Advances in Intelligent Systems and Computing 991

Hoai An Le Thi
Hoai Minh Le
Tao Pham Dinh *Editors*

Optimization of Complex Systems: Theory, Models, Algorithms and Applications

Springer

Advances in Intelligent Systems and Computing

Volume 991

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ISSN 2194-5357 ISSN 2194-5365 (electronic)
Advances in Intelligent Systems and Computing
ISBN 978-3-030-21802-7 ISBN 978-3-030-21803-4 (eBook)
https://doi.org/10.1007/978-3-030-21803-4

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Preface

WCGO 2019 was the sixth event in the series of World Congress on Global Optimization conferences, and it took place on July 8–10, 2019 at Metz, France. The conference aims to bring together most leading specialists in both theoretical and algorithmic aspects as well as a variety of application domains of nonconvex programming and global optimization to highlight recent advances, trends, challenges and discuss how to expand the role of these fields in several potential high-impact application areas.

The WCGO conference series is a biennial conference of the International Society of Global Optimization (iSoGO). The first event WCGO 2009 took place in Hunan, China. The second event, WCGO 2011, was held in Chania, Greece, followed by the third event, WCGO 2013, in Huangshan, China. The fourth event, WCGO 2015, took place in Florida, USA, while the fifth event was held in Texas, USA. One of the highlights of this biannual meeting is the announcement of Constantin Carathéodory Prize of iSoGO awarded in recognition of lifetime contributions to the field of global optimization.

WCGO 2019 was attended by about 180 scientists and practitioners from 40 countries. The scientific program includes the oral presentation of 112 selected full papers as well as several selected abstracts covering all main topic areas. In addition, the conference program was enriched by six plenary lectures that were given by Prof. Aharon Ben-Tal (Israel Institute of Technology, Israel), Prof. Immanuel M. Bomze (University of Vienna, Austria), Prof. Masao Fukushima (Nanzan University, Japan), Prof. Anna Nagurney (University of Massachusetts Amherst, USA), Prof. Panos M. Pardalos (University of Florida, USA), and Prof. Anatoly Zhigljavsky (Cardiff University, UK).

This book contains 112 papers selected from about 250 submissions to WCGO 2019. Each paper was peer-reviewed by at least two members of the International Program Committee and the International Reviewer Board. The book covers both theoretical and algorithmic aspects of nonconvex programming and global optimization, as well as its applications to modeling and solving decision problems in various domains. The book is composed of ten parts, and each of them deals with either the theory and/or methods in a branch of optimization such as continuous optimization, DC programming and DCA, discrete optimization and network optimization, multiobjective programming, optimization under uncertainty, or models and optimization methods in a specific application area including data science, economics and finance, energy and water management, engineering systems, transportation, logistics, resource allocation and production management. We hope that the researchers and practitioners working in nonconvex optimization and several application areas can find here many inspiring ideas and useful tools and techniques for their works.

We would like to thank the chairs and the members of International Program Committee as well as the reviewers for their hard work in the review process, which helped us to guarantee the highest quality of the selected papers for the conference. We cordially thank the organizers and chairs of special sessions for their contributions to the success of the conference. Thanks are also due to the plenary lecturers for their interesting and informative talks of a world-class standard.

The conference was organized by the Computer Science and Applications Department, LGIPM, University of Lorraine, France. We wish to especially thank all members of the Organizing Committee for their excellent work to make the conference a success. The conference would not have been possible without their considerable effort.

We would like to express our sincere thanks to our main sponsors: Réseau de Transport d'Électricité (France), Conseil régional du Grand Est (France), Metz Métropole (France), Conseil départemental de la Moselle (France), Université de Lorraine (France), Laboratoire de Génie Informatique, de Production et de Maintenance (LGIPM) - Université de Lorraine, UFR Mathématique Informatique Mécanique Automatique - Université de Lorraine, and DCA Solutions (Vietnam).

Our special thanks go to all the authors for their valuable contributions, and to the other participants who enriched the conference success.

Finally, we cordially thank Springer for their help in publishing this book.

July 2019

Hoai An Le Thi
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Contents

Continuous Optimization

A Hybrid Simplex Search for Global Optimization with Representation Formula and Genetic Algorithm 3
Hafid Zidani, Rachid Ellaia, and Eduardo Souza de Cursi

A Population-Based Stochastic Coordinate Descent Method 16
Ana Maria A. C. Rocha, M. Fernanda P. Costa, and Edite M. G. P. Fernandes

A Sequential Linear Programming Algorithm for Continuous and Mixed-Integer Nonconvex Quadratic Programming 26
Mohand Bentobache, Mohamed Telli, and Abdelkader Mokhtari

A Survey of Surrogate Approaches for Expensive Constrained Black-Box Optimization 37
Rommel G. Regis

Adaptive Global Optimization Based on Nested Dimensionality Reduction 48
Konstantin Barkalov and Ilya Lebedev

A B-Spline Global Optimization Algorithm for Optimal Power Flow Problem 58
Deepak D. Gawali, Bhagyesh V. Patil, Ahmed Zidna, and Paluri S. V. Nataraj

Concurrent Topological Optimization of a Multi-component Arm for a Tube Bending Machine 68
Federico Ballo, Massimiliano Gobbi, and Giorgio Previati

Discrete Interval Adjoints in Unconstrained Global Optimization 78
Jens Deussen and Uwe Naumann

Diving for Sparse Partially-Reflexive Generalized Inverses 89
Victor K. Fuentes, Marcia Fampa, and Jon Lee

Filtering Domains of Factorable Functions Using Interval Contractors 99
Laurent Granvilliers

Leveraging Local Optima Network Properties for Memetic Differential Evolution 109
Viktor Homolya and Tamás Vinkó

Maximization of a Convex Quadratic Form on a Polytope: Factorization and the Chebyshev Norm Bounds 119
Milan Hladík and David Hartman

New Dynamic Programming Approach to Global Optimization 128
Anna Kaźmierczak and Andrzej Nowakowski

On Chebyshev Center of the Intersection of Two Ellipsoids 135
Xiaoli Cen, Yong Xia, Runxuan Gao, and Tianzhi Yang

On Conic Relaxations of Generalization of the Extended Trust Region Subproblem 145
Rujun Jiang and Duan Li

On Constrained Optimization Problems Solved Using the Canonical Duality Theory 155
Constantin Zălinescu

On Controlled Variational Inequalities Involving Convex Functionals 164
Savin Treanţă

On Lagrange Duality for Several Classes of Nonconvex Optimization Problems 175
Ewa M. Bednarczuk and Monika Syga

On Monotone Maps: Semidifferentiable Case 182
Shashi Kant Mishra, Sanjeev Kumar Singh, and Avanish Shahi

Parallel Multi-memetic Global Optimization Algorithm for Optimal Control of Polyarylenephthalide's Thermally-Stimulated Luminescence 191
Maxim Sakharov and Anatoly Karpenko

Proper Choice of Control Parameters for CoDE Algorithm 202
Petr Bujok, Daniela Einšpiglová, and Hana Zámečníková

Semidefinite Programming Based Convex Relaxation for Nonconvex Quadratically Constrained Quadratic Programming 213
Rujun Jiang and Duan Li

Solving a Type of the Tikhonov Regularization of the Total Least Squares by a New S-Lemma 221
Huu-Quang Nguyen, Ruey-Lin Sheu, and Yong Xia

Solving Mathematical Programs with Complementarity Constraints with a Penalization Approach 228
Lina Abdallah, Tangi Migot, and Mounir Haddou

Stochastic Tunneling for Improving the Efficiency of Stochastic Efficient Global Optimization 238
Fábio Nascentes, Rafael Holdorf Lopez, Rubens Sampaio, and Eduardo Souza de Cursi

The Bernstein Polynomials Based Globally Optimal Nonlinear Model Predictive Control 247
Bhagyesh V. Patil, Ashok Krishnan, Foo Y. S. Eddy, and Ahmed Zidna

Towards the Biconjugate of Bivariate Piecewise Quadratic Functions 257
Deepak Kumar and Yves Lucet

Tractable Relaxations for the Cubic One-Spherical Optimization Problem 267
Christoph Buchheim, Marcia Fampa, and Orlando Sarmiento

DC Programming and DCA

A DC Algorithm for Solving Multiobjective Stochatic Problem via Exponential Utility Functions 279
Ramzi Kasri and Fatima Bellahcene

A DCA-Based Approach for Outage Constrained Robust Secure Power-Splitting SWIPT MISO System 289
Phuong Anh Nguyen and Hoai An Le Thi

DCA-Like, GA and MBO: A Novel Hybrid Approach for Binary Quadratic Programs 299
Sara Samir, Hoai An Le Thi, and Mohammed Yagouni

Low-Rank Matrix Recovery with Ky Fan 2-*k*-Norm 310
Xuan Vinh Doan and Stephen Vavasis

Online DCA for Times Series Forecasting Using Artificial Neural Network 320
Viet Anh Nguyen and Hoai An Le Thi

Parallel DC Cutting Plane Algorithms for Mixed Binary Linear Program 330
Yi-Shuai Niu, Yu You, and Wen-Zhuo Liu

Sentence Compression via DC Programming Approach 341
Yi-Shuai Niu, Xi-Wei Hu, Yu You, Faouzi Mohamed Benammour, and Hu Zhang

Discrete Optimization and Network Optimization

A Horizontal Method of Localizing Values of a Linear Function in Permutation-Based Optimization 355
Liudmyla Koliechkina and Oksana Pichugina

An Experimental Comparison of Heuristic Coloring Algorithms in Terms of Found Color Classes on Random Graphs 365
Deniss Kumlander and Aleksei Kulitškov

Cliques for Multi-term Linearization of 0–1 Multilinear Program for Boolean Logical Pattern Generation 376
Kedong Yan and Hong Seo Ryoo

Gaining or Losing Perspective 387
Jon Lee, Daphne Skipper, and Emily Speakman

Game Equilibria and Transition Dynamics with Networks Unification 398
Alexei Korolev and Ilia Garmashov

Local Search Approaches with Different Problem-Specific Steps for Sensor Network Coverage Optimization 407
Krzysztof Trojanowski and Artur Mikitiuk

Modelling Dynamic Programming-Based Global Constraints in Constraint Programming 417
Andrea Visentin, Steven D. Prestwich, Roberto Rossi, and Armagan Tarim

Modified Extended Cutting Plane Algorithm for Mixed Integer Nonlinear Programming 428
Wendel Melo, Marcia Fampa, and Fernanda Raupp

On Proximity for k-Regular Mixed-Integer Linear Optimization 438
Luze Xu and Jon Lee

On Solving Nonconvex MINLP Problems with SHOT 448
Andreas Lundell and Jan Kronqvist

Reversed Search Maximum Clique Algorithm Based on Recoloring 458
Deniss Kumlander and Aleksandr Porošin

Sifting Edges to Accelerate the Computation of Absolute 1-Center in Graphs 468
Wei Ding and Ke Qiu

Solving an MINLP with Chance Constraint Using a Zhang's Copula Family . . . 477
Adriano Delfino

Stochastic Greedy Algorithm Is Still Good: Maximizing Submodular + Supermodular Functions . . . 488
Sai Ji, Dachuan Xu, Min Li, Yishui Wang, and Dongmei Zhang

Towards Multi-tree Methods for Large-Scale Global Optimization . . . 498
Pavlo Muts and Ivo Nowak

Optimization under Uncertainty

Fuzzy Pareto Solutions in Fully Fuzzy Multiobjective Linear Programming . . . 509
Manuel Arana-Jiménez

Minimax Inequalities and Variational Equations . . . 518
Maria Isabel Berenguer, Domingo Gámez, A. I. Garralda–Guillem, and M. Ruiz Galán

Optimization of Real-Life Integrated Solar Desalination Water Supply System with Probability Functions . . . 526
Bayrammyrat Myradov

Social Strategy of Particles in Optimization Problems . . . 537
Bożena Borowska

Statistics of Pareto Fronts . . . 547
Mohamed Bassi, E. Pagnacco, Eduardo Souza de Cursi, and R. Ellaia

Uncertainty Quantification in Optimization . . . 557
Eduardo Souza de Cursi and Rafael Holdorf Lopez

Uncertainty Quantification in Serviceability of Impacted Steel Pipe . . . 567
Renata Troian, Didier Lemosse, Leila Khalij, Christophe Gautrelet, and Eduardo Souza de Cursi

Multiobjective Programming

A Global Optimization Algorithm for the Solution of Tri-Level Mixed-Integer Quadratic Programming Problems . . . 579
Styliani Avraamidou and Efstratios N. Pistikopoulos

A Method for Solving Some Class of Multilevel Multi-leader Multi-follower Programming Problems . . . 589
Addis Belete Zewde and Semu Mitiku Kassa

A Mixture Design of Experiments Approach for Genetic Algorithm Tuning Applied to Multi-objective Optimization 600
Taynara Incerti de Paula, Guilherme Ferreira Gomes, José Henrique de Freitas Gomes, and Anderson Paulo de Paiva

A Numerical Study on MIP Approaches over the Efficient Set 611
Kuan Lu, Shinji Mizuno, and Jianming Shi

Analytics-Based Decomposition of a Class of Bilevel Problems 617
Adejuyigbe Fajemisin, Laura Climent, and Steven D. Prestwich

KMCGO: Kriging-Assisted Multi-objective Constrained Global Optimization 627
Yaohui Li, Yizhong Wu, Yuanmin Zhang, and Shuting Wang

Multistage Global Search Using Various Scalarization Schemes in Multicriteria Optimization Problems 638
Victor Gergel and Evgeniy Kozinov

Necessary Optimality Condition for Nonlinear Interval Vector Programming Problem Under B-Arcwise Connected Functions 649
Mohan Bir Subba and Vinay Singh

On the Applications of Nonsmooth Vector Optimization Problems to Solve Generalized Vector Variational Inequalities Using Convexificators 660
Balendu Bhooshan Upadhyay, Priyanka Mishra, Ram N. Mohapatra, and Shashi Kant Mishra

SOP-Hybrid: A Parallel Surrogate-Based Candidate Search Algorithm for Expensive Optimization on Large Parallel Clusters 672
Taimoor Akhtar and Christine A. Shoemaker

Surrogate Many Objective Optimization: Combining Evolutionary Search, ϵ-Dominance and Connected Restarts 681
Taimoor Akhtar, Christine A. Shoemaker, and Wenyu Wang

Tropical Analogues of a Dempe-Franke Bilevel Optimization Problem 691
Sergeĭ Sergeev and Zhengliang Liu

$\Phi-$Weak Slater Constraint Qualification in Nonsmooth Multiobjective Semi-infinite Programming 702
Ali Sadeghieh, David Barilla, Giuseppe Caristi, and Nader Kanzi

Data science: Machine Learning, Data Analysis, Big Data and Computer Vision

A Discretization Algorithm for *k*-Means with Capacity Constraints . . . 713
Yicheng Xu, Dachuan Xu, Dongmei Zhang, and Yong Zhang

A Gray-Box Approach for Curriculum Learning 720
Francesco Foglino, Matteo Leonetti, Simone Sagratella, and Ruggiero Seccia

A Study on Graph-Structured Recurrent Neural Networks and Sparsification with Application to Epidemic Forecasting 730
Zhijian Li, Xiyang Luo, Bao Wang, Andrea L. Bertozzi, and Jack Xin

Automatic Identification of Intracranial Hemorrhage on CT/MRI Image Using Meta-Architectures Improved from Region-Based CNN 740
Thi-Hoang-Yen Le, Anh-Cang Phan, Hung-Phi Cao, and Thuong-Cang Phan

Bayesian Optimization for Recommender System 751
Bruno Giovanni Galuzzi, Ilaria Giordani, A. Candelieri, Riccardo Perego, and Francesco Archetti

Creation of Data Classification System for Local Administration 761
Raissa Uskenbayeva, Aiman Moldagulova, and Nurzhan K. Mukazhanov

Face Recognition Using Gabor Wavelet in MapReduce and Spark . . . 769
Anh-Cang Phan, Hung-Phi Cao, Ho-Dat Tran, and Thuong-Cang Phan

Globally Optimal Parsimoniously Lifting a Fuzzy Query Set Over a Taxonomy Tree 779
Dmitry Frolov, Boris Mirkin, Susana Nascimento, and Trevor Fenner

K-Medoids Clustering Is Solvable in Polynomial Time for a 2d Pareto Front 790
Nicolas Dupin, Frank Nielsen, and El-Ghazali Talbi

Learning Sparse Neural Networks via ℓ_0 and Tℓ_1 by a Relaxed Variable Splitting Method with Application to Multi-scale Curve Classification 800
Fanghui Xue and Jack Xin

Pattern Recognition with Using Effective Algorithms and Methods of Computer Vision Library 810
S. B. Mukhanov and Raissa Uskenbayeva

The Practice of Moving to Big Data on the Case of the NoSQL Database, Clickhouse 820
Baktagul Imasheva, Nakispekov Azamat, Andrey Sidelkovskiy, and Ainur Sidelkovskaya

Economics and Finance

Asymptotically Exact Minimizations for Optimal Management of Public Finances 831
Jean Koudi, Babacar Mbaye Ndiaye, and Guy Degla

Features of Administrative and Management Processes Modeling 842
Ryskhan Satybaldiyeva, Raissa Uskenbayeva, Aiman Moldagulova, Zuldyz Kalpeyeva, and Aygerim Aitim

Optimization Problems of Economic Structural Adjustment and Problem of Stability 850
Abdykappar Ashimov, Yuriy Borovskiy, and Mukhit Onalbekov

Research of the Relationship Between Business Processes in Production and Logistics Based on Local Models 861
Raissa Uskenbayeva, Kuandykov Abu, Rakhmetulayeva Sabina, and Bolshibayeva Aigerim

Sparsity and Performance Enhanced Markowitz Portfolios Using Second-Order Cone Programming 871
Noam Goldberg and Ishy Zagdoun

Managing Business Process Based on the Tonality of the Output Information 882
Raissa Uskenbayeva, Rakhmetulayeva Sabina, and Bolshibayeva Aigerim

Energy and Water Management

Customer Clustering of French Transmission System Operator (RTE) Based on Their Electricity Consumption 893
Gabriel Da Silva, Hoai Minh Le, Hoai An Le Thi, Vincent Lefieux, and Bach Tran

Data-Driven Beetle Antennae Search Algorithm for Electrical Power Modeling of a Combined Cycle Power Plant 906
Tamal Ghosh, Kristian Martinsen, and Pranab K Dan

Finding Global-Optimal Gearbox Designs for Battery Electric Vehicles 916
Philipp Leise, Lena C. Altherr, Nicolai Simon, and Peter F. Pelz

Location Optimization of Gas Power Plants by a Z-Number Data Envelopment Analysis 926
Farnoosh Fakhari, R. Tavakkoli-Moghaddam, M. Tohidifard, and Seyed Farid Ghaderi

Optimization of Power Plant Operation via Stochastic Programming with Recourse 937
Tomoki Fukuba, Takayuki Shiina, Ken-ichi Tokoro, and Tetsuya Sato

Randomized-Variants Lower Bounds for Gas Turbines Aircraft Engines 949
Mahdi Jemmali, Loai Kayed B. Melhim, and Mafawez Alharbi

Robust Design of Pumping Stations in Water Distribution Networks 957
Gratien Bonvin, Sophie Demassey, and Welington de Oliveira

Engineering Systems

Application of PLS Technique to Optimization of the Formulation of a Geo-Eco-Material 971
S. Imanzadeh, Armelle Jarno, and S. Taibi

Databases Coupling for Morphed-Mesh Simulations and Application on Fan Optimal Design 981
Zebin Zhang, Martin Buisson, Pascal Ferrand, and Manuel Henner

Kriging-Based Reliability-Based Design Optimization Using Single Loop Approach 991
Hongbo Zhang, Younes Aoues, Hao Bai, Didier Lemosse, and Eduardo Souza de Cursi

Sensitivity Analysis of Load Application Methods for Shell Finite Element Models 1001
Wilson Javier Veloz Parra, Younes Aoues, and Didier Lemosse

Transportation, Logistics, Resource Allocation and Production Management

A Continuous Competitive Facility Location and Design Problem for Firm Expansion 1013
Boglárka G.-Tóth, Laura Anton-Sanchez, José Fernández, Juana L. Redondo, and Pilar M. Ortigosa

A Genetic Algorithm for Solving the Truck-Drone-ATV Routing Problem 1023
Mahdi Moeini and Hagen Salewski

A Planning Problem with Resource Constraints in Health Simulation Center 1033
Simon Caillard, Laure Brisoux Devendeville, and Corinne Lucet

Edges Elimination for Traveling Salesman Problem Based on Frequency K_5s 1043
Yong Wang

Industrial Symbioses: Bi-objective Model and Solution Method 1054
Sophie Hennequin, Vinh Thanh Ho, Hoai An Le Thi, Hajar Nouinou, and Daniel Roy

Intelligent Solution System Towards Parts Logistics Optimization 1067
Yaoting Huang, Boyu Chen, Wenlian Lu, Zhong-Xiao Jin, and Ren Zheng

Optimal Air Traffic Flow Management with Carbon Emissions Considerations 1078
Sadeque Hamdan, Oualid Jouini, Ali Cheaitou, Zied Jemai, Imad Alsyouf, and Maamar Bettayeb

Scheduling Three Identical Parallel Machines with Capacity Constraints 1089
Jian Sun, Dachuan Xu, Ran Ma, and Xiaoyan Zhang

Solving the Problem of Coordination and Control of Multiple UAVs by Using the Column Generation Method 1097
Duc Manh Nguyen, Frédéric Dambreville, Abdelmalek Toumi, Jean-Christophe Cexus, and Ali Khenchaf

Spare Parts Management in the Automotive Industry Considering Sustainability 1109
David Alejandro Baez Diaz, Sophie Hennequin, and Daniel Roy

The Method for Managing Inventory Accounting 1119
Duisebekova Kulanda, Kuandykov Abu, Rakhmetulayeva Sabina, and Kozhamzharova Dinara

The Traveling Salesman Drone Station Location Problem 1129
Daniel Schermer, Mahdi Moeini, and Oliver Wendt

Two-Machine Flow Shop with a Dynamic Storage Space and UET Operations 1139
Joanna Berlińska, Alexander Kononov, and Yakov Zinder

Correction to: The Practice of Moving to Big Data on the Case of the NoSQL Database, Clickhouse C1
Baktagul Imasheva, Nakispekov Azamat, Andrey Sidelkovskiy, and Ainur Sidelkovskaya

Author Index 1149

Continuous Optimization

A Hybrid Simplex Search for Global Optimization with Representation Formula and Genetic Algorithm

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Abstract. We consider the problem of minimizing a given function $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ on a regular not empty closed set S. When f attains a global minimum at exactly one point $x^* \in S$, for a convenient random variable X and a convenient function $g : \mathbb{R}^2 \longrightarrow \mathbb{R}$. In this paper, we propose to use this Representation Formula (RF) to numerically generate an initial population. In order to obtain a more accurate results, the Representation Formula has been coupled with other algorithms:

- Classical Genetic Algorithm (GA). We obtain a new algorithm called (RFGA),
- Genetic Algorithm using Nelder Mead algorithm at the mutation stage (GANM). We obtain a new algorithm called (RFGANM),
- Nelder Mead Algorithm. We obtain a new algorithm called (RFNM).

All these six algorithms (RF, GA, RFGA, GANM, RFGANM, RFNM) were tested on 21 benchmark functions with a complete analysis of the effect of different parameters of the methods. The experiments show that the RFNM is the most successful algorithm. Its performance was compared with the other algorithms, and observed to be the more effective, robust, and stable than the others.

Keywords: Global optimization · Genetic algorithm · Representation formula · Nelder Mead algorithm

1 Introduction

In the context of the resolution of engineering problems, many optimization algorithms have been proposed, tested and analyzed in the last decades. However, optimization in engineering remains an active research field, since many real-world engineering optimization problems remain very complex in nature and quite difficult to be solved by the existing algorithms. The existing literature presents intensive research efforts to solve some difficulty points, which remains

H. A. Le Thi et al. (Eds.): WCGO 2019, AISC 991, pp. 3–15, 2020.
https://doi.org/10.1007/978-3-030-21803-4_1

still incompletely solved and for which only partial response has been obtained. Among these, we may cite: handling of non convexity - specially when optimization restrictions are involved, working with incomplete or erroneous evaluation of the functions and restrictions, increasing the number of optimization variables up to those of realistic designs in practical situations, dealing with non regular (discontinuous or non-differentiable) functions, determining convenient starting points for iterative methods. Floudas [5] We observe that the difficulties concerning non-convexity and the determination of starting points are connected: efficient methods for the optimization of regular functions are often deterministic and involve gradients, but depends strongly on the initial point - they can be trapped by local minima if a non convenient initial guess is used. Alternatively, methods based on the exploration of the space of the design variables usually involve a stochastic aspect - thus, a significant increase in the computational cost - and are less dependent of the initial choice, but improvements in their performance request combination with deterministic methods and may introduce a dependence on the initial choice. This last approach tends to the use of hybrid procedures involving both approaches and try to benefit from the best of each method - by these reasons, the literature about mixed stochastic/deterministic methods has grown in the last years [2]. Those hybrid algorithms perform better if the initial point belongs to an attraction area of the optimum. This shows the importance of the initial guesses in optimization algorithm [8]. Hence, we would like in this paper to use a representation formula to provide a convenient initial guess of the solution. Let S denote a closed bounded regular domain of the n-dimensional Euclidean space $\mathbb{R}^n$, and let f be a continuous function defined on S and taking its values on $\mathbb{R}$. An unconstrained optimization problem can be formulated, in general, as follows:

$$x^* = Arg \min_{x \in S} \; f(x) \;, \tag{1}$$

In the literature, representation formulas have been introduced in order to characterize explicitly solutions of the problem 1. In general, these representations assume that S contains a single optimal point $\mathbf{x}^*$ (but many local minima may exist on S). For instance, Pincus [9] has proposed the representation formula:

$$x^\star = \lim_{\lambda \to +\infty} \frac{\int_S x \; e^{-\lambda f(x)} dx}{\int_S e^{-\lambda f(x)} dx}.$$

More recently, the original representation proposed by Pincus has been reformulated by Souza de Cursi [3] as follows: let X be a random variable taking its values on S and $g : \mathbb{R}^2 \longrightarrow \mathbb{R}$ be a function. If these elements are conveniently chosen, then

$$x^* = \lim_{\lambda \to +\infty} \frac{E\left(X \;\, g\left(\lambda, f(X)\right)\right)}{E\left(g\left(\lambda, f(X)\right)\right)} \tag{2}$$

The formulation of Pincus corresponds to $g\left(\lambda, s\right) = e^{-\lambda s}$, what is a convenient choice. The general properties of X and g are detailed, for instance, in [4]. An extension to infinite dimensional situations can be found in [4]. In this work, we propose the use of the representation given by Eq. (3) hybridized with the Nelder Mead algorithm and a genetic algorithm, for the global optimization of multimodal functions.

2 Hybrid Simplex Search with Representation Formula and Genetic Algorithm

Hybrid methods have been introduced to keep the flexibility of the stochastic methods and the efficiency of the deterministic one. In our paper, the hybrid method for solving optimization problems is a coupling of the representation formula proposed by Pincus [9] and Nelder Mead algorithm and genetic algorithm. The representation formula is used first to find the region containing the global solution, based on the generating of finite samples of the random variables involved in the expression and an approximation of the limits. For instance, we may choose λ large enough and generate a sample by using standard random number generators.

The generation of points can be done either using a normal distribution or a Gaussian one. In the case of Gaussian distribution, when a trial point generated lies outside S, it has been projected in order to get admissible point.

In order to obtain a more accurate results, it is convenient to use the improvement by the following algorithms:

- Classical Genetic Algorithm (GA). We obtain a new algorithm called (RFGA).
- Genetic Algorithm using Nelder Mead algorithm at the mutation stage (GANM). We obtain a new algorithm called (RFGANM).
- Nelder Mead Algorithm. We obtain a new algorithm called (RFNM).

2.1 Representation Formula

As previously observed, if f attains its global minimum at exactly one point x^* on S, we have

$$x^* = \lim_{\lambda \to +\infty} \frac{E\left(X\ g\left(\lambda, f(X)\right)\right)}{E\left(g\left(\lambda, f(X)\right)\right)}, \tag{3}$$

where $g : \mathbb{R}^2 \longrightarrow \mathbb{R}$ is continuous and strictly positive, $s \longrightarrow g : (\lambda, s)$ is strictly decreasing for any $s \in f(S)$ and $\lambda > 0$, while X is a convenient random variable. These conditions are fullfilled, for instance, when X is uniformly distributed or gaussian and $g\left(\lambda, s\right) = e^{-\lambda s}$ (what corresponds to the classical choice of Pincus). We use these particular choices in the sequel.

A numerical implementation can be performed by taking a large fixed value of λ in order to represent the limit $\lambda \to +\infty$. In order to prevent an overflow, λ should be increased gradually up to the desired value and it may be convenient to use positive functions f (for instance, by adding a constant to the original f). A finite sample of X is generated, according to a probability P - this consists simply in generating N admissible points $(x_1, x_2, ..., x_N) \in S$ - and estimations of the means are used to approximate the exact means, what leads to

$$x^* \simeq x_c^* = \frac{\sum_{i=1}^{N} x_i \quad g\left(\lambda, f(x_i)\right)}{\sum_{i=1}^{n} g\left(\lambda, f(x_i)\right)}$$

3 Test Bed

To demonstrate the efficiency and the accuracy of the hybrid algorithms, 21 typical benchmark functions of different levels of complexity and multimodality were chosen from the global optimization literature [6].
One hundred runs have been performed for each test function to estimate the probability of success of the used methods. The used test functions are: Bohachevsky 1 **BO1**, Bohachevsky 2 **BO2**, Branin function **BR**, Camel function **CA**, Cosine mixture function **CO**, DeJoung function **DE**, Goldstein and price function **GO**, Griewank function **GR**, Hansen function **HN**, Hartman 3 function **HR3**, Hartman 6 function **HR6**, Rastrigin function **RA**, Rosenbrock function **RO**, Shekel 5 **SK5**, Shekel 7 **SK7**, Shekel 10 **SK10**, Shubert 1 function **SH1**, Shubert 2 function **SH2**, Shubert 3 function **SH3**, Shubert 4 function **SH4** and Wolfe nondifferentiable function **WO**.

4 Numerical Results

In this section we focus on the efficiency of the six algorithms, i.e. Representation Formula (RF), Classical Genetic Algorithm (GA), Representation Formula with GA (RFGA), Genetic Algorithm using Nelder Mead algorithm at the mutation stage (GANM), Representation Formula with GA and Nelder Mead (RFGANM), and Representation Formula with Nelder Mead (RFNM).

A series of experiments have been done to make some performance analysis about them. To avoid attributing the optimization results to the choice of a particular conditions and to conduct fair comparisons, we have performed each test 100 times, starting from various randomly selected points in the hyper rectangular search domain.

The used parameters in genetic algorithm are: Population size: from 2 to 50, the mutation rate is set to 0.2, the selection method is rank weighting, the stopping criteria are maximum iteration (set to 2000 for GA) and the maximum number of continuous iterations without improving the solution, and it is set to 1000 for GA.

Concerning NM we adopted the standard parameters recommended by the authors, the used stoping criteria are: Maximum function evaluation, $maxfun = 50000$, maximum of iteration, $maxiter = 10000$, termination tolerance on the function value $tolf = 10^{-5}$ and termination on the tolerance on $xtolx = 10^{-6}$.

Extensive experimentations concerning the effect of different parameters have been performed: influence of the Pincus function, influence of the population size for GA, influence of the sample size for RF and comparison with others methods. Because of the limitation of space, only a few experiments are presented, chosen in order to illustrate significant aspects.

The following abbreviations are introduced to lighten the text:

TestF: Test Function; Dim: Dimension; PopGA: Genetic algorithm population size; SS: Sample size used in RF; SR: Success rate; SD: Standard deviation; CPUT: CPU Time (in seconds); NEvalF: Number of function evaluations; The reported results are in terms of the rate of successful optimizations (SR), the standard deviation, the CPU time and the average of the function evaluation number. The term SR is the number of successful run i.e. when the algorithm generates a solution with a required accuracy, where the 'required accuracy' is a given maximum value that is calculated as the absolute difference between the solution found and the global optimum divided by the global solution (when it is non zero). The chosen accuracy for all the tests is 10^{-4}.

4.1 Influence of the Pincus Function

In the representation formula, the Pincus expression corresponds to $g(\lambda, s) = e^{-\lambda s}$, which is a convenient choice. In our experiments, four other descent functions (continuous and strictly decreasing) have been used for solving the benchmark functions with the proposed algorithms: $\frac{1}{\lambda\ s^3}$, $\frac{1}{\lambda\ ln(s)}$, $\frac{e^{-\lambda}}{s^3}$, *and* $10^{(-\lambda\ s)}$.

The tests show that the choice of the function has no significant effect on the solution quality and algorithms performance, a small difference in terms of execution time has been observed.

4.2 Influence of the Population Size Used in GA

To examine the effect of the population size on the solution quality as well as the computational efforts for acquiring the optimal solution using GA, GANM, RFGA and RFGANM, eight levels of size (2, 4, 6, 8, 12, 16, 20 and 50) are examined and the experimental results are reported in Table 1 for GA and GANM methods and Table 2 for RFGA and RFGANM. The sample size for RF depends on the function dimension: SS = 60 for Dim = 1 or 2, SS = 100 for Dim = 3, SS = 300 for Dim = 4 or 5, and SS=500 for Dim =6 or 10. One of the general observations is that as the population size increase, all of them require more time (more number of function evaluations) and tend to find better solution, as indicated by smaller standard deviation and bigger success rate. In the case of more complex problem (case Rosenbrock functions) the GA and RFGA failed to find a solution with the required accuracy ($SR = 0\%$) even for $PopGA = 200$. A slight improvement is obtained by the representation formula (RFGA).
A worsening of the solution obtained by GA is observed for GANM (except for SK10, BO2, GR and RO). The results obtained by RFGANM are the best in term of success rate (SR=100% for PopGA=50) with additional number of function evaluation.

Table 1. Influence of the population size for GA and GANM

TestF	Dim	PopGA	GA				GANM			
			SR	SD	CPUT	NEvalF	SR	SD	CPUT	NEvalF
SH4	4	12	60%	2,13E-02	0,35	23 613	4%	2,68E-01	14,88	213 782
SH4	4	20	94%	2,13E-02	0,43	39 860	19%	3,16E-01	25,05	360 290
SH4	4	50	100%	5,77E-06	0,72	98 557	77%	1,11E-01	74,01	1 060 096
SK10	4	12	27%	3,52E-01	0,80	23 812	83%	2,08E-01	68,77	218 926
SK10	4	20	33%	3,33E-01	0,88	39 762	94%	1,22E-01	120,73	381 753
SK10	4	50	61%	3,31E-01	1,12	95 898	100%	5,64E-08	341,22	1 079 693
BO2	2	12	24%	1,07E-01	0,23	20 562	100%	5,72E-11	0,29	6 359
BO2	2	20	53%	1,06E-01	0,25	31 919	100%	2,68E-11	0,48	10 824
BO2	2	50	85%	7,61E-02	0,37	72 035	100%	3,47E-12	1,24	28 147
CA	2	12	93%	1,00E-04	0,24	20 489	98%	1,11E-01	3,39	64 233
CA	2	20	100%	1,39E-05	0,28	33 515	100%	3,72E-08	5,74	109 435
CA	2	50	100%	6,06E-06	0,43	77 729	100%	3,72E-08	15,85	303 313
CO	4	12	100%	1,16E-05	0,30	23 818	29%	2,46E-01	1,87	35 774
CO	4	20	100%	1,22E-06	0,35	39 336	65%	2,02E-01	1,91	38 478
CO	4	50	100%	1,09E-06	0,56	93 199	100%	6,88E-08	2,41	54 340
GR	5	12	0%	1,87E-01	0,33	23 998	93%	4,17E-02	24,97	376 071
GR	5	20	1%	1,75E-01	0,39	39 884	99%	1,46E-02	42,64	642 752
GR	5	50	44%	1,18E-01	0,67	102 050	100%	1,95E-13	113,94	1 716 703
HN	2	12	81%	5,06E-02	0,27	21 712	52%	2,62E-01	4,69	80 671
HN	2	20	94%	3,85E-02	0,32	36 018	76%	1,29E-01	8,29	142 888
HN	2	50	100%	3,19E-06	0,47	82 607	99%	1,76E-02	22,37	384 416
RA	2	12	27%	5,05E-03	0,22	19 169	51%	1,24E+00	1,35	26 337
RA	2	20	64%	3,62E-04	0,26	32 278	67%	6,11E-01	1,57	31 274
RA	2	50	93%	1,11E-04	0,34	67 520	98%	1,40E-01	1,61	35 270
RO	10	12	0%	3,45E+01	0,33	24 012	100%	7,88E-12	44,78	683 973
RO	10	20	0%	3,37E+01	0,40	40 020	100%	5,49E-12	75,83	1 161 545
RO	10	50	0%	2,92E+01	0,69	102 050	100%	2,99E-12	200,54	3 071 508
WO	2	12	1%	4,63E-02	0,23	20 512	100%	0,00E+00	2,04	39 624
WO	2	20	6%	2,59E-02	0,25	31 105	100%	0,00E+00	3,55	69 076
WO	2	50	43%	1,07E-02	0,37	70 768	100%	0,00E+00	9,42	183 422

4.3 Influence of the Sample Size for RF

To investigate the effect of the simple size used in the representation formula on searching quality of the hybrid algorithms, we choose six level of sample sizes ($SS = 60, 100, 500,\ 1000, 2000\ and\ 5000$) and we set the value of population size to 6. The experiments results are reported in Table 3. Because of the limitation of space, only 10 test functions are presented, chosen in order to illustrate significant aspects.

We observe that the success rate and the number of function evaluation increase with the sample size. The RF method failed in almost the tests ($SR = 0\%$ in the most cases) except in the case of HR3 function ($SR = 100\%\ for\ SS =$