Bienvenido Barraza Martínez
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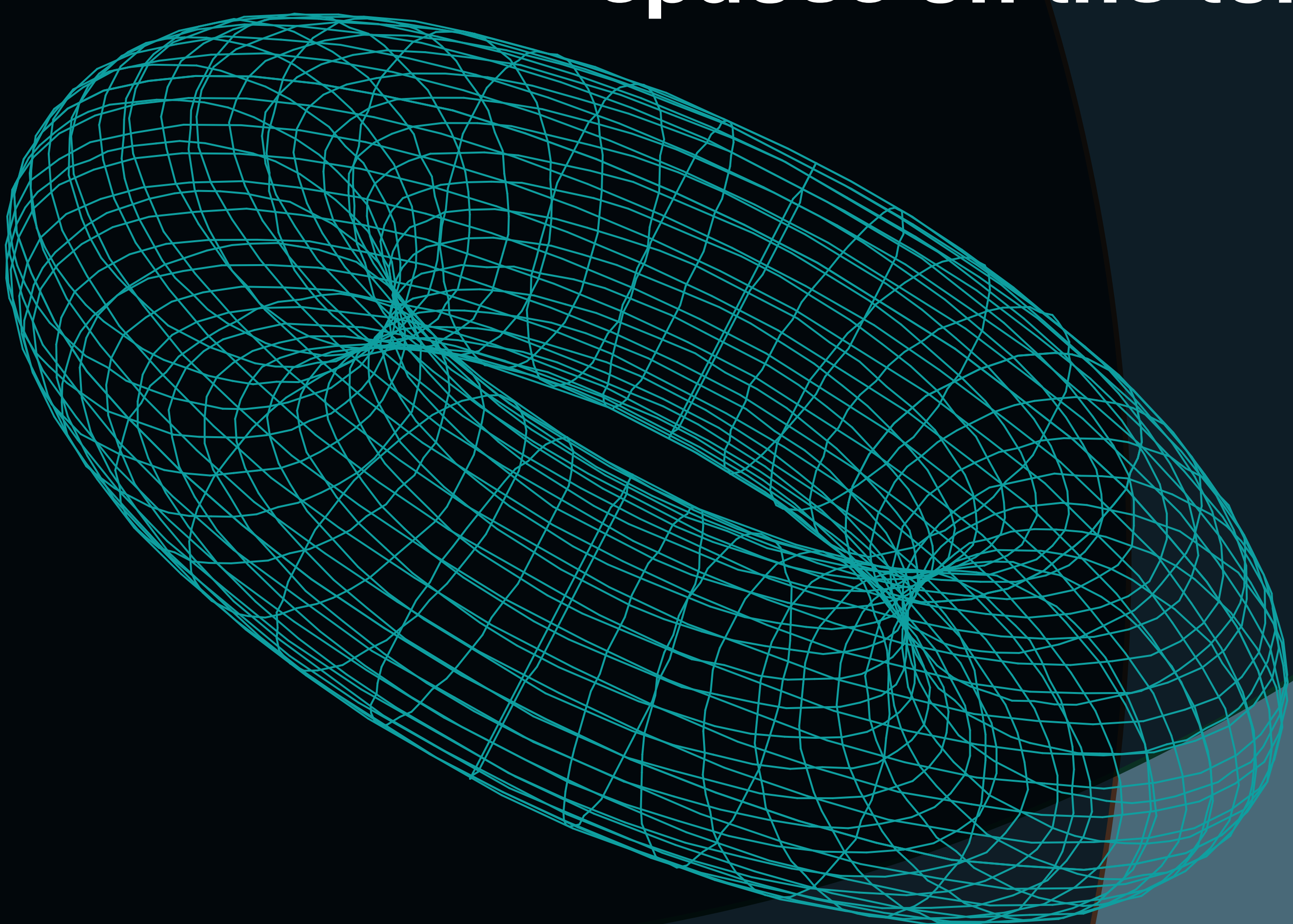
Vector-valued function and distribution spaces on the torus



Universidad del Norte
Editorial

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Prefacio

Este libro contiene detalles de las propiedades que satisfacen ciertos espacios de funciones y distribuciones vector valuadas definidas en el toro n-dimensional. En particular, el texto aborda un estudio introductorio de los espacios de Besov toroidales, los cuales hacen su aparición en muchas aplicaciones a las ecuaciones diferenciales parciales (EDPs).

El libro surge de la motivación que dejó el proyecto de investigación titulado *Operadores Pseudodiferenciales Banach vector-valuados en el toro n-dimensional* (financionado por Colciencias 2013-2015, código 121556933488, que tuvo los productos [9], [7] y [14], entre otros, y que contó con la colaboración de Robert Denk y su grupo de investigación adscrito al Departamento de Matemáticas y Estadística de la Universidad de Konstanz, Alemania). En aquel momento, en la búsqueda de la literatura pudimos encontrar célebres y excelentes textos alrededor de la temática de los espacios de Besov toroidales (ver [6], [30] y [32]), pero a diferencia de dichos textos, aquí se incluye el caso vector valuado sobre el toro n-dimensional $\mathbb{T}^n$ $(n \geq 1)$ y se hace un esfuerzo por presentar con claridad y en detalle todo lo que corresponde al toro, y de paso brindar al lector una entrada más agradable a esta temática que es introductoria, por ejemplo, en el estudio del análisis armónico y las EDPs con condiciones periódicas. En lo concerniente a $\mathbb{R}^n$ no entramos en muchos detalles porque eso conllevaría a muchas páginas de explicaciones, lo cual está por fuera de nuestra manifestada intención, además de existir mucha literatura al respecto.

Esperamos que este libro sea de gran interés para estudiantes de pregrado y posgrado en matemáticas, y también para matemáticos investigadores que quieran tener un primer contacto con la temática arriba expuesta.

Como apoyo al lector para ubicar fácilmente información relevante en el texto, ubicamos al final de este una lista bibliográfica, una lista de símbolos y un índice alfabético.

Agradecemos de antemano los comentarios, sugerencias y eventuales correcciones que los lectores nos puedan enviar vía email para tenerlos en cuenta en una posible versión mejorada.

Por último, pero no menos importante, extendemos nuestro agradecimiento a la Universidad del Norte por brindarnos el espacio y las herramientas para poder elaborar este libro y a su editorial por la publicación de los frutos de nuestra investigación.

Barranquilla, enero de 2019 — Los autores

Preface

This book contains details of the properties that satisfy certain function spaces and vector-valued distributions defined in the n-dimensional torus. In particular, the text deals with an introductory study of toroidal Besov spaces, which make their appearance in many applications to partial equations differential (PDEs).

The book is born out of the motivation left by the research project titled *Operadores Pseudodiferenciales Banach vector-valuados en el toro n-dimensional* (financed by Colciencias 2013-2015, code 121556933488, which included the products [9], [7] and [14], among others, and which had the collaboration of Robert Denk and his research group attached to the Department of Mathematics and Statistics at the University of Konstanz, Germany). At that time, in a literary search, we found several well-known and excellent texts on the subject of Besovs toroidal spaces (see [6], [30] and [32]), but in contrast to these texts, here we have included the vector valued case on the n-dimensional torus $\mathbb{T}^n$ $(n \geq 1)$ and an effort is made to present clearly and in detail everything that corresponds to the torus. Our goal is to provide the reader with a more pleasant introduction to this subject, for example, in the study of harmonic analysis and PDEs with periodic conditions. Concerning $\mathbb{R}^n$ we do not discuss many details because that would lead to many pages of explanations, which is outside of our stated intention, and a significant amount of literature on the subject already exists.

We expect that this book will be of great interest to undergraduate and graduate students in mathematics, as well as to mathematical researchers who would like an introduction to the previously mentioned topic.

To support the reader in easily locating relevant information in the text, a bibliographic list, a list of symbols and an alphabetical index are located at the end of the text.

We would like to thank readers in advance for any comments, suggestions and possible corrections that may be considered for a future version.

Last, but not least, we extend our thanks to Universidad del Norte for providing us the space and tools to produce this book and to its publisher for publishing the fruits of our research.

Barranquilla, January 2019 The authors

Introduction

The interest of considering spaces of functions on the torus is because these can be identified with 2π-periodic functions, so it is natural to look for solu-tions of EDPs with periodic border conditions in these spaces. Now, boundary conditions are understood in the sense of the trace, which is related to the trace method and the trace spaces in the theory of interpolation. A. Lunardi in his book [20] establishes the existing relationship between trace spaces and certain real interpolation spaces, which some authors use to define Besov spaces. In this text, we define these spaces using the standard method known as *dyadic decomposition*; however, we introduce a chapter on real interpola-tion theory in order to establish that real interpolation between vector valued toroidal Besov spaces also results in Besov spaces. It is important to mention that better regularity results can be established for the solution of parabolic evolution equations when the data is in certain interpolation spaces (as can be seen in the book by H. Amann [3]) and that the theory of interpolation can be used to demonstrate the regularity of the solution to these problems of evolution (as can be seen in the work of Lunardi [21]).

The Besov spaces on $\mathbb{R}^n$ owe their appearance to the Russian mathematician, Oleg Vladimirovich Besov (born 1933), who introduced them into his text *Integral Representations of Functions and Embedding Theorems*, Nauka-Moscow 1975. Forty-three years have passed since the creation of these spaces and the literature on the subject is becoming more and more extensive. These are elements with which mathematicians work hard and whose efforts are reflected in the production of articles around this theme. A quick search of the literature will provide plenty of evidence to this fact. It is important to mention that certain Besov spaces coincide with certain Sobolev spaces and are related to certain spaces of well-known functions in literature such as: L^p spaces, Hölder-Zygmund spaces, Bessel-Potencial spaces (or Lebesgue or Liouville), Hardy spaces, Lipschitz spaces and Slobodeckij spaces, see Section 3.5.4 from [30] and Section 2.2.2 from [32].