

Francesco dell'Isola

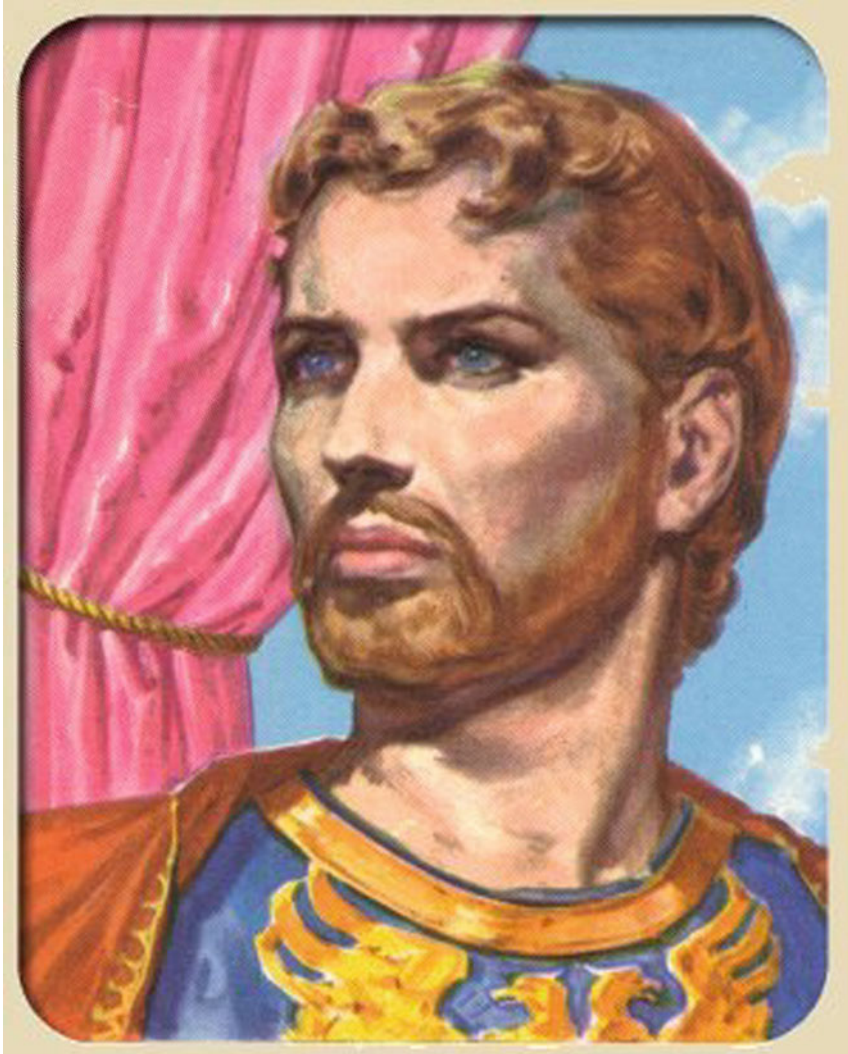
# Big-(Wo)men, Tyrants, Chiefs, Dictators, Emperors and Presidents

Towards the Mathematical  
Understanding of Social Groups



Springer

Big-(Wo)men, Tyrants, Chiefs, Dictators, Emperors  
and Presidents



**“Intentio vero nostra est manifestare ea quae sunt sicut sunt”  
“In truth, it is our intention to describe the things which are exactly as they are”  
Frederick II Hohenstaufen “Stupor Mundi”.**

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Towards the Mathematical Understanding  
of Social Groups

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*This work is dedicated to the memory of my beloved uncle Luigi De Luca. He was Professor of Classical Languages—Grammar and Literature and Principal of High Schools and transmitted to many generations of students the pleasure of understanding the logic intrinsic in reality. He was capable to explain to everybody, in a precise and rigorous way, every abstract idea, even the most difficult. He has taught me nearly every concept which I later needed in my scientific career, including the basics of set theory. I will never forget his lecture about Giambattista Vico, where he let me understand Vico's dream of transforming history into the phenomenological evidence predicted by A New Science (Una Scienza Nuova). I hope that his pedagogical spirit will revive in this work.*

# Foreword

When I met the author, he was graduating in physics and I was the professor lecturing his class in measure theory. Since then, I have never ceased to be surprised by his enthusiasm for studies. This was enough, for me, to have a good reason for reading what he has to say, and indeed the exposition in his book is accurate and pervaded by a contagious enthusiasm.

The manipulations that, as for example in the last years in Italy, the electoral systems underwent for purposes not always aimed at the collective well-being show that democracy is difficult to achieve. The situation in which the world navigates today suggests that democracy is sometimes not the best way to make a decision. The persuasive means that criminal and governmental organisations use to guide people's opinions towards their own interests are so refined and achieve such effects as to suggest that, in reality, a dictator actually does exist very often. Sometimes, when faced with the inability of a democratic decision, a dictator is even desired. Therefore, an analysis such as that presented by Francesco dell'Isola, which is easy to read but is facing the problems with competence, depth, rigour and a pinch of imagination, is to be welcomed.

In fact, his analysis is difficult to find elsewhere. It contains several historical references, widely developed and consistent with the subject of discussion, which are conducted in such a way that the reader, while reading this book, builds and refines his knowledge and his understanding of the subject. Thus, among curious, sometimes-questionable!-but-never-trivial speculations, stories and observations, one arrives at more than half of the volume, before finding a parenthesis on the mathematical theory of games, which at this point seems very opportune. Nothing highly technical, like almost the whole essay (except, perhaps, the second appendix on Arrow's theorem).

The text deserves to be read. It could leave the reader not completely satisfied, as the subject is difficult and the limit on the number of pages was stringent, but it will increase his intellectual curiosity towards some challenging parts of modern science. That was not a small feat.

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# Preface

**Exact Sciences, as evolved from Hellenistic Science, are the tool, which humans have invented to understand all natural phenomena.**

Exact sciences are based on the formulation of mathematical models, which “mimic” natural systems. Using mathematical techniques, one then gets the solution of some “problems” which, finally, allow for the prediction and control of natural phenomena.

In this book, we try to present, in a friendly way, the mathematical ideas which made us understand some important aspects of the dynamics of social groups. We believe that they are very similar to those used to describe the behaviour of Lagrangian dynamical systems! This similarity may attract the attention of the layman having the curiosity to understand the intrinsic unity of natural phenomena.

The reader is warned: although the scientific method has been extremely successful in the description of physical phenomena, human beings often refrain from applying it to the study of themselves as species, social groups or individuals. However, also human behaviour is based on specific natural laws. In this work, we dare to overcome the just mentioned taboo by exploiting the visionary understanding of the structure of social groups gained thanks to the intellectual inventions of two giants in applied mathematics: *Le Marquis de Condorcet* and *Kenneth Arrow*.

**Some questions are very important if one wants to understand rationally the structure of human groups:**

Why power is always concentrated in the hands of few?

Why every social group has always a chief?

Is it possible to establish rules for a true democracy?

Why men and women incessantly look for power?

Are there differences between the exercise of power by men and women?

What about The Platonic Republic, i.e. a human group directed by Philosophers?

Will it remain a dream forever?

## **Did humans have always refused to study themselves and their societies?**

Not always! Indeed, in the period immediately before French Revolution and during the revolution itself humanity has probably experienced the greatest cultural and scientific advancement ever. The dream of French revolutionaries was to employ rationality in every aspect of intellectual activity, including the study of society and human behaviour.

Marie Jean Antoine Nicolas de Caritat, marquis de Condorcet has been one of the main scientists and mathematicians of that period. He tried to use his outstanding mathematical skills to design laws and societies. Of course, the optimisation of this design, in the intention of Condorcet, was aimed to reach the general happiness and the highest degree of human progress. In 1785, Condorcet wrote the *Essai sur l'application de l'analyse à la probabilité des décisions rendues à la pluralité des voix* (Essay on the Application of Analysis to the Probability of Majority Decisions). This is maybe one of the most important works in the history of applied mathematics and one of the first which applied mathematics to social sciences: in it, one finds one fundamental result and one important conjecture.

Condorcet's main result is called, since then,

### **The Condorcet's Jury Theorem.**

It states that:

*if every member of a voting group is "more clever than average" (i.e. if he has more chances to cast the "correct" vote than to cast the "wrong" one) then the voting group is "much more clever than the average" higher and higher is the number of its members.*

This result seems to be a strong argument in favour of democracy. The reader should, however, pay careful attention to the hypotheses of the theorem.

It happens very often that applied mathematicians (and their epigones) forget about the hypotheses of a theorem and assume that the theses of proven theorems are always true. Instead, one can only state that: if the theorem's condition is true, then its consequence is assured. This is the structure of an implication. Unfortunately, the certitude of mathematics has been, too often, misunderstood. Mathematics never claims that a consequence is unconditionally true. Instead, it claims that a thesis is always true when the hypotheses of the implication are true.

The important hypothesis in the Jury Theorem is that «every member of a voting group is "more clever than average"». Therefore, the democratic choice is more effective than the choice of every single elector under this condition only. Condorcet himself was aware of the fact that, usually, the electoral bodies do not verify his hypothesis.

Unfortunately, even if one manages (and nobody knows how to do this!) to form an electoral body where all electors are "more clever than the average", usually the decision to be made is not between two options. This plurality of options is one of the hypotheses of the so-called

### **Condorcet Conjecture.**

Its informal statement we quote here:

*if the alternatives for a social choice are more than two, then it is not possible to establish a rule for obtaining such a social choice from the individuals' choices which is truly democratic.*

In a specific sense, Condorcet was right. Indeed, a theorem, whose formulation makes his conjecture precise, was proven rigorously by Arrow many years later (during XX century).

Before being killed by the Revolution, which he had greatly supported, Condorcet wrote the *Esquisse d'un tableau historique des progrès de l'esprit humain* (Sketch for a Historical Picture of the Progress of the Human Spirit) which is surely one of the major texts of the enlightenment and of historical thought. In this work, Condorcet narrates the history of progress in science, civilisation and technology and argues that there is a strict connection between scientific progress and the acceptance and respect of human rights and justice: he dreams a future and rational society whose structure is shaped by means of the knowledge acquired with science. Condorcet was clearly a follower of Plato's utopia.

### **The spirit of Condorcet has been fully recovered by Arrow.**

They both maintain that a careful and objective classification of observable social phenomena is required, together with a serious mathematical modelling effort, if one wants to understand the reasons for which human societies "are organised in the way in which they are organised".

Hopefully, the reader will agree that, also in this kind of investigations, we must fully accept the spirit of that declaration by Frederick II Hohenstaufen, which we have used as epigraph for this essay.

In order to have some simplified phenomena to study and to model, we will also describe some examples of social groups, which are simpler than human societies. We refer to those groups, which are constituted by chimpanzees, gorillas and bonobos: indeed, they represent a less complex version of human groups which show, however, some of their main features. The phenomena observed and the theories developed by evolutionary ethologists and mathematicians will be described, in this essay, by refraining to present explicitly their most formal mathematical structure or the most technical details. We however believe that it is possible, notwithstanding this choice, to convey the main ideas, which constitute the present knowledge in the field.

### **The mathematical theory of leadership is far from being complete and exhaustive.**

Condorcet's and Arrow's results opened a new field of intellectual challenges to be explored. Many new open problems are confronting now their successors. Moreover, the phenomenology of the leadership is nearly completely unexplored and the rigorous description of its more important aspects is far from being obtained

with methods as precise as those used in the mathematical analysis developed by Arrow.

To make clear the difficulties to be faced by the new generations of mathematicians and social scientists, some stories, characters and behaviours will be described in this essay. These characters will be embedded in their social context, in order to try to understand better the reasons of their existence. When it is possible, these stories are interrupted by some explanations based on what it is believed now to be the most reasonable and updated explanation of the described behaviours. Our final intent is to try to spread, also in our *époque*, those ideas which dominated the century of enlightenment, obviously updated in their most advanced forms.

***L'esprit des Lumières* (The spirit of Enlightenment) seems to be more and more needed, especially nowadays, to direct human history in the right direction.**

Rome/L'Aquila, Italy

Francesco dell'Isola

# Acknowledgements

I did not try to hide my cultural roots, in writing this essay. I was educated in Magna Graecia, where a wonderful melting pot mixed Greek philosophy, Roman pragmatism, Longobard proud sense of freedom, Byzantine duplicity and culture, Arab initiative, inventive and tolerance, Normans loyalty and determination, French sophisticated traditions and sense of State, Spanish opportunism, Piedmontese administrative ideas. My English style reflects my education and evokes the European common Greek and Latin roots. I did not look for sophistication, and I simply tried to maintain my original way of expressing, when the English dictionaries allowed me to do so.

I must also thank all those persons, whose name is not suitable to list here, as one may need too many pages, who allowed me to understand the true meaning of Condorcet Conjecture and Arrow's theorem.

Since I was a child, I was surprised by the apparently inexplicable behaviour of humans. Therefore, in order to understand why they are doing that which they are doing, I was obliged to discover the most beautiful part of modern mathematics.

A scientific study of human behaviour is possible: we must develop this part of knowledge without being afraid of what we could discover and without taboos. Once more Frederick II Hohenstaufen is showing us the right way.

# Reviewer Note

Big-(Wo)men, Tyrants, Chiefs, Dictators, Emperors and Presidents.  
*Towards the mathematical understanding of social groups.*

By Francesco dell'Isola

Reviewed by Nicola L. Rizzi, University Roma Tre

Note: Quotations from the book are reported in *italic*; quotations from another author in *courier*.

The author starts with the claim that every social group cannot survive without a dictator (or leader)—the meaning of this word is explained in Appendix 1—and the aim of his book is clearly declared at its beginning, as follows:

*Why a dictator must always exist?*

*Arrow's theorem, based on the conjecture of Condorcet, provides us with a precise and rational answer to these questions.*

This statement is discussed, essentially, from the point of view of a mathematician with the help of the Arrow and Nash theorems applied, in a sense, to the Condorcet theory.

Nevertheless, the book is not a technical one because, without missing the mathematical rigour when necessary, it is written in layman's terms and full of examples that help the reader to grasp, without great effort, also complex concepts.

The role and the dynamics of the leadership in many environments and situation, namely zoology, history, academy, family and so on, is investigated.

For each one of the cases, the author gives prototypal examples referred, e.g. for zoology to the primate groups; for history to the Italian process of unity; for academy to some fictional examples of academic behaviour.

It must be stressed that the discussion of the examples is always based on a deep knowledge of the matter being, in many cases, the result of original researches with the evidence of some aspects, generally disregarded, that question the clichés by original and upstream insight. This, in my opinion, is one of the most interesting features of the book.

The reading of the book is a long and exciting travel in which the reader is accompanied by honourable men like Carlo Filangeri, filthy figures like Liborio Romano or archetypical characters like Don Pasquale.

At the end of this travel, pointing out the tendency of the humankind *to enslave themselves*, the author goes further and comes up with the concept of Brittle Morality (or Brittle Honesty) of the humans as the primary cause of the existence of dictators, opposed to the (very small diffuse) resilience and ductility that is the necessary condition for opposing resistance to tyrants.

This is synthesised in the statement

*As suggested by La Boétie the only way to stop the power of the dictators belonging to the class so beautifully exemplified by Don Pasquale and Caesar Sacristy is to stop having any interaction with them, by refusing any contact and compromise.*

and the author comes to the following pessimistic (but we have to recognise full realistic) conclusion

*Unfortunately this strategy is not easy to implement, as its success requires the support of a large percentage of group members.*

This conclusion is worsened by the observation that

*There are, however, many Big-Men and Big-Women who altruistically worked for improving the quality of life of their group. Everybody did meet in his life this kind of person. The most miserable among us did exclusively exploit them and their help.*

At this point, the reader could conclude that the book is the expression of a very pessimistic view of life. Actually, this is not the true conclusion of the book as the author leaves us a hope when, in the final part writes

*For concluding this essay we want to explicitly state that the search for democracy should not be considered a hopeless endeavor*

in force of the following explanation:

*How can we solve the problem of designing a democracy, given that a single «democratic» function of choice does not exist?*

*We could say that this problem can probably (and hopefully) be solved as Charles-Louis de Secondat, Baron of La Brède and Montesquieu had imagined: by means of a system of successive approximations!*

In fact, the author writes

*We do not want here to claim that the models developed up to now by applied mathematicians are sufficient to describe, as effectively as done for physical phenomena, also social and economical phenomena. We simply believe that we are in a transitory phase, which is very similar to the one when Galileo started to understand physical natural phenomena by means of geometry and mathematical theorems.*

*We believe that eventually a scientific understanding of social phenomena will be attained: of course it is very difficult to imagine when this result will be actually attained.*

This crucial and final message can be summarised as follows: (i) the hope (or confidence) in the progress of the scientific knowledge (mathematics, for the

author) should lead to a better understanding of the social organisation; (ii) the process of *successive approximations* can be a way to improve the social organisation, and this goal can be reached!

The message, in my opinion, gives more remarkable food for thought to the reader, given also that it strengthens and is strengthened by an analogous feeling, coming from another side of the science, the science of language, expressed in the following quotation:

It is sobering to realize—as I believe we must—how little we have progressed in our knowledge of human beings and society, or even in formulating clearly the problems that might be seriously studied. But there are, I think, a few footholds that seem fairly firm. I like to believe that the intensive study of one aspect of human psychology—human language—may contribute to a humanistic social science that will serve, as well, as an instrument for social action. It must, needless to say, be stressed that social action cannot await a firmly established theory of human nature and society, nor can the validity of the latter be determined by our hopes and moral judgements. The two—speculation and action—must progress as best they can, looking forward to the day when theoretical inquiry will provide a firm guide to the unending, often grim, but never hopeless struggle for freedom and social justice.

(Noam Chomsky: *Language and Freedom*, Lecture at the University Freedom and the Human Sciences Symposium, Loyola University, Chicago, 8–9 January 1970)



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