventory follows shortage and shortage follows inventory, respectively.

Notice that if demand is higher than the production rate, then shortages appear. In this context, the manufacturing firm decides to cover this shortage. This research work deals with an inventory model in which it is allowed fully backlogged shortages with fully two-level trade credit scheme. The inventory model is formulated as a nonlinear constrained optimization problem. In order to exemplify and certify the inventory model, five examples are presented and solved.

This research work is designed in the following manner. Section 1.2 presents the suppositions and notation. Section 1.3 develops the inventory model. Section 1.4 solves five instances. Section 1.5 gives a sensitivity analysis. Section 1.6 exposes conclusions and research guidelines.

Table 1.1 Some related EOQ/EPQ inventory models with deterioration

Author(s)	Deterioration	Demand rate	Shortages	Level of permissible delay in payments	Inventory policies	EOQ/EPQ
Shah et al. [16]	Yes	Selling price-dependent	No	No	–	EOQ
Wu et al. [21]	Yes	Constant	No	Two levels	–	EOQ
Chen et al. [7]	Yes	Constant	No	Two levels	–	EOQ
Bhunia and Shaikh [2]	Yes	Selling price-dependent	Partial backlogging	Single level	IFS & SFI	EOQ
Bhunia et al. [3]	Yes	Linearly time-dependent	Partial backlogging	No	IFS & SFI	EOQ
Shah and Cárdenas-Barrón [17]	Yes	Constant	No	Two levels	–	EOQ
Bhunia et al. [4]	Yes	Stock-dependent	Partial backlogging	Single level	IFS	EOQ
Bhunia et al. [1]	Yes	Linearly time-dependent	Partial backlogging	Single level	IFS & SFI	EOQ
This research work	Yes	Constant demand	Full backlogging	Two-level trade credit policy	IFS	EPQ

1.2 Suppositions and Notation

1.2.1 Suppositions

The inventory model is based on the suppositions listed below:

1. Demand rate is known and constant.
2. The planning horizon is infinite.
3. Replenishment rate is instantaneous.
4. Stockout is permissible, and unsatisfied demand is fully backlogged.
5. The trade credit policy applies to both retailer and customer.

1.2.2 Notation

The following symbols are utilized during the inventory model development:

Symbol	Units	Description
c_o	\$/order	Replenishment cost
c	\$/unit	Purchasing cost
p	\$/unit	Selling price
c_h	\$/unit/unit time	Holding cost
c_b	\$/unit/unit time	Shortage cost
θ	$\theta \in (0, 1)$	Deterioration rate
P	Units/unit time	Production rate
D	Units/unit time	Demand rate
t_1	Unit time	Time when stock level attains its maximum level
t_2	Unit time	Time when the stock level touches zero
t_3	Unit time	Time when the inventory level achieves its maximum shortage level
T	Unit time	Replenishment cycle
$I(t)$	Units	Inventory level at time t; $0 \leq t \leq T$
M	Unit time	The retailer's trade credit period given by the supplier
N	Unit time	Customer's trade credit period given by the retailer
I_e	%/unit time	Interest earned by the retailer
I_p	%/unit time	Interest paid by the retailer
$TC_i(S, R)$	\$/unit time	The total cost where $i = 1, 2 \ldots, 5$
Decision variables		
S	Units	Order quantity
R	Units	Shortage level

1.3 Inventory Model Formulation

The differential equations that define the inventory level thru time t during the period $[0, T]$ are expressed below:

$$\frac{dI(t)}{dt} + \theta I(t) = P - D, \quad t \in [0, t_1] \tag{1.1}$$

$$\frac{dI(t)}{dt} + \theta I(t) = -D, \quad t \in (t_1, t_2] \tag{1.2}$$

$$\frac{dI(t)}{dt} = -D, \quad t \in (t_2, t_3] \tag{1.3}$$

$$\frac{dI(t)}{dt} + \theta I(t) = P - D, \quad t \in (t_3, T] \tag{1.4}$$

The following results are obtained when differential equations (1.1)–(1.4) are solved taking into account the boundary conditions $I(0) = 0, I(t_1) = S, I(t_3) = -R$, and $I(t)$ is continuous at $t = t_1, t_2$ and t_3:

$$I(t) = \frac{P - D}{\theta}\left(1 - e^{-\theta t}\right), \quad t \in [0, t_1] \tag{1.5}$$

$$I(t) = \frac{D}{\theta}\left(e^{\theta(t_2 - t)} - 1\right), \quad t \in [0, t_1] \tag{1.6}$$

$$I(t) = D(t_2 - t), \quad t \in (t_2, t_3] \tag{1.7}$$

$$I(t) = \frac{P - D}{\theta}\left(1 - e^{\theta(T - t)}\right), \quad t \in (t_3, T] \tag{1.8}$$

Using the boundary and continuity conditions, then the following results are obtained:

$$t_1 = -\frac{1}{\theta}\log\frac{P - D - S\theta}{P - D} \tag{1.9}$$

$$t_2 = t_1 + \frac{1}{\theta}\log\frac{D + S\theta}{D} \tag{1.10}$$

$$t_3 = t_2 + \frac{R}{D} \tag{1.11}$$

$$T = t_3 + \frac{1}{\theta}\log\frac{(P - D) + R\theta}{P - D} \tag{1.12}$$

The total cost of the inventory model is comprised of the terms listed below:

(a) The ordering cost (OC):

$$OC = c_o \tag{1.13}$$

(b) The inventory holding cost (*HC*):

$$HC = c_h\left(\int_0^{t_1} I(t)dt + \int_{t_1}^{t_2} I(t)dt\right) = c_h\left[\frac{(P-D)}{\theta}\left\{t_1 + \left\{e^{-\theta t_1} - 1\right\}\right\} + \frac{D}{\theta^2}\left\{e^{\theta(t_2-t_1)} - \theta(t_2 - t_1) - 1\right\}\right] \tag{1.14}$$

(c) The purchase cost (*PC*):

$$PC = c(S + R) \tag{1.15}$$

(d) The shortage cost (*SC*):

$$SC = c_b\left[\int_{t_2}^{t_3} [-I(t)]dt + \int_{t_3}^{T} [-I(t)]dt\right] = \left[\frac{D}{2}(t_3 - t_2)^2 + \frac{(P-D)}{\theta^2}\left\{e^{\theta(T-t_3)} - \theta(T - t_3) - 1\right\}\right] \tag{1.16}$$

Taking into consideration the credit period given by supplier to its retailer (M) and credit period provided by the retailer to his/her customer (N), the following five cases are identified: **Case 1**: $N < M \le t_1 < t_2$, **Case 2**: $N < t_1 \le M \le t_2$, **Case 3**: $t_1 \le N < M \le t_2$, **Case 4**: $t_1 \le N < t_2 \le M$, and **Case 5**: $t_1 < t_2 \le N < M$. These are explained below.

Case 1: $N < M \le t_1 < t_2$ (Fig. 1.1)

Here, the customer's credit period (N) is less than the retailer credit period (M), where M is less than or equal to t_1. So, the retailer needs to pay the interest during $[M, t_2]$. Owing to customer credit period retailer can earn interest in the period $[N, M]$. Therefore, the interest paid (*IP*) is determined with $cI_c\left[\int_M^{t_1} I(t)dt + \int_{t_1}^{t_2} I(t)dt\right]$.

Hence,

$$IP = cI_c\left[\frac{(P-D)}{\theta}\left\{(t_1 - M) + \frac{1}{\theta}\left(e^{-\theta t_1} - e^{-\theta M}\right)\right\} + \frac{D}{\theta^2}\left\{e^{\theta(t_2-t_1)} - \theta(t_2 - t_1) - 1\right\}\right] \tag{1.17}$$

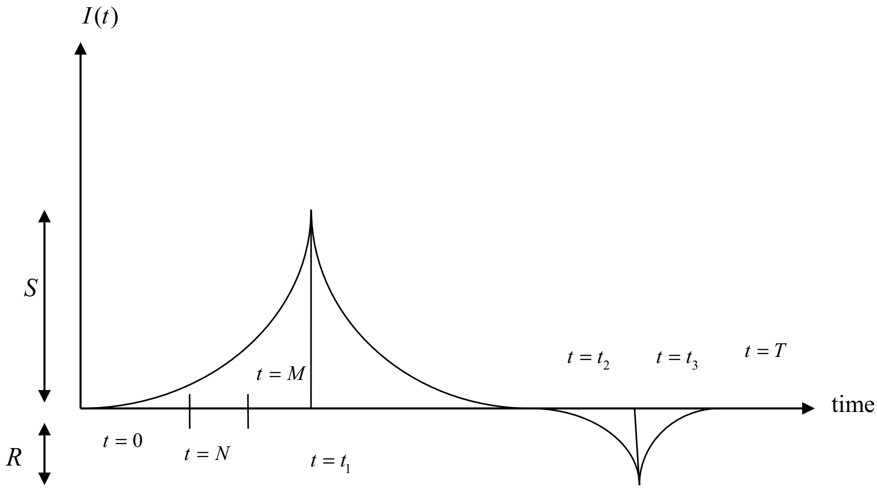

Fig. 1.1 When $N < M \leq t_1 < t_2$

Moreover, the interest earned (IE) is calculated with $pI_e \int_N^M \int_0^t Ddudt$.

Thus,

$$IE = \frac{pI_e D(M^2 - N^2)}{2} \tag{1.18}$$

Consequently, the total cost

$$TC_1(S, R) = \frac{X_1}{T} \tag{1.19}$$

where

$$X_1 = c_o + c(S + R) + HC + SC + IP - IE \tag{1.20}$$

Now, the optimization problem is formulated as follows:
Problem 1

$$\left.\begin{array}{ll} \text{Minimize} & TC_1(S, R) = \dfrac{X_1}{T} \\ \text{subject to} & N < M \leq t_1 < t_2 \end{array}\right\} \tag{1.21}$$

Case 2: $N < t_1 \leq M \leq t_2$ (Fig. 1.2)

The customer's credit period (N) is less than the retailer credit period (M) specified by the supplier where M is greater than or equal to t_1. For that reason, it is necessary

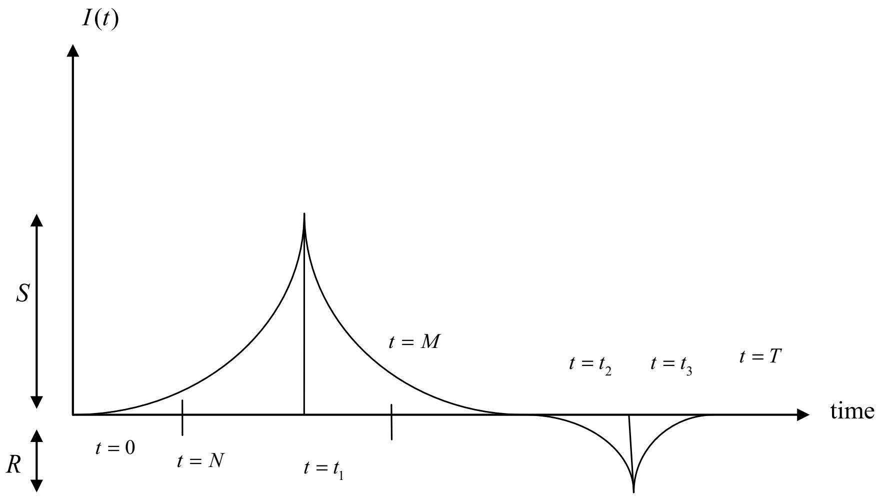

Fig. 1.2 When $N < t_1 \leq M \leq t_2$

that the retailer pays the interest for the duration of the following interval $[M, t_2]$. On the other hand, owing to customer credit period the retailer obtains interest within of the interval $[N, M]$. Then, the interest paid is obtained with $cI_c\left[\int_M^{t_2} I(t)dt\right]$.

Thus,

$$IP = cI_c\left[\frac{D}{\theta^2}\left\{e^{\theta(t_2-M)} - \theta(t_2 - M) - 1\right\}\right] \tag{1.22}$$

and the interest earned is calculated with $pI_e \int_N^M \int_0^t Ddudt$.

Therefore,

$$IE = \frac{pI_e D(M^2 - N^2)}{2} \tag{1.23}$$

Consequently, the total cost is written as

$$TC_2(S, R) = \frac{X_2}{T} \tag{1.24}$$

where

$$X_2 = c_o + c(S + R) + HC + SC + IP - IE \tag{1.25}$$

Hence, the optimization problem is

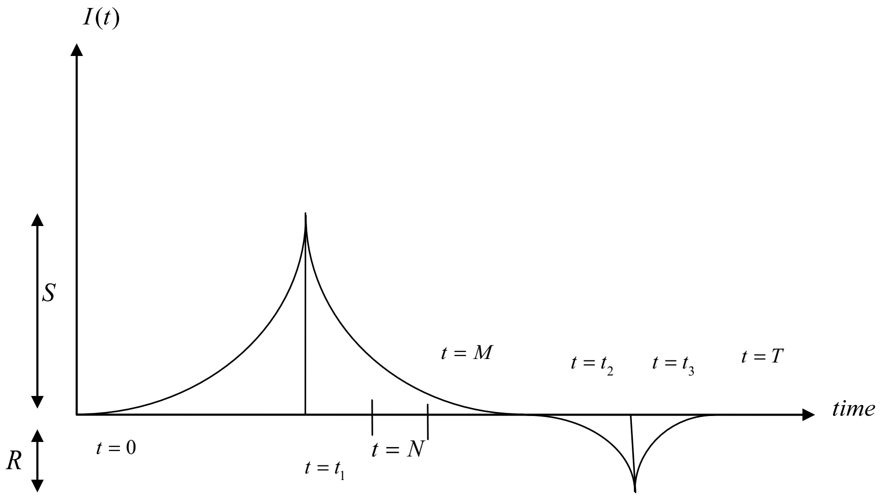

Fig. 1.3 When $t_1 \leq N < M \leq t_2$

Problem 2

$$\left.\begin{array}{ll}\text{Minimize} & TC_2(S, R) = \dfrac{X_2}{T} \\ \text{subject to} & N < t_1 \leq M < t_2\end{array}\right\} \tag{1.26}$$

Case 3: $t_1 \leq N < M \leq t_2$ (Fig. 1.3)

Notice that the customer's credit period (N) settled by the retailer is less than the retailer's credit period (M) established by the supplier where M is less than or equal to t_2 and greater than t_1. Thus, the retailer needs to cover the interest for the time period $[M, t_2]$. In contrast, due to the customer's credit interval, the retailer gets interest through the time period $[N, M]$. So, the interest paid is computed with $cI_c\left[\int_M^{t_2} I(t)dt\right]$.

Then

$$IP = cI_c\left[\frac{D}{\theta^2}\left\{e^{\theta(t_2-M)} - \theta(t_2 - M) - 1\right\}\right] \tag{1.27}$$

and interest earned is given by $pI_e \int_N^M \int_0^t Ddudt$

$$IE = \frac{pI_e D(M^2 - N^2)}{2} \tag{1.28}$$

For that reason, the total cost is expressed as

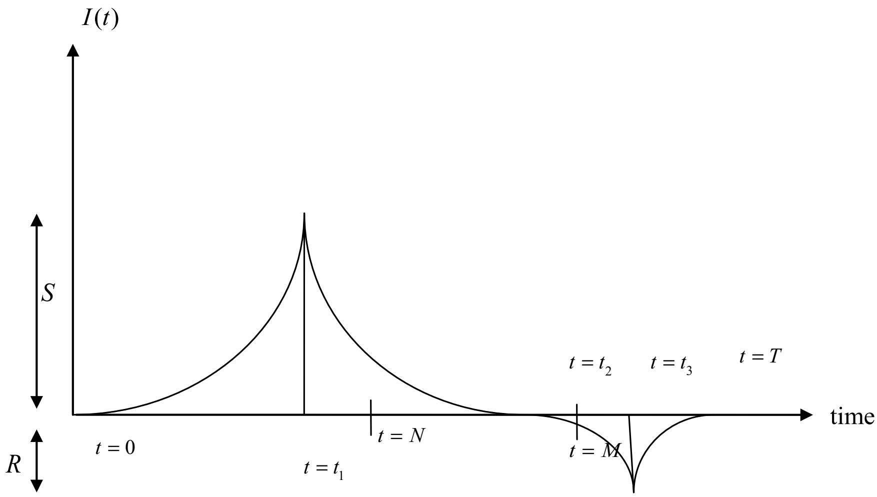

Fig. 1.4 When $t_1 \le N < t_2 \le M$

$$TC_3(S, R) = \frac{X_3}{T} \tag{1.29}$$

where

$$X_3 = c_o + c(S + R) + HC + SC + IP - IE \tag{1.30}$$

Here, the optimization problem is presented as follows:

Problem 3

$$\left.\begin{array}{r}\text{Minimize} \quad TC_3(S, R) = \dfrac{X_3}{T} \\ \text{subject to} \quad t_1 \le N < M \le t_2\end{array}\right\} \tag{1.31}$$

Case 4: $t_1 \le N < t_2 \le M$ (Fig. 1.4)

Here, the retailer's credit period (M) is greater than or equal to (t_2), so it is not necessary that the retailer pay the interest. Consequently, the interest paid is $IP = 0$. But retailer gains interest, and it is calculated as follows: $pI_e \int_N^M \int_0^t D du dt$.

Thus,

$$IE = \frac{pI_e D(M^2 - N^2)}{2} \tag{1.32}$$

The total cost is written as

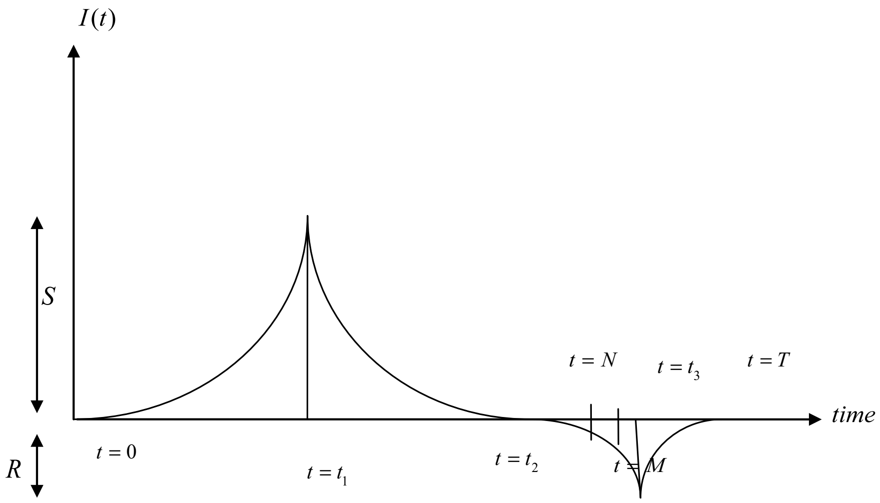

Fig. 1.5 When $t_1 < t_2 \leq N < M$

$$TC_4(S, R) = \frac{X_4}{T} \tag{1.33}$$

where

$$X_4 = c_o + c(S + R) + HC + SC + IP - IE \tag{1.34}$$

Now, the optimization problem becomes

Problem 4

$$\left.\begin{aligned} &\text{Minimize} \quad TC_4(S, R) = \frac{X_4}{T} \\ &\text{subject to} \quad t_1 \leq N < t_2 \leq M \end{aligned}\right\} \tag{1.35}$$

Case 5: When $t_1 < t_2 \leq N < M$ (Fig. 1.5)

Both the retailer credit period (M) and customer credit period (N) are greater than (t_2). So, the retailer does not need to pay interest. Therefore, the interest paid is $IP = 0$. But retailer wins interest, and it is $pI_e \int_N^M \int_0^t Ddudt$.

Thus,

$$IE = \frac{pI_e D(M^2 - N^2)}{2} \tag{1.32}$$

Here, the total cost is determined by

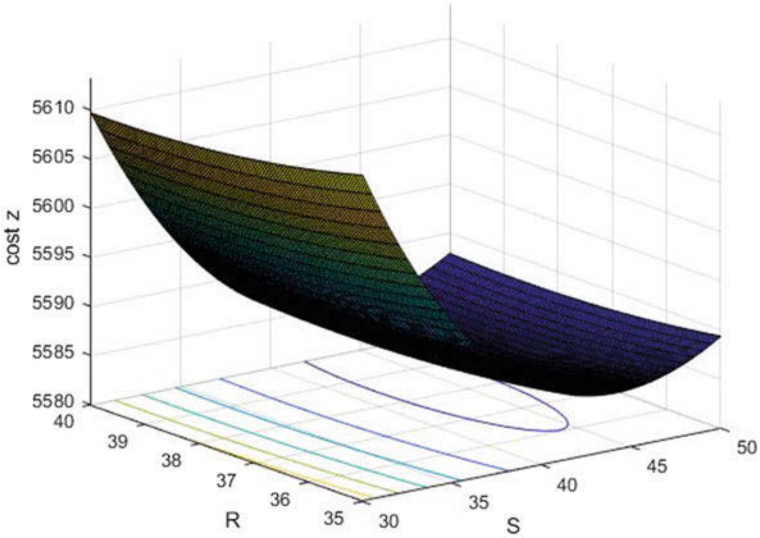

Fig. 1.6 Convexity of the objective function for Case 1

$$TC_5(S, R) = \frac{X_5}{T} \tag{1.33}$$

where

$$X_5 = c_o + c(S + R) + HC + SC + IP - IE \tag{1.34}$$

Here, the optimization problem is stated as

Problem 5

$$\left.\begin{array}{ll} Minimize & TC_5(S, R) = \frac{X_5}{T} \\ subject\, to & t_1 < t_2 \leq N < M \end{array}\right\} \tag{1.35}$$

The necessary conditions for optimality of objective function are $\frac{\partial TC_i()}{\partial S} = 0$ and $\frac{\partial TC_i()}{\partial R} = 0$. The sufficiency conditions are

$$\frac{\partial T^2 C_i()}{\partial S^2} \geq 0, \quad \frac{\partial T^2 C_i()}{\partial R^2} \geq 0$$

$$\text{and} \left(\frac{\partial T^2 C_i()}{\partial S^2}\right)\left(\frac{\partial T^2 C_i()}{\partial R^2}\right) - \left(\frac{\partial T^2 C_i()}{\partial S \partial R}\right)^2 \geq 0$$

The corresponding optimization problem is highly nonlinear in nature. So, it is difficult to prove the optimality analytically. For that reason, the convexity is shown graphically (see Fig. 1.6).

1.4 Numerical Examples

With the intention of exemplifying and certify the inventory model, five instances are presented and solved. Each example illustrates one case of the inventory model. The data of these instances are given in Table 1.2.

The generalized reduced gradient method (GRG) is applied in order to solve the five numerical examples. Table 1.3 presents the optimal solution for all examples.

1.5 Sensitivity Analysis

The instance 1 is utilized to analyze the influence of over/underestimation of input data on the optimal solution of the initial stock level (S), maximum shortage level (R), cycle length (T), the total cost (TC), and time periods: t_1, t_2, and t_3. The analysis is performed by modifying (decreasing/increasing) the input data by $+20\%$ to -20%. The results are computed by varying one input datum and maintaining the other data with original value. Table 1.4 displays the results of the sensitivity analysis.

From Table 1.4, the following observations are mentioned:

(i) With the increase in the value of replenishment cost c_o, the total cost (TC), highest stock (S), shortage level (R), and the replenishment cycle (T) increase, which is an obvious result.

(ii) As the holding cost c_h increases, the total cost (TC) and shortage level (R) increase whereas the highest stock (S) and the replenishment cycle (T) decrease. On the other hand, with the increase in production rate (P), the total cost (TC) and highest stock (S) increase but the replenishment cycle (T) decreases.

(iii) Higher the deterioration rate θ, higher the total cost (TC) but lesser the highest stock (S) and the replenishment cycle (T). A higher deterioration rate means more deteriorated items, which results in an increase in deterioration cost.

(iv) An increment in the shortage cost means more total cost (TC) as well as more highest stock (S). Whereas for an increase in the value of purchasing cost (c), the total cost (TC) and shortage level (R) increase but the highest stock (S) and the replenishment cycle decrease. Moreover, as the value of selling price increases (p), the total cost (TC), highest stock (S), shortage level (R), as well as the replenishment cycle (T) decrease significantly.

Table 1.2 Data for the instances

Instance	c_o \$/order	D units/year	P units/year	p \$/unit	c \$/unit	h \$/unit/year	I_p %/year	I_e %/year	M year	N year	θ	c_b \$/unit/year
1	150	600	3000	45	20	15	15	9	0.1	0.05	0.05	20
2	150	500	1000	40	30	15	15	9	0.1	0.05	0.05	20
3	100	500	1000	40	25	15	12	9	0.1	0.07	0.05	20
4	200	550	900	45	30	15	12	9	0.2	0.8	0.05	25
5	170	600	800	45	35	20	12	7	0.12	0.07	0.07	30

Table 1.3 The optimal solution for all instances

Instance	Case	S	R	t_1	t_2	t_3	T	TC
1	$N < M \leq t_1 < t_2$	44.0621	38.5184	0.1105	0.1838	0.2480	0.3440	5584.208
2	$N < t_1 \leq M \leq t_2$	43.2274	39.88456	0.0866	0.1729	0.2527	0.3323	8312.618
3	$t_1 \leq N < M \leq t_2$	34.9388	31.9287	N	0.1398	0.2036	0.2674	6898.536
4	$t_1 \leq N < t_2 \leq M$	27.9441	27.9441	N	0.1307	0.1815	0.2612	7319.905
5	$t_1 < t_2 \leq N < M$	10.4871	10.4871	0.0525	N	0.0874	0.13981	6533.081

Table 1.4 Sensitivity analysis for instance 1

Parameters	% of variation in parameter	TC^*	% of the change in					
			S^*	R^*	t_1^*	t_2^*	t_3^*	T^*
c_o	−20	−1.66	−11.82	−11.82	−11.85	−11.83	−11.83	−11.82
	−10	0.80	−5.72	−5.72	−5.74	−5.73	−5.73	−5.72
	10	0.76	5.41	5.41	5.43	5.42	5.42	5.41
	20	1.48	10.56	10.56	10.60	10.57	10.57	10.56
c_h	−20	−0.70	14.25	−5.10	14.29	14.26	9.27	5.29
	−10	−0.33	6.61	−2.37	6.63	6.62	4.29	2.23
	10	0.30	−5.80	2.14	−5.81	−5.80	−3.74	−2.10
	20	0.57	−10.92	4.09	−10.95	−10.93	−7.04	−3.94
P	−20	−35.08	−18.90	−21.05	62.48	30.03	16.81	28.22
	−10	−15.49	−7.62	−8.69	23.26	10.95	5.86	10.28
	10	12.58	5.46	6.43	−15.67	−7.24	−3.70	−6.81
	20	23.00	9.52	11.35	−27.04	−12.46	−6.30	−11.72
D	−20	3.75	1.17	2.87	−22.47	−3.13	5.08	−2.16
	−10	3.27	0.85	2.19	−12.06	−2.31	1.79	−1.81
	10	−6.07	−2.75	−3.76	14.46	4.07	−0.22	3.52
	20	−14.95	−7.29	−9.24	32.57	10.52	1.49	9.33
θ	−20	−0.02	−0.25	0.33	−0.31	−0.27	−0.12	0.02
	−10	−0.01	−0.13	0.16	−0.15	−0.14	−0.06	0.01
	10	0.01	0.13	−0.16	0.15	0.14	0.06	−0.01

(continued)

Table 1.4 (continued)

Parameters	% of variation in parameter	TC^*	% of the change in					
			S^*	R^*	t_1^*	t_2^*	t_3^*	T^*
	20	0.02	0.25	−0.33	0.31	0.27	0.12	−0.02
c_b	−20	−0.74	−5.25	17.92	−5.26	−5.25	0.75	5.53
	−10	−0.34	−2.44	8.18	−2.45	−2.45	0.31	2.50
	10	0.30	2.15	−6.99	2.16	2.15	−0.21	−2.10
	20	0.57	4.06	−13.03	4.07	4.06	−0.36	−3.89
c	−20	−17.26	2.73	−0.15	2.74	2.74	1.99	1.39
	−10	−8.63	1.35	−0.07	1.35	1.35	0.98	0.69
	10	8.63	−1.32	0.06	−1.32	−1.32	−0.96	−0.68
	20	17.26	−2.60	0.11	−2.61	−2.61	−1.90	−1.34
p	−20	0.09	0.67	0.67	0.68	0.67	0.67	0.67
	−10	0.05	0.34	0.34	0.34	0.34	0.34	0.34
	10	−0.05	−0.34	−0.34	−0.34	−0.34	−0.34	−0.34
	20	−0.1	−0.68	−0.68	−0.68	−0.68	−0.68	−0.68
M	−20	0.34	2.39	2.39	2.40	2.39	2.39	2.39
	−10	0.18	1.27	1.27	1.27	1.27	1.27	1.27
	10	−0.20	−1.42	−1.42	−1.43	−1.42	−1.42	−1.42
	20	−0.42	−3.00	−3.00	−3.01	−3.01	−3.00	−3.00
N	−20	−0.06	−0.41	−0.41	−0.41	−0.41	−0.41	−0.41
	−10	−0.03	−0.21	−0.21	−0.21	−0.21	−0.21	−0.21
	10	0.03	0.24	0.24	0.24	0.24	0.24	0.24
	20	0.07	0.49	0.49	0.50	0.49	0.49	0.49

(v) As the supplier increases, the credit period (M) for its retailer, the total cost (TC), highest stock (S), shortage level (R), and the replenishment cycle (T) decrease.

(vi) As the retailer increases, the credit period (N) for its customer, the total cost (TC), highest stock (S), shortage level (R), and the replenishment cycle (T) increase.

(vii) The time periods (t_1, t_2, t_3) are highly sensitive regarding parameters c_o, c_h, P and D; moderately sensitive regarding c, c_b and M; and less sensitive regarding the rest parameters.

(viii) The cycle length (T) is very sensitive pertaining to the data of c_o, c_h and P; discreetly sensitive relating to D, c_b, c and M; and less sensitive in regard to the other data.

1.6 Conclusion

This research work develops a production-inventory model for an article that deteriorates considering fully backlogged shortages and full two-level trade credit scheme. Here, it is supposed that demand function and production rate are known and constant. A solution method for the inventory model is developed. The validation and effectiveness of the proposed inventory model are assessed through numerical examples. The findings suggest significance importance of the proposed inventory model to the retail industry for decision-making under realistic scenarios.

This inventory model can be extended by considering stock-dependent demand, price-dependent demand, stochastic demand, partial backlogging, inflation, partial trade credit policy, fuzzy-valued inventory cost, interval valued inventory costs, overtime production rate, and imperfect production processes. These are some interesting topics that researchers can explore.

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Chapter 2
An Economic Order Quantity (EOQ) Inventory Model for a Deteriorating Item with Interval-Valued Inventory Costs, Price-Dependent Demand, Two-Level Credit Policy, and Shortages

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Abstract In today's competitive environment, every leading organization wishes to improve the pricing strategies in order to increase revenue, credit policy is one of the best tools of it. This research work develops an economic order quantity (EOQ) inventory model for a deteriorating item that considers interval-valued inventory costs, price dependent demand, two-level credit policy, and shortages. Due to high and uncertainty in demand, sometimes organizations have to face the situation of stock out. So, keeping this scenario in mind, this work considers the situation of partially backlogging. Here, it is developed an EOQ inventory model by considering a non-linear interval-valued constrained optimization problem. Two types of particle swarm optimization (PSO) algorithm are used to resolve it, and then the results are compared. Sensitivity analysis is done in order to investigate the impact of key parameters on decision-making. Finally, conclusions along with some managerial insights are given.

Keywords Inventory · Deterioration · Price-dependent demand · Partial shortages · Interval-valued inventory costs · Two-level credit policy

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2.1 Introduction

In the inventory management literature, very little research work has been done in relation to considering that the inventory costs are represented in an interval-valued. Many researchers assume that the inventory costs such that the ordering cost, inventory carrying cost, and purchasing cost are expressed as a fixed value known. Nevertheless, in reality, all of the mentioned costs are imprecise numbers in nature instead of a fixed value due to the fact that generally, the inventory costs fluctuate by reason of several factors such as changes in prices. In order to explain why it needs to use an interval number rather than the fixed value number, the following reason is mentioned. Normally, the inventory carrying cost is distinct during the seasons of the year. For example, the deterioration rate is different in summer and winter. During summer time, it is necessary to use preservation technology with the intention of decreasing the deterioration percentage of some perishable products and therefore the holding cost is different from holding cost in the winter time. Another cost that also varies is the labor charges, which change over the period of time.

To overcome the problem of imprecise numbers, the researchers and academicians use the following approaches: (i) stochastic, (ii) fuzzy, and (iii) fuzzy-stochastic. In the case of the stochastic approach, the inventory data are considered as random variables with a given and known probability distribution. In the case of the fuzzy approach, the data of the inventory system and the constraints are expressed with fuzzy sets with a given and known membership function. In the case of fuzzy-stochastic approach, some inventory data are supposed to be represented by fuzzy sets and the rest of the inventory parameters are assumed random variables. But it is not an easy task to select the most suitable membership function or probability distribution.

With the aim of avoiding the complexity in the selection of the right membership function or the right probability distribution, it is suggested to use interval numbers. With this, the imprecise problem is converted to an interval-valued problem, which can be solved, by any soft computing optimization technique, such as the different versions of particle swarm optimization (PSO) or genetic algorithm (GA). In this connection, the reader can see the related works, which apply interval number into the area of inventory control. Gupta et al. [1] applied the interval concept in the field of inventory theory. They resolved an inventory problem with interval-valued inventory costs using a genetic algorithm approach. After that, Dey et al. [2] formulated an inventory model considering interval-valued lead time. Again, Gupta et al. [3] developed an inventory model using interval-valued inventory costs. Bhunia et al. [4] solved a stock-dependent inventory model with interval-valued inventory costs using particle swarm optimization (PSO). Afterward, Bhunia and Shaikh [5] built a two-warehouse inventory model with inflation, and they solved it using particle swarm optimization.

In the current competitive markets, the permissible delay in payment has a vital role in promoting the business. Normally, the suppliers give different types of facilities to retailers, and the retailers give some facilities to their direct customers. This