SECOND EDITION

ESSENTIAL Algorithms

A PRACTICAL APPROACH TO COMPUTER ALGORITHMS USING PYTHON® AND C#

Rod Stephens

WILEY

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A Practical Approach to Computer Algorithms Using Python® and C#

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For Maki

About the Author

Rod Stephens started out as a mathematician, but while studying at MIT, he discovered how much fun algorithms are. He took every algorithms course MIT offered, and he has been writing complex algorithms ever since.

During his career, Rod has worked on an eclectic assortment of applications in fields such as telephone switching, billing, repair dispatching, tax processing, wastewater treatment, concert ticket sales, cartography, and training for professional football players.

Rod was a Microsoft Visual Basic Most Valuable Professional (MVP) for 15 years and has taught introductory programming courses. He has written more than 30 books that have been translated into languages from all over the world. He has also written more than 250 magazine articles covering C#, Visual Basic, Visual Basic for Applications, Delphi, and Java.

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