

SECOND EDITION

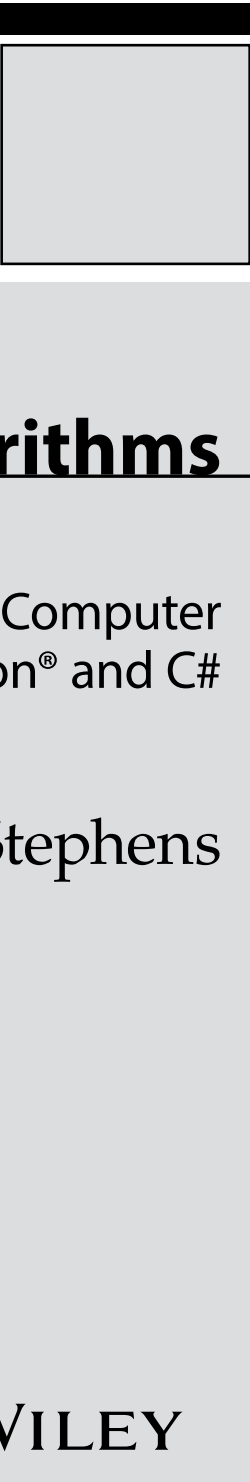
ESSENTIAL Algorithms

A PRACTICAL APPROACH
TO COMPUTER ALGORITHMS
USING PYTHON* AND C#

Rod Stephens

WILEY

Essential Algorithms



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A Practical Approach to Computer
Algorithms Using Python® and C#

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For Maki



About the Author

Rod Stephens started out as a mathematician, but while studying at MIT, he discovered how much fun algorithms are. He took every algorithms course MIT offered, and he has been writing complex algorithms ever since.

During his career, Rod has worked on an eclectic assortment of applications in fields such as telephone switching, billing, repair dispatching, tax processing, wastewater treatment, concert ticket sales, cartography, and training for professional football players.

Rod was a Microsoft Visual Basic Most Valuable Professional (MVP) for 15 years and has taught introductory programming courses. He has written more than 30 books that have been translated into languages from all over the world. He has also written more than 250 magazine articles covering C#, Visual Basic, Visual Basic for Applications, Delphi, and Java.

Rod's popular C# Helper website (<http://www.csharpHelper.com>) receives millions of hits per year and contains tips, tricks, and example programs for C# programmers. His VB Helper website (<http://www.vb-helper.com>) contains similar material for Visual Basic programmers.

You can contact Rod at: RodStephens@csharpHelper.com.



About the Technical Editor

John Mueller is a freelance author and technical editor. He has writing in his blood, having produced 112 books and more than 600 articles to date. The topics range from networking to artificial intelligence and from database management to heads-down programming. Some of his current books include discussions of data science, machine learning, and algorithms. His technical editing skills have helped more than 70 authors refine the content of their manuscripts. John has provided technical editing services to numerous magazines, performed various types of consulting, and he writes certification exams as well.

Be sure to read John's blog at: <http://blog.johnmuellerbooks.com/>. You can reach John on the Internet at John@JohnMuellerBooks.com. John also has a website at <http://www.johnmuellerbooks.com/>. Be sure to follow John on Amazon at <https://www.amazon.com/John-Mueller/>.



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