Lecture Notes in Mechanical Engineering

Andrey A. Radionov
Oleg A. Kravchenko
Victor I. Guzeev
Yurij V. Rozhdestvenskiy *Editors*

Proceedings of the 5th International Conference on Industrial Engineering (ICIE 2019)

Volume II

Springer

Lecture Notes in Mechanical Engineering

Lecture Notes in Mechanical Engineering (LNME) publishes the latest developments in Mechanical Engineering—quickly, informally and with high quality. Original research reported in proceedings and post-proceedings represents the core of LNME. Volumes published in LNME embrace all aspects, subfields and new challenges of mechanical engineering. Topics in the series include:

- Engineering Design
- Machinery and Machine Elements
- Mechanical Structures and Stress Analysis
- Automotive Engineering
- Engine Technology
- Aerospace Technology and Astronautics
- Nanotechnology and Microengineering
- Control, Robotics, Mechatronics
- MEMS
- Theoretical and Applied Mechanics
- Dynamical Systems, Control
- Fluid Mechanics
- Engineering Thermodynamics, Heat and Mass Transfer
- Manufacturing
- Precision Engineering, Instrumentation, Measurement
- Materials Engineering
- Tribology and Surface Technology

To submit a proposal or request further information, please contact the Springer Editor in your country:

China: Li Shen at li.shen@springer.com
India: Dr. Akash Chakraborty at akash.chakraborty@springernature.com
Rest of Asia, Australia, New Zealand: Swati Meherishi at swati.meherishi@springer.com
All other countries: Dr. Leontina Di Cecco at Leontina.dicecco@springer.com

To submit a proposal for a monograph, please check our Springer Tracts in Mechanical Engineering at http://www.springer.com/series/11693 or contact Leontina.dicecco@springer.com

Indexed by SCOPUS. The books of the series are submitted for indexing to Web of Science.

More information about this series at http://www.springer.com/series/11236

Andrey A. Radionov · Oleg A. Kravchenko ·
Victor I. Guzeev · Yurij V. Rozhdestvenskiy
Editors

Proceedings of the 5th International Conference on Industrial Engineering (ICIE 2019)

Volume II

Springer

Editors
Andrey A. Radionov
South Ural State University
Chelyabinsk, Russia

Victor I. Guzeev
South Ural State University
Chelyabinsk, Russia

Oleg A. Kravchenko
Platov South Russian State Polytechnic University
Novocherkassk, Russia

Yurij V. Rozhdestvenskiy
South Ural State University
Chelyabinsk, Russia

ISSN 2195-4356 ISSN 2195-4364 (electronic)
Lecture Notes in Mechanical Engineering
ISBN 978-3-030-22062-4 ISBN 978-3-030-22063-1 (eBook)
https://doi.org/10.1007/978-3-030-22063-1

© Springer Nature Switzerland AG 2020
This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.
The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.
The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, expressed or implied, with respect to the material contained herein or for any errors or omissions that may have been made. The publisher remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Preface

International Conference on Industrial Engineering took place on March 25–29, 2019, in Sochi, Russian Federation. The conference was organized by four universities—South Ural State University (National Research University), Moscow Polytechnic University, Platov South-Russian State Polytechnic University, and Volgograd State Technical University.

The conference was carried out under financial support of the South Ural State University (National Research University).

The conference was really large-scaled and international. The international program committee has selected more than 500 reports. The conferees represented 63 Russian cities from the western and central parts to the Far East regions. International participants represented such countries as China, Germany, Kazakhstan, Kyrgyzstan, Portugal, Saudi Arabia, Tajikistan, Ukraine, USA, Uzbekistan.

The conference participants submitted papers reflecting recent advances in the field of Industrial Engineering, in Russian and English. The conference was organized into 13 sections, including Part 1 "Mechanical Engineering" (Machinery and Mechanism Design; Dynamics of Machines and Working Processes; Friction, Wear, and Lubrication in Machines; Design and Manufacturing Engineering of Industrial Facilities; Transport and Technological Machines; Mechanical Treatment of Materials; Industrial Hydraulic Systems; Green Manufacturing) and Part 2 "Materials Engineering and Technologies for Production and Processing" (Polymers, Composites and Ceramics; Steels and Alloys, Metallurgical and Metalworking Technologies; Chemical and Hydrometallurgical Technologies; Surface Engineering and Coatings; Processing and Controlling Technologies).

The international program committee selected 294 papers from Part 1 of the conference technical sections for publication in book series "Lecture Notes in Mechanical Engineering."

The organizing committee would like to express our sincere appreciation to everybody who has contributed to the conference. Heartfelt thanks are due to authors, reviewers, participants and to all the team of organizers for their support and enthusiasm which granted success to the conference.

Chelyabinsk, Russia Andrey A. Radionov

Contents

Part 1

Part 2

Modeling of Roll Roughness Transfer Process to Strip During Skin-Pass Rolling

N. N. Ogarkov, S. I. Platov and E. U. Zvyagina

Abstract In this paper, the modeling of a roll roughness transfer process to the strip in terms of skin-passing conditions was carried out. In the present paper, the analysis of the strain–stress state of material imprinted into microcavities of roll face is made, for which reason the latter is presented in the form of V-shaped grooves. The model of roll roughness transfer to the strip was developed by taking into account the real distribution of material in the rough layer of the roll and the strip, as well as according to the type of roll processing and skin-pass conditions that allows evaluating the degree of filling a singular microcavity under known pressure values of the strip on contact with a roll, friction coefficient, roll roughness parameters, and skin-pass conditions. The findings afford to forecast a reproduction of roll microrelief of the surface of deformed strip.

Keywords Microrelief · Microrelief transfer mechanism · Reproduction · Microcavity · Temper-rolled strip · Coefficient of imprinting

1 Introduction

One of the understudied key problems to be solved relates to relating the rolling conditions and roll roughness parameters and cold-rolled strip to the necessary required surface microgeometry of the finished product with account of the consumer performance [1–8].

The formation of the prescriptive strip microrelief directly near the roll pass is carried out by means of transfer mechanism of roll face microrelief to the strip and transformation of the initial microrelief [9–14].

N. N. Ogarkov · S. I. Platov · E. U. Zvyagina (✉)
Magnitogorsk State Technical University named after G.I. Nosov, 38, Lenin Av, Magnitogorsk 455000, Russia
e-mail: zviagina_mmf@mail.ru

A. A. Radionov et al. (eds.), *Proceedings of the 5th International Conference on Industrial Engineering (ICIE 2019)*, Lecture Notes in Mechanical Engineering,
https://doi.org/10.1007/978-3-030-22063-1_1

2 Theoretical Research

We define the quantitative evaluation of imprinting process of roll roughness to the skin-passed strip by the coefficient of imprinting K which is represented by ratio of material flow value of the skin-rolled strip z_f to the size of roll microcavity characterized by a parameter R_z (Fig. 1).

The problem will be solved for triangular roll microcavity the stress condition in which while filling while skin-rolled steel is shown in Fig. 1.

Please do not alter the formatting and style layouts which have been set up in this template document. As indicated in the template, papers should be prepared in single column format suitable for direct printing onto paper. Do not number pages on the front, as page numbers will be added separately for the preprints and the proceedings. Leave a line clear between paragraphs. All the required style templates are provided in the file "MS Word Template" with the appropriate name supplied, e.g. choose "heading1" for your first-order heading text, "abstract" for the abstract text, etc.

The determination of dimensionless stress necessary for filling the material into V-shaped cavities will be fulfilled by using Bocharov formula [15], which relative to adjacent toward one other roughness cavities takes the following form:

$$\frac{P}{\sigma_S} = \frac{4}{\sqrt{3\pi}}\left[3\ln\left(\frac{1}{1-\frac{z_f}{R_Z}}\right)+\frac{\pi S}{h}\right], \tag{1}$$

where P—rolls pressure during skin-pass rolling; σ_S—yield strength of skin-rolled steel; R_z and S—height and stepwise roll face roughness parameter [16, 17]; h—thickness of outlet skin-pass strip.

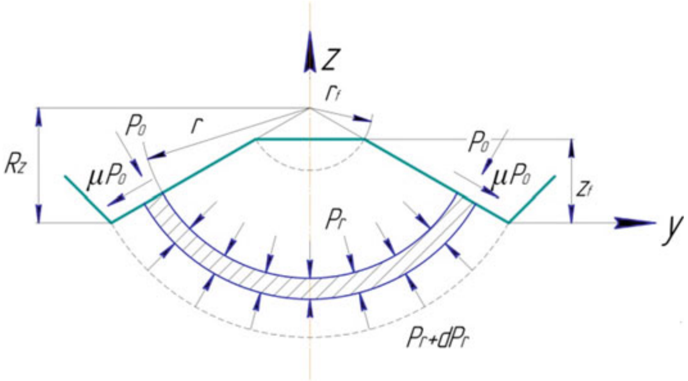

Fig. 1 Stress condition diagram by filling V-shaped roll microcavity with skin-rolled steel

Considering that $\frac{z_f}{R_z} = K$ and solving an Eq. (1) in relation to K we shall obtain:

$$1 - \frac{1}{\exp \frac{1}{3}\left[\frac{\sqrt{3}\pi P_{cp}}{4\sigma_S} - \frac{\pi S}{h}\right]}. \tag{2}$$

To evaluate the rolls' pressure, we shall rely on the features of skin-pass rolling process and admit the following assumptions:

- working rolls of the skin-rolling mill are drive, of the same diameter with the same roll face roughness;
- form of roll contact with skin-rolled strip is flat;
- dry skin-pass rolling is carried out with the friction coefficient at the deformation area … $\mu = 0,25\ldots0,35$.

Due to higher friction coefficient values, the neutral point at the deformation area is located near the center of contact line.

With due account of the assumptions, the pressure at the deformation area without regard to roughness contacting will be determined by the following equation:

$$P = \sigma_d \left[\frac{h_0(1-\varepsilon)}{\mu L}\right] \left\{ \exp\left[\frac{\mu L}{h_0(1-\varepsilon)}\right] - 1 \right\}, \tag{3}$$

where ε—skin-roll strip draft; L—contact length a roll—a skin-roll strip; μ—coefficient of friction at the roll pass; h_0—thickness of skin-roll strip in input.

Relating to the contact of roll deformed strip with large friction coefficient, it was shown in the papers [3, 18, 19] that a contact length a roll—a strip can be determined on the following formula:

$$L = \frac{1}{4}\left[D\varepsilon\mu + \sqrt{(D\varepsilon\mu)^2 + 8Dh_0\varepsilon}\right], \tag{4}$$

where D—work roll diameter.

The minimum pressure of skin-roll passing required for strip deformation shall be determined from the following formula [20]:

$$\sigma_d = 1.15(\sigma_S + d \lg 1000\dot{\varepsilon}) - \sigma_t, \tag{5}$$

where σ_s—yield stress determined by tension testing at a standard strain rate; d—dynamic coefficient considering the velocity effect when magnified ten times; $\dot{\varepsilon}$—strain rate during skin-roll passing; σ_t—tensile stress at the roll pass determined by strip tension stresses between a uncoiler and mill stand and a mill stand and coiler.

The average strain rate $\dot{\varepsilon}$, at which a skin-roll passing is carried out, can be determined by approximation:

$$\dot{\varepsilon} = \frac{2V}{D\mu}, \quad (6)$$

where V—velocity of roll periphery.

The shortcoming of the Eq. (3) is that it includes the friction coefficient at roll pass, but it does not consider the fact that an imprinting of work roll surface roughness to skin-rolled strip is carried out within the limits of their rough layers and depends significantly on the fact to what extent the deformation processes of the skin-roll strip at roughness contact zone interact against each other and with the «base», on which they are located.

During skin-roll passing, the pressure that defines the bearing power of rough layers shall be determined by their bearing surfaces. The regularity of change in bearing surfaces when approaching of roughness is determined by their supporting curves. Considering that the degree of involvement of roll face micro-irregularities and the strip is different, it is reasonable to use their medium integral values [21] that determine the relative quantity of the steel in rough layers.

The approach of roll rough surfaces and skin-rolled strip will lead to the occurrence of total supporting surface of rough layers. We suppose that the total supporting surface is to be determined by a ratio:

$$q = \frac{q_1 + q_2}{2}, \quad (7)$$

where q_1 and q_2—the percentage of material in rough layers of the roll and the strip.

The parameter points q_1 are presented in Table 1.

The parameter q_2 while strip rolling by sizing rolls shall be 0.43–0.59, depending on roll surfaces condition.

In consideration of the foregoing, the actual contact pressure of rough surfaces at the roll pass shall be

$$P_{sh} = P/q. \quad (8)$$

Table 1 Parameter points q_1 for different cold roll surface finishing

Processing type	$R_{\max}$/μm	q_1
Grinding	4.7/3.2	0.402/0.485
	2.4/1.6	0.423/0.519
	1.2/1.0	0.550/0.566
Wheel-blasting	8.6/5.8	0.482/0.538
	4.8/3.2	0.503/0.564
	1.2/0.8	0.548/0.576
Electroerosion texturizing	12.1/7.2	0.489/0.543
	9.6/4.9	0.533/0.564
	4.2/1.2	0.567/0.576

The dimensionless steel pressure of the skin-pass mill rolls with account of formulas (3), (4), (5), and (6, 7 and 8) shall be written as follows:

$$\frac{P_{sh}}{\sigma_s} = \frac{1}{q}\left[1.15\left(1 + \frac{d \lg 1000^{\frac{2V}{D\mu}}}{\sigma_s}\right) - \frac{\sigma_t}{\sigma_s}\right]\left[\frac{h_0(1-\varepsilon)}{\mu L}\right]\left\{\exp\left[\frac{\mu L}{h_0(1-\varepsilon)}\right] - 1\right\}. \quad (9)$$

The formula used for calculation the coefficient of imprinting will be finally written as follows:

$$K = 1 - \frac{1}{\exp\left\{\frac{\sqrt{\pi}}{6q}\left[\left(1 + \frac{d \lg 1000^{\frac{2V}{D\mu}}}{\sigma_s}\right) - \frac{\sigma_t}{\sigma_s}\right]\left[\frac{h_0(1-\varepsilon)}{\mu L}\right]\left\{\exp\left[\frac{\mu L}{h_0(1-\varepsilon)}\right] - 1\right\} - \frac{\pi S}{3h}\right\}}. \quad (10)$$

The application of the formula (10) allows to determinate the coefficient of imprinting of the roll roughness on the strip including the real distribution of material in a rough layer of the roll and the strip depending on the original roughness of roll face, the thickness of roll skin strip, rolling speed, tension stress, and drafting.

The target values of the coefficient of imprinting by variances of skin-pass rolling are given in Fig. 2.

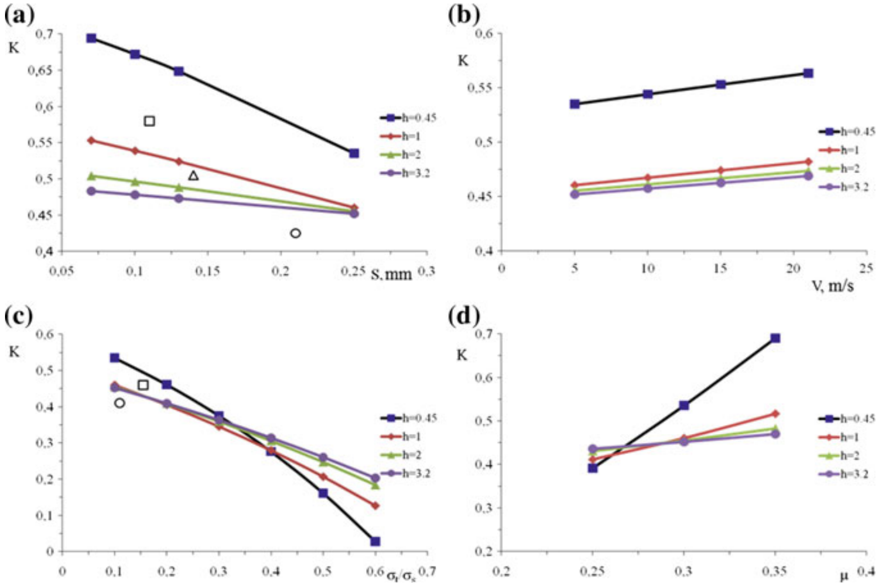

Fig. 2 Dependence of coefficient of imprinting *K* compared to experimental data—Δ □ ○ by skin-pass rolling of a strip of different thickness with original data *D* = 500 mm, ε = 0.02, *q* = 0.5, *d* = 12 on: **a** stage of microroughness; **b** velocity of roll periphery; **c** tension capacity; **d** coefficient of friction

The resulting dependence allows concluding that the best reproduction of the roll microrelief on the skin-rolled strip is evidenced during “dry” skin-pass rolling of thin strip with little microroughness width.

The resulting data are recommended for dry skin-pass rolling of the strips and for wet skin-pass rolling to a limited extent [22, 23].

The increase in strip tension dramatically reduces the imprinting of roll microrelief at the surface of skin-rolled strip.

By increasing in the roll velocity, there is a minor effect on change in coefficient of imprinting especially for thick strips.

3 Conclusions

The roll roughness transfer model to the strip was developed with account of the roll processing type and skin-pass rolling conditions that allow estimating the degree of filling of microrelief of singular microcavity at the known pressure values at the contact of the strip with the roll, friction coefficient, roll roughness parameters, and skin-pass rolling conditions.

The resulting data can be used in amendments of the existing and developments of new skin-pass rolling conditions for cold-rolled strips to obtain the required microrelief according to the consumer properties by taking into account the coefficient of microgeometry of roll face of the skin-pass rolling mill.

References

1. Polukhin PI (1974) Contact interaction of steel and work tool during rolling. Metallurgy, Moscow
2. Ogarkov NN, Khalin SV (2003) Regulation of rolled products surface roughness depending on its consumer performance. In: Processes and machinery of metallurgic production: interregional collection of studies. Edition of Nosov Magnitogorsk State Technical University, Magnitogorsk, p 131
3. Roberts VL Cold rolling of steel: translated from English. Metallurgy, Moscow
4. Zvyagina EU (2017) Equipment upgrading and technological advancement of roll ragging for improvement of automotive sheet surface quality. Dissertation, Nosov Magnitogorsk State Technical University
5. Ogarkov NN, Zvyagina EU, Zaletov UD (2006) Effect of roll face processing technique on the quality of cold-rolled sheet. In: Modern methods of design and technology of mechanic engineering: international collection of studies. Edition of Nosov Magnitogorsk State Technical University, Magnitogorsk, p 39
6. Belov BK, Begletsov DO Modelling of generation process of surface microtopography during skin-pass rolling of automotive sheet. In: Salganik VM (ed) Modelling and development of process of metal treatment under pressure: interregional collection of studies. Edition of Magnitogorsk State Technical University, Magnitogorsk, p 58
7. Utsch M, Vinke P (2004) Roll texturing technology as a base of modern surfaces in automotive cold mill flat products. In: MS&T: Conference Proceeding, p 599

8. Ogarkov NN, Zaletov UD, Laskov SA, Zvyagina EU, Pozhidaev UA (2010) Improvement of wheel blasting of mill rolls for production of automotive sheet, vol 2(30). Vestnik of Nosov Magnitogorsk State Technical University, p 41
9. Rasp W, Wichern CM (2002) Effects of surface-topography directionality and lubrication condition on frictional behavior during plastic deformation. J Mater Process Technol 125:379
10. Kuznetsov LA, Mamyshev AB (1989) Theoretical determination of coefficient of imprinting of roll microrelief into the strip during cold rolling, vol 6. News of Higher Educational Institutions: Iron and Steel Industry, p 38
11. Gorbunov AB, Radionov AF, Belov BK et al (2007) Production of automotive sheets with specified surface microtopography, vol 4. Rolling, p 15
12. Marigue C, Bragard A (1983) Surface roughness and user properties of cold rolled steel sheets. American Society for Metals, London, 11–13 May 1983, p 242
13. Kuznetsov LA, Mamyshev AB (1989) Theoretical determination of coefficient of imprinting (impression) of roll microrelief into the strip during cold rolling, vol 6. News of Higher Educational Institutions: Iron and Steel Industry, p 38
14. Mukhin UA, Ryblov AB, Bobkov EB, Tcherny BA (2014) Formation of surface microgeometry of cold rolled strips: study guide. Edition Lipetsk Technical University Press, Lipetsk
15. Bocharov Y, Kobayashi S, Thomsen EG (1962) The mechanics of the coining process. Trans ASME Series B J Eng Ind 84:491
16. Ogarkov NN, Zvyagina EYu, Zaletov YuD, Khomenko NN, Kerimova LF (2016) Improvements in shot blasting to increase the surface peak density of auto-industry steel sheet. Steel Transl 46(12):847
17. Tang Jing Gang (2004) Determination and analysis of surface roughness of cold rolled steel sheets, vol 2. Iron Steel Vanadium Titanium, p 66
18. Aliev IC (2002) Study of contact plastic friction factor. In: Upgrading of processes and machinery for pressure shaping in steel industry and mechanic engineering: collection of studies. DGMA, Kramatorsk, p 112
19. Vasiliev JD, Dementienko AB (2001) Study of contact line with a roll during cold rolling. In: Ferrous Metals, vol 7. News of Higher Educational Institutions, p 21
20. Roberts WL (1972) An approximate theory of temper rolling. In: Iron and steel engineer year book, p 530
21. Ogarkov NN (1996) Roughness formation of the rolled products with high-quality surface finishing by means of regulation control of roll surface coating condition. Dissertation, Nosov Magnitogorsk State Technical University
22. Terentiev DV, Ogarkov NN, Platov SI, Kozlov AB (2018) Effect of operational conditions and oil consumption of contact areas on the thickness of lubricating film in heavy duty friction assemblies of melting facilities. In: Ferrous metals, p 60
23. Ogarkov NN, Platov SI, Shemetova ES, Tepentev DV, Nekit VA, Samodurova MN (2017) Oil absorption capacity of the contact surfaces in metal-forming processes. Metallurgist 61(1–2):58

Working Surface Calculation of Teeth Bevel Gear Helical-Bevel Gearing at Milling with Hob

E. A. Poluektov, B. A. Lopatin and S. V. Plotnikova

Abstract The paper considers an internal gearing of a bevel gear and a spur gear, shaped according to a conventional technology. There is a technological problem of obtaining the working profile of the bevel gear teeth in the internal spur–bevel gearing when the working profiles are being shaped. Obtaining a theoretically accurate surface of the gear teeth is troublesome due to the difficulty of making cutters with internal teeth and practical realization of machine gearing with the axis inclination of the workpiece or the cutter axis. This is related to the need of producing a shaping cutter with internal teeth for each gear train so that its geometry would be identical to that of the spur gear in the gear train. The paper describes a method for shaping an approximate teeth profile of a bevel gear in a spur–bevel gearing with a rack-type tool. The proposed method provides a sufficient degree of approximation of the shaped surface to the theoretically accurate one, which makes it applicable. In the paper, the equations are presented describing the teeth surface, as a result of a two-parameter bending with a hob of the surface with conical billet for evaluation. This makes it possible to evaluate the degree of approximation received for the working surface of the teeth to the theoretically accurate surface.

Keywords Spur–bevel gearing · Internal meshing · Non-involute gear · Gear milling

---

E. A. Poluektov · B. A. Lopatin (✉) · S. V. Plotnikova
Zlatoust Branch of South Ural State University (National Research University),
16, Turgenev Street, Zlatoust 456209, Russia
e-mail: lopatinba@susu.ru

A. A. Radionov et al. (eds.), *Proceedings of the 5th International Conference on Industrial Engineering (ICIE 2019)*, Lecture Notes in Mechanical Engineering,
https://doi.org/10.1007/978-3-030-22063-1_2

1 Introduction

A spur–bevel gearing is a gear train, in which one of the gears has a cylindrical blank, and the other—a conical one [1–5]. To provide a linear teeth contact in the internal spur–bevel gearing, the generating gear in the cutter-blank meshing has to be an involute spur gear identical to the spur gear of the gear train [6–9].

2 Main Part

Figure 1 shows a diagram of the internal spur–bevel meshing with a generating involute spur gear. The teeth flank of the spur gear is an involute cylinder. This surface (see Fig. 1) in the moving reference frame $X_2Y_2W_2$, associated with the spur gear, is described by the equations:

$$\begin{aligned} X_2 &= r_{b2}[\sin(v_{y2} - \psi_{b2}) - v_{y2} \cdot \cos(v_{y2} - \psi_{b2})] \\ Y_2 &= r_{b2}[\cos(v_{y2} - \psi_{b2}) + v_{y2} \cdot \sin(v_{y2} - \psi_{b2})] \\ W_2 &= u, \end{aligned} \tag{1}$$

where r_{b2} is the radius of the base spur gear cylinder, v_{y2} is the roll angle of the involute curve, ψ_{b2} is half of the angular thickness of the tooth dedendum on the rolling circle of the spur gear, u is the z-coordinate of the face section of the spur gear.

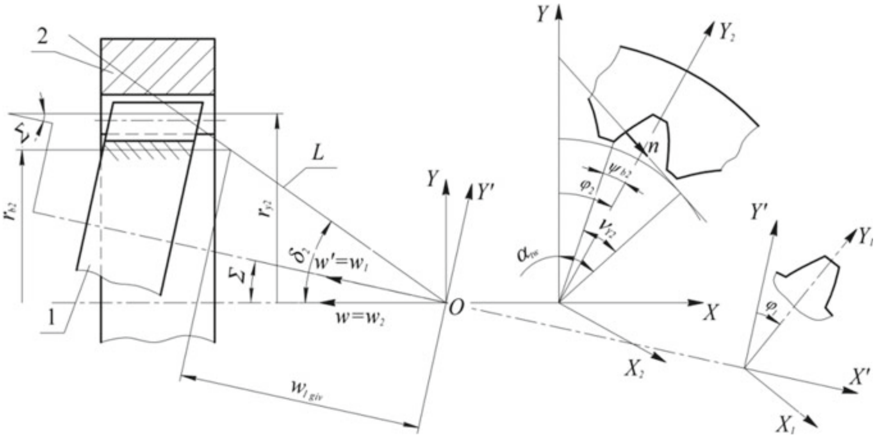

Fig. 1 Meshing of the generating: 1—spur gear, 2—bevel gear

The angle v_{y2} for the given radius r_{y2} is determined by the expression

$$v_{y2} = \mathrm{tg} \cdot \arccos\left(\frac{r_{b2}}{r_{y2}}\right). \tag{2}$$

The angle ψ_{b2} is calculated according to the dependence

$$\psi_{b2} = \frac{\pi}{2 \cdot z_2} + \frac{2 \cdot x_2 \cdot \mathrm{tg}\,\alpha}{z_2} + \mathrm{inv}\,\alpha \tag{3}$$

where x_2 is the profile shift coefficient of the spur gear.

The flank surface of the bevel gear teeth is an envelope of the generating surface and is non-involute [2]. This surface in the moving reference system $X_1Y_1W_1$, associated with the bevel gear is described by the equations:

$$\begin{aligned} X_1 &= r_{b1}[\cos\phi_2(\sin\alpha_{\mathrm{tw}} - v_{y1}\cos\alpha_{\mathrm{tw}}) - \sin\phi_2\cos\Sigma(\cos\alpha_{\mathrm{tw}} + v_{y1}\sin\alpha_{\mathrm{tw}})] \\ &\quad + u\sin\phi_2\sin\Sigma \\ Y_1 &= r_{b1}[\sin\phi_2(\sin\alpha_{\mathrm{tw}} - v_{y1}\cos\alpha_{\mathrm{tw}}) - \cos\phi_2\cos\Sigma(\cos\alpha_{\mathrm{tw}} + v_{y1}\sin\alpha_{\mathrm{tw}})] \\ &\quad - u\cos\phi_2\sin\Sigma \\ W_1 &= r_{b1}\sin\Sigma(\cos\alpha_{\mathrm{tw}} + v_{y1}\sin\alpha_{\mathrm{tw}}) + u\cos\Sigma\cos\alpha_{\mathrm{tw}} = \cos(v_{y1} - \psi_{b1} - \phi_1) \\ &= \frac{r_{b1}}{u \cdot \mathrm{tg}\,\delta_1}; \phi_1 = \phi_2 \cdot i_{12} \end{aligned} \tag{4}$$

where Σ is the shaft angle, α_{tw} is the gearing angle in the face section of the spur gear, φ_1, φ_2 are the turning angles of the bevel gear and the spur gear.

Theoretically, exact flank surface of the bevel gear teeth can be formed by a shaping cutter with internal teeth. In this case, a geometrical gearing diagram of the internal conjugate spur–bevel meshing is implemented in the cutter-blank meshing. However, due to the complex manufacture of shaping cutters with internal teeth, this tool is not produced on the industrial level. Thus, currently it seems impossible to form a theoretically exact profile using this method.

An approximate profile can be shaped with gear-cutting equipment using a rack-type cutting tool [10]. In this case, the rack-type cutting tool (a milling cutter, a grinding wheel) moves along the blank gear axis according to a certain law. Such cutting is carried out by standard gear milling machines equipped with a follow-up device or by CNC machines. Figure 2 shows the cutting diagram. By selecting the cutter path, we can obtain a tooth similar in shape to the exact non-involute gear tooth [11, 12].

To calculate the coordinates of the envelope curve points, we used the mathematical apparatus of the involute bevel gearing developed by Bezrukov [13] and took the tool angle δ_{0i} as variable in each section. The tool shifting coefficient in an arbitrary face section of the bevel gear in the midpoint of the tooth depth is determined according to the expression.