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Joe Rosen

SYMMETRY RULES

How Science and Nature Are Founded on Symmetry

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For Mira

Preface

Ernest Rutherford (New Zealand–British physicist, 1871–1937), the 1908 Nobel Laureate who discovered the existence of atomic nuclei, is famously quoted as having said: “Physics is the only real science. All the rest is butterfly collecting.” Or something to that effect. I like to include this quote in my introductory remarks at the first class meetings of the physics courses I teach.

I have seen that there are those who interpret this as a put-down of amateurs (butterfly collectors) in science. However, my own interpretation of Rutherford’s statement is that he is claiming that, except for physics, all of the rest of science is involved merely in collecting facts and classifying them (butterfly collecting). It is physics, unique among the sciences, that is attempting to find *explanations* for the classified data.

The periodic table of the chemical elements, originally proposed by Dmitri Ivanovich Mendeleev (Russian chemist, 1834–1907), presents an example of this. Chemists toiled to discover the chemical elements and their properties and then classified the elements in the scheme that is expressed by the periodic table. Here was the chemists’ butterfly collecting. It took physicists to *explain* the periodic table by means of quantum theory.

Rutherford’s assessment of science might well have held a large degree of validity in the 19th and early 20th centuries. But since then other fields of science than physics have developed ‘physics envy’ and they too are now busy searching for explanations. For example, chemistry finds its explanations in physics. And explanations in biology are found, on one level, in evolution theory and, on another level, in chemistry and physics.

I differ with Rutherford, though, in his narrow conception of science. To be sure, science involves searching for explanations. But production and collection of data through experimentation and observation and classification of the data supply the raw material for science to attempt to explain. Without them there would be nothing to explain and no 'real science' in Rutherford's sense. So I include butterfly collecting in my broad conception of science.

The point of all that, for the purpose of this book, is to lead to the notion that science – even in its broad conception – not only makes much use of symmetry, but is essentially and fundamentally based on symmetry. Indeed, science rests firmly on the triple foundation of reproducibility, predictability, and reduction, all of which are symmetries, with additional support from analogy and objectivity, which are symmetries too. So it is not much of an exaggeration to claim that science is symmetry. Or perhaps in somewhat more detail, science is our view of nature through symmetry spectacles. That is one component of the main thesis of this book.

In addition to an exposition and justification of this central idea, that science is founded on symmetry, we also look into how symmetry is used in science in general and in physics in particular (Rutherford's 'real science'). And we find: symmetry of evolution (symmetry of the laws of nature), symmetry of states of physical systems, gauge symmetry of the fundamental interactions, and the symmetry inherent to quantum theory. So not only do we *view* nature through symmetry spectacles, but we *understand* nature in the language of symmetry. That is another component of this book's main thesis.

All that leads to deep questions that await clarification. What is the source of all this symmetry? What is nature telling us? Is nature symmetry, at least in some sense? If not at the level that physics is presently investigating, are deeper levels of reality involved with symmetry in a very major way? Or even, will symmetry turn out to be what those fundamental levels are *all* about? Is symmetry the foundational principle of the Universe?

Such ideas lurk in the back of many physicists' minds, and some physicists express them outrightly. Brian Greene, for one, states in Chap. 8 of [1]: "From our modern perspective, symmetries are the foundation from which laws spring." And Stenger [2] adds his vote.

Speaking of the Universe, it is shown in this book that the Universe cannot possess exact symmetry. This connects to conceptual problems with symmetry breaking at 'phase transitions' in the evolution of the

Universe according to big-bang type cosmological schemes. Such and related matters are discussed, including the nature of the 'quantum era' that is assumed to form the first evolutionary stage in big-bang type schemes. But many questions remain for future elucidation. Are big-bang type cosmological schemes the best models for the evolution of the Universe? If so, did the Universe pass through distinct eras separated by transitions that might be characterized as 'phase transitions'? What were the properties of the eras and of the transitions? Was there a 'quantum era'? If there was, can it be meaningfully described? And can present-day high-energy physics reflect the properties of earlier stages in the evolution of the Universe? If it can, what will the results of experiments soon to be performed at high-energy laboratories, such as CERN's Large Hadron Collider, reveal about the earlier Universe? And what will they tell us about today's physics? Will they help clarify or will they sow confusion?

Here is the order of presentation: We start in Chap. 1 with a brief introduction to the concept of symmetry, including an analysis of the intimate relation between symmetry and asymmetry – especially that symmetry implies asymmetry – and a discussion of analogy and classification as symmetry. We then see in Chap. 2 what science is, how it makes use of symmetry, and how it is based solidly on symmetry. So solidly, in fact, that one might well view science as symmetry. In Chap. 3 we consider a number of ways in which physics, in particular, additionally makes use of symmetry. Since physics underlies the other sciences, we find that science is based even more solidly on symmetry, and perhaps nature will turn out to possess a symmetry foundation as well. The symmetry principle, also known as Curie's principle, is derived in its various versions in Chap. 4. We see in Chap. 5 two ways in which the symmetry principle is very usefully applied in science. In Chaps. 6 and 7 we discuss the ideas of imperfect symmetry and symmetry in general and as applied to the Universe and its evolution, as well as related ideas.

There then follows the more formal part of the book, in which we develop a formalism of symmetry. Chapters 8 and 9 form a brief introduction to group theory, the mathematical language of symmetry, which is indispensable for serious quantitative, as well as qualitative, applications of symmetry in science, mostly in physics and chemistry. Nevertheless, in spite of that indispensability, Chaps. 8 and 9 can be skipped without too much harm to those preferring a more conceptual approach. Chapter 10 develops the language and formalism that underlie the application of symmetry. Group theory is unavoidable there, but I try to allow the reader to make sense of the ideas even without group theory. And finally, in Chap. 11 we apply symmetry considerations and the symmetry formalism to physical processes and derive the symmetry principles that apply to them.

Chapter 12 brings together and summarizes the principles of symmetry that are developed and presented in this book.

I would like to express my thanks to my friends and colleagues Avshalom C. Elitzur and Lawrence W. Fagg, who kindly read the manuscript of this book and helped me with their comments and suggestions. And especially, I thank my wife, Mira Frost, for her unflagging support and for putting up with my disappearances into my study to work on this book.

Rockville, Maryland, *J. Rosen*
August 2007

Contents

1

The Concept of Symmetry

1.1 The Essence of Symmetry

Everyone has some idea of what *symmetry* is. We recognize the bilateral (left-right) symmetry of the human body, of the bodies of many other animals, and of numerous objects in our environment. We enjoy the rotation symmetry of many kinds of flower. We consider a scalene triangle, one with all sides unequal, to be completely lacking in symmetry, while we see symmetry in an isosceles triangle and even more symmetry in an equilateral triangle. That is only for starters. Any reader of this book can easily point out many more kinds and examples of symmetry.

In science, of course, our recognition and utilization of symmetry is often more sophisticated, sometimes very much more. But what symmetry actually boils down to in the final analysis is that *the situation possesses the possibility of a change that leaves some aspect of the situation unchanged*.

A bilaterally symmetric body can be reflected through its midplane, through the (imaginary) plane separating the body's two similar halves. Think of a two-sided mirror positioned in that plane. Such a reflection is a change. Yet the reflected body looks the same as the original one; it coincides with the original: the reflected right and left hands, paws, or hooves coincide, respectively, with the original left and right ones, and similarly with the feet, ears, and other paired parts (see Fig. 1.1).

For the triangles let us for simplicity confine ourselves to rotations and reflections within the plane of the figures. Then a rotation is made about a point in the plane, which is the point of intersection of the

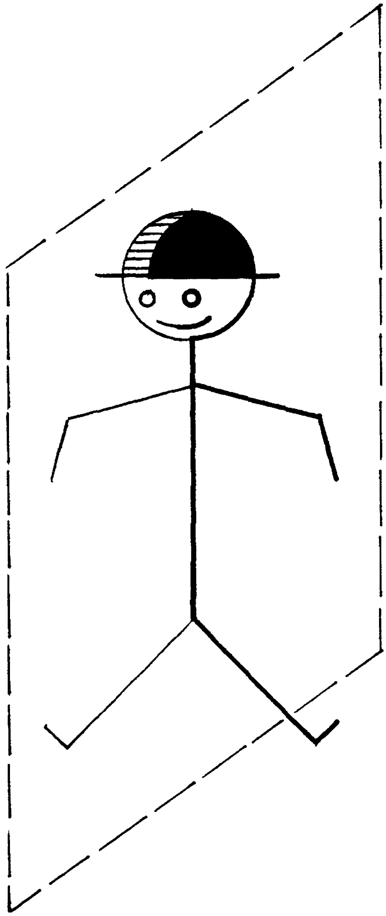

Fig. 1.1. Bilateral symmetry

axis of rotation that is perpendicular to the plane. A reflection is made through a line in the plane, where the line is where a two-sided mirror that is perpendicular to the plane intersects the plane. An infinite number of such changes can be performed on any triangle. But for an equilateral triangle there are only a finite number of them that can be made on it and that nevertheless leave its appearance unchanged, i.e., rotations and reflections for which the changed triangle coincides with the original. They are rotations about the triangle's center by 120° and by 240°, and reflections through each of the triangle's three heights, five changes altogether (see Fig. 1.2). (For the present we do not count rotations by multiples of 360°, which are considered to be no change at all.)

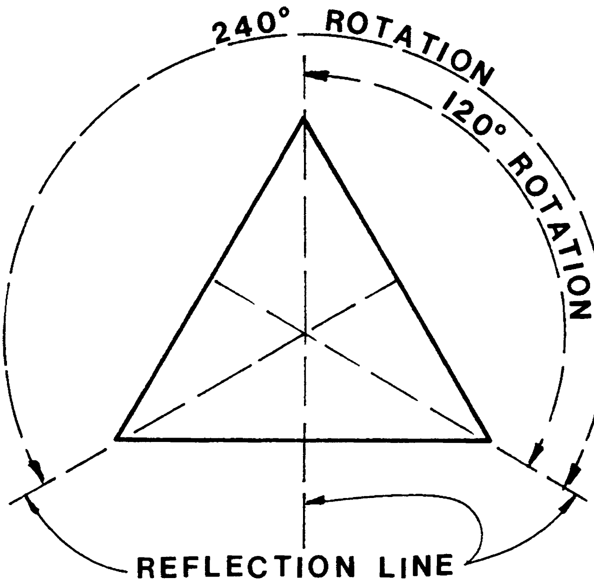

Fig. 1.2. Changes bringing an equilateral triangle into coincidence with itself

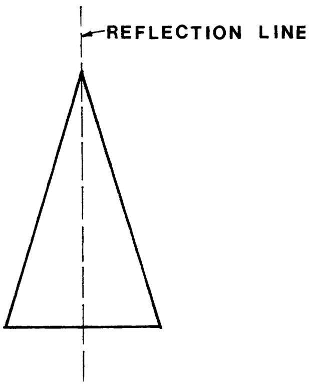

Fig. 1.3. Change bringing an isosceles triangle into coincidence with itself

Although an infinity of planar rotations and reflections can also be performed on any isosceles triangle, there is only a single such change that preserves the appearance of such a triangle, that leaves the triangle coinciding with itself. It is reflection through the height on its base (see Fig. 1.3). And a scalene triangle cannot be made to coincide with itself by any planar rotation or reflection, once again not counting rotations by multiples of 360° (see Fig. 1.4).

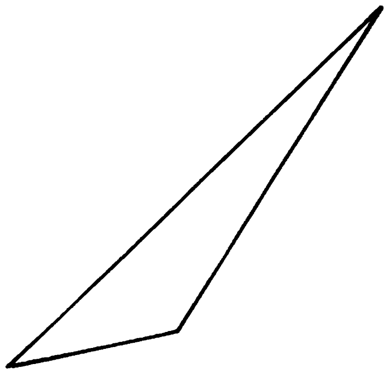

Fig. 1.4. No change brings a scalene triangle into coincidence with itself

I stated above that symmetry is in essence that the situation possesses the possibility of a change that nevertheless leaves some aspect of the situation unchanged. That can be concisely formulated as this precise definition of symmetry:

Symmetry is immunity to a possible change.

When we have a situation for which it is possible to make a change under which some aspect of the situation remains unchanged, i.e., is immune to the change, then the situation can be said to be *symmetric under the change with respect to that aspect*. For example, a bilaterally symmetric body is symmetric under reflection through its midplane with respect to appearance. Its external appearance is immune to midplane reflection. (The arrangement of its internal organs, however, most usually does not have that symmetry. The human heart, for instance, is normally left of center.) For very simple animals, their bilateral symmetry might also hold with respect to physiological function as well. That is not true for more complex animals.

An equilateral triangle is symmetric with respect to appearance under the rotations and reflections we mentioned above. An isosceles triangle is symmetric with respect to appearance under reflection through the height on its base. But a scalene triangle is not symmetric with respect to appearance under any planar rotation or reflection.

Note the two essential components of symmetry:

1. *Possibility of a change*. It must be possible to perform a change, although the change does not actually have to be performed.
2. *Immunity*. Some aspect of the situation would remain unchanged, if the change were performed.

If a change is possible but some aspect of the situation is not immune to it, we have *asymmetry*. Then the situation can be said to be *asymmetric under the change with respect to that aspect*. For example, a scalene triangle is asymmetric with respect to appearance under all planar rotations and reflections. All triangles are asymmetric with respect to appearance under 45° rotations. While equilateral triangles are symmetric with respect to appearance under 120° rotations about their center, isosceles triangles do not possess this symmetry; they are asymmetric under 120° rotations with respect to appearance. And while a triangle might be symmetric or asymmetric with respect to appearance under a given rotation or reflection, all triangles are symmetric under all rotations and reflections with respect to their area; rotations and reflections do not change area. On the other hand, all plane figures are asymmetric with respect to area under dilation, which is enlargement (or reduction) of all linear dimensions by the same factor. The area then increases (or diminishes) by the square of that factor.

If there is no possibility of a change, then the very concepts of symmetry and asymmetry will be inapplicable. For example, if the property of color is not an ingredient of the specification of a plane figure, then the change of, say, color interchange will not be a possible change for such a figure. Thus color interchange symmetry or asymmetry will not be conceptually applicable to the situation. Or alternatively, one might say that such a plane figure will possess trivial symmetry under such a change. One might say that all its aspects will be trivially immune to such a change. It is a matter of taste, but I tend to prefer calling it inapplicability rather than triviality.

If, however, color *is* included in the specification of a figure, then color interchange will become a possible change for it. For example, if the figure is black and white, it will be symmetric under red–green interchange with respect to appearance. Interchange red and green, and nothing will happen to the figure. If the figure is black and green, it will be asymmetric under the same change with respect to the same aspect. Interchange red and green, and the figure will become black and red, which is not the same as black and green.

As an example, consider the black–white figure of Fig. 1.5. What symmetries can we find lurking here? For simplicity let us confine ourselves to the plane of the figure, as we did earlier. If we consider only the geometric properties of the figure and ignore its coloring, then the figure possesses the symmetry of the square with respect to its appearance: it is symmetric under rotations by 90°, 180°, and 270°,

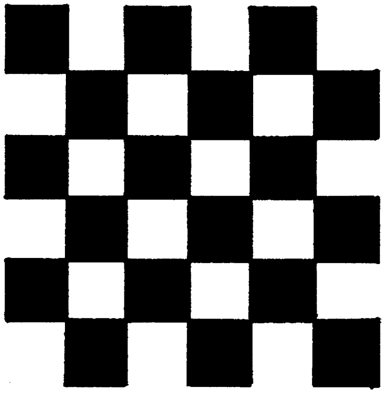

Fig. 1.5. Black–white figure as an example

and under reflections through each of its diagonals and through each of its vertical and horizontal midlines. (You might want to indicate these four lines in the figure.) That adds up to seven changes under which the figure is symmetric with respect to appearance, ignoring its coloration.

Let color enter our considerations. Some of the changes that did not change appearance before, now do make a difference. They are rotations by 90° and 270° and reflections through the vertical and horizontal midlines. That leaves the colored figure symmetric with respect to appearance only under rotation by 180° and reflections through each of the two diagonals, three changes. With color now in the picture, we can consider black–white interchange. However, the figure is asymmetric under this change. Nevertheless, we can still find symmetry under black–white interchange if we combine the interchange with a geometric change to form a compound change. Thus the figure is symmetric with respect to appearance under the compound changes consisting of black–white interchange together with rotation by 90°, with rotation by 270°, with reflection through the vertical midline, and with reflection through the horizontal midline, making four compound changes.

Approximate symmetry is approximate immunity to a possible change. There is no approximation in the change or in its possibility; it must indeed be possible to perform a change. The approximation is in the immunity. Some aspect of the situation must change by only a little, however that is evaluated, when some change is performed. Then the situation can be said to be *approximately symmetric under the change with respect to that aspect.*

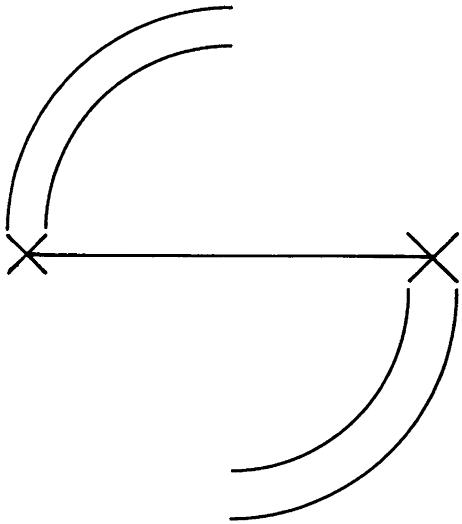

Fig. 1.6. Approximate two-fold rotation symmetry

For example, the figure of Fig. 1.6 possesses approximate two-fold rotation symmetry with respect to its appearance. Under 180° rotation about its center its appearance changes, but only by a little. And the bilateral symmetry of humans and other animals is in reality also only approximate. Not only do the internal organs not all possess that symmetry, but even for external appearance the symmetry is never exact. For instance, the fingerprints of one hand are not the mirror images of the corresponding fingerprints of the other hand, and the hand and foot of one side (usually the right side for right-handed people) are almost always slightly longer than those of the other side.

Approximate symmetry is a softening of the hard dichotomy between symmetry and asymmetry. The extent of deviation from exact symmetry that can still be considered approximate symmetry will depend on the context and the application and could very well be a matter of personal taste. The same figure, for example that of Fig. 1.6, might be considered approximately symmetric (or slightly asymmetric) by some observers, while others might consider it very asymmetric (or nowhere near symmetric). We will discuss approximate symmetry in more detail in Sect. 6.1 and will formalize the notion in Sect. 10.6.

1.2 Symmetry Implies Asymmetry

Change is the bringing about of something different. For a difference to exist, in the sense of having physical meaning, a physical gauge for the difference, a *reference frame*, is needed. A reference frame serves as a standard against which putative changes are evaluated: You think you have performed a change. OK, let us gauge the situations before and after what you think was a change. They come up different, the gauge distinguishes between them? You have indeed made a change. The gauge shows no difference, does not distinguish between them? Your 'change' is no change. Thus, *the existence of a reference frame is necessary to give existence to the difference and to the possibility of change*. And *the nonexistence of an appropriate reference frame makes a supposed change impossible*. In Sect. 3.3 we discuss the idea of reference frame in detail.

To illustrate this, think of an object floating in otherwise empty space. Now consider the change of moving the object by some distance in some direction, i.e., spatial displacement. All right, the object is now displaced, at least we might think it is. Picture the result of the move: again an object floating in otherwise empty space. Is the result of the displacement physically (as opposed to, say, philosophically) different from the original situation? No. There is no way to distinguish between the original and the displaced situations. They are identical. A displacement involves a change of location. But in otherwise empty space all locations are identical, since there is no reference frame for position in the space. So no change has taken place and the supposed change is not a possible change. The case is similar for rotation and for reflection.

In order to be capable of gauging a difference, a reference frame cannot be immune to the change that brings about the difference. It cannot be immune to the change for which it is intended to serve as reference. Otherwise it could not serve its purpose.

For example, the change of spatial displacement brings about a difference in location. A set of tape measures as coordinate axes could serve as a reference frame for that, but not if the tapes are themselves immune to displacement, i.e., not if the tapes are infinite and homogeneous (unmarked). Imagine sliding such a tape parallel to itself and compare the final and original situations of the tape with each other. There is no difference. Marked axes, which can indeed gauge differences in location and can thus serve as a reference frame for displacements, are themselves affected by displacements. Imagine sliding a marked tape, like an ordinary tape measure, parallel to itself and compare its final and initial situations with each other. They are clearly different. If you imagine sliding the tape eight meters in the positive direction, then in its final state the tape's zero mark will align with the eight meter mark of the tape in its original state.

A reference frame is a changeable aspect of a situation. Now, any changeable aspect of a situation can serve as a reference frame for that change in the situation, since it is tautological that a changeable aspect of a situation is not immune to its own change. A changeable aspect of a situation allows the possibility of a change. Indeed, we can say that it *represents* the possibility of a change and that any possibility of a change is represented by a changeable aspect of the situation. So a situation will possess symmetry if and only if it has *both* an aspect that can change – giving the possibility of a change – *and* an aspect that does not change concomitantly – giving the immunity to the possible change. In other words, the possibility of a change, which is a necessary component of symmetry, is contingent upon the existence of an asymmetry of the situation under the change. And hence the succinct result:

Symmetry implies asymmetry.

This is discussed further in [3]. Another, less succinct way of expressing the relation between symmetry and asymmetry is this:

> *Symmetry requires a reference frame, which is necessarily asymmetric. The absence of a reference frame implies identity, hence no possibility of change, and hence the inapplicability of the concept of symmetry.*

Consider, for example, the equilateral triangle of Fig. 1.2. Its appearance is an aspect of it that is immune to 120° rotation about its center in its plane, so it possesses symmetry under 120° rotation with respect to appearance. Or so we might blithely think. But what is this change we call '120° rotation'? Do what we will to the triangle – twist it, twirl it, twitch it, swivel it – when is it rotated by 120°, or rotated at all for that matter? Unless we have a reference frame to endow an orientation difference with existence and thus give significance to a rotation, all our actions amount to nil, nothing is accomplished. The situations before and after our efforts remain identical. We then have no rotation at all, nor do we have even the conceptual possibility of rotation. So the concept of rotation symmetry is inapplicable to the triangle in the absence of an appropriate reference frame.

However, the triangle is not a universe in itself. The *total* situation, that of the equilateral triangle together with its environment, does possess aspects that are not immune to 120° rotation and that can thereby serve as a reference frame for 120° rotation. The walls of the room, for instance, could serve as reference frame, since they are asymmetric under 120° rotation. Thus, rotation by 120° is indeed a change. The equilateral triangle is symmetric in the context of its environment. It is symmetric under 120° rotation thanks to its environment's lack of immunity to 120° rotation, thanks to the asymmetry of the total situation – triangle plus environment – under the rotation.

The above results concerning symmetry, change, immunity, reference frame, and asymmetry can be summarized by the following diagram, where arrows denote implication:

$$\text{symmetry} \begin{array}{l} \nearrow \begin{array}{l}\text{possibility}\\ \text{of a change}\end{array} \longrightarrow \begin{array}{l}\text{reference for}\\ \text{the change}\end{array} \longrightarrow \begin{array}{l}\text{asymmetry}\\ \text{under the change}\end{array} \\ \\ \searrow \begin{array}{l}\text{immunity to}\\ \text{the change}\end{array} \end{array}$$

Thus, *for there to be symmetry, there must concomitantly exist asymmetry under the same change that is involved in the symmetry.* For every symmetry there is an asymmetry tucked away somewhere in the Universe.

So symmetry implies asymmetry. This relation is not symmetric, since asymmetry does not imply symmetry, at least not in the same sense that symmetry implies asymmetry, in the sense that actual symmetry implies actual asymmetry, as was demonstrated above. However, asymmetry does imply symmetry in the limited sense that the lack of immunity to a possible change implies the conceptual possibility of immunity to the change. Thus actual asymmetry implies merely the conceptual possibility of, but not actual, symmetry.

1.3 Analogy and Classification Are Symmetry

A very important kind of symmetry for science, one that is not often thought of as symmetry, is *analogy*. Analogy is the immunity of the validity of a relation to changes of the elements involved in it.

To see what we actually have here, consider, for example, the relation expressed by the statement, 'An animal has a relatively long tail'. This relation involves two elements, an animal and a relatively long tail. The former element is not unique, since there are more than just a single animal in the world. Indeed, one can say, for example, 'This deer has a relatively long tail', or 'That squirrel has a relatively long tail'. But the relation is not valid for all animals. Deer do not have relatively long tails, while squirrels do. Nevertheless, there are, in fact, more than just a single animal for which the statement is valid. Squirrel A has a relatively long tail, squirrel B also has a relatively long tail, squirrel C does too, so does squirrel D, and so on for all squirrels as well as for certain other animals. The relation defines an analogy among animals: All relatively long-tailed animals are analogous in that, whatever their differences, they all possess the common property of having a relatively long tail. And as a fringe benefit we have that all relatively non-long-tailed (i.e., medium-, short-, and no-tailed) animals are analogous in that, whatever their differences, they all possess the common property that their tails are not relatively long.

We see that this analogy is symmetry by noting:

1. There is the possibility of a change. Since the relation is formulated for more than a single animal, the animal to which it is applied can be switched.
2. The validity of the relation is immune to certain such changes. The relation holds just as well for squirrel A, for squirrel B, for C, etc., who each proudly waves a relatively long tail.

For another, similar analogy consider the relation expressed by 'An astronomical body moves along an elliptical orbit with the Sun at one of its foci'. This statement, too, can be viewed as involving two elements, an astronomical body and an elliptical orbit with the Sun at one of its foci. The former element is not unique; there are more than a single astronomical body in the cosmos. For example, one can say, 'The Moon moves . . . ', or 'Venus moves . . . '. But the relation is not valid for all astronomical bodies. The Moon does not move in such a way, while Venus does. There are, however, more than a single astronomical body for which the statement is valid. They include all the planets of the solar system, for whom the statement becomes Kepler's first law of planetary motion (Johannes Kepler, German astronomer and mathematician, 1571–1630), as well as the asteroids, dwarf planets, some of the comets, etc. The statement is an expression of analogy among astronomical bodies: All those bodies that move along elliptical orbits with the Sun at one of their foci are analogous in that, whatever their differences, they all move in just that way. And we also have that all the other astronomical bodies, such as stars and moons, are analogous in that, whatever their differences, they all do not move along ellipses with the Sun at one of the foci.

This analogy, too, is symmetry:

1. There is the possibility of a change. Since the relation is applicable to more than a single astronomical body, the body to which it is applied can be switched.
2. The validity of the relation is immune to certain such changes. The statement is valid just as well for Mars, for Neptune, for Uranus, etc., each of which moves along an elliptical orbit with the Sun at one of its foci.

Now, consider a relation involving a pair of changeable elements, 'X is the locus of all points equidistant from a given point (its center), all points lying in Y'. The relation involves a pair of elements, (X, Y), where X and Y can be any geometric objects and are certainly not unique. For instance, one can say, 'A triangle is the locus ... lying in an ellipsoid', or 'A circle is the locus ... lying in a plane'. But the relation is not valid for all pairs of geometric objects. It is not true for the pair (a triangle, an ellipsoid), while it does hold for the pair (a circle, a plane). There are more than one pair (X, Y) for which it is valid. Three of them are: (a pair of points, a line), (a circle, a plane), and (a spherical surface, space). This relation between X and Y defines an analogy among pairs of geometric objects: All those pairs (X, Y) whose elements X and Y fulfill the relation as stated are analogous in that, whatever their differences, they all fulfill the relation. And in addition, all those pairs whose elements do not fulfill the relation are analogous in that, whatever their differences, they do not fulfill the relation.

Analogies involving pairs of changeable elements are often put in the form: A is to B as C is to D as .... For the present example this form is: A pair of points is to a line as a circle is to a plane as a spherical surface is to space .... (This form of expressing an analogy can easily be generalized for relations involving any number of changeable elements.) The symmetry here is:

1. Since the relation is applicable to more than one pair of geometric objects, the pair to which it is applied can be switched.
2. The validity of the relation is immune to certain such changes.

Now, consider the experimental setup of a given sphere rolling down a fixed inclined plane, with the experimental procedure of releasing the sphere from rest, letting it roll for any time interval t, and noting the distance d the sphere rolls in this time interval. Performing n such experiments, we collect n data pairs (or data points) (t_1, d_1), ..., (t_n, d_n). The pairs obey the relation $d_k = bt_k^2$, where b is a positive proportionality constant, for $k = 1, \dots, n$. This is a relation involving two changeable elements, as in the preceding example. The n data pairs, as well as an infinity of potential data pairs, are analogous in that they all obey the same relation, $d = bt^2$, and in that sense t_1 is to d_1 as t_2 is to d_2 as .... All other (t, d) pairs, which do not obey the relation $d = bt^2$, are also analogous in that they do not obey the relation. The symmetry is that we can switch among actual and potential data pairs, and however we switch among them, the relation between t and d remains the same.

Additional physics analogies can by found in [4].

With the help of the four examples above we now see how analogy, as the immunity of the validity of a relation under changes of the elements involved in it, is indeed what we thought we understood by the term 'analogy' before we found ourselves hopelessly confused by such a weird definition. The reason for such a definition of analogy, besides its being a good one, is that it directly exposes the symmetry that is analogy, since it implies:

1. the possibility of a change, the change of elements involved in the relation,
2. the immunity of the validity of the relation to certain such changes.

Note that analogy implies and is implied by *classification*. An analogy imposes a classification by decomposing the set of elements or element pairs, triples, etc., to which the relation is applicable into classes of analogous elements or pairs, triples, etc. For example, among animals the relation, 'An animal has a relatively long tail', separates all animals into a class of relatively long-tailed animals, those animals for which the relation is valid, and a class of relatively non-long-tailed animals, those for which the statement is false. In the astronomical example the relation, 'An astronomical body moves along an elliptical orbit with the Sun at one of its foci', decomposes the set of all astronomical bodies into a class of those for which the relation is valid, the most notable of which are the planets of the solar system, and a class of astronomical bodies that do not move according to the statement, which includes the planetary moons and all the stars, among others.

In the geometric example the relation, ‘X is the locus of all points equidistant from a given point (its center), all points lying in Y’, separates all pairs of geometric objects into a class of those pairs for which the relation holds, the best known of which are (a pair of points, a line), (a circle, a plane), and (a spherical surface, space), and a class of those that do not fulfill the relation, such as (a triangle, an ellipsoid) and (a hyperboloid, space). And in the laboratory example the relation $d = bt^2$ decomposes all (t, d) pairs into a class of those obeying the relation, i.e., all actual and potential data pairs for the experiment, and a class of those for which $d \neq bt^2$, those that cannot be data for the experiment.

Conversely, a classification defines the analogy of belonging to the same class. If any set is decomposed into mutually exclusive classes, then the very property of belonging to the same class defines an analogy among the elements of the set. For instance, the kids in a school can be, and for administrative purposes are, classified by grade. That makes all pupils in the same grade analogous. Or, motor vehicles can be classified by the number of axles. This classification makes all vehicles with the same number of axles analogous, which might find expression in the toll rate on toll roads.

For a detailed example, consider the classification of the chemical elements that is expressed by the periodic table, originally proposed by Mendeleev. Each column of the table comprises a group of elements possessing similar chemical properties. There is the noble gas group (helium, neon, argon, . . .), the halogen group (fluorine, chlorine, bromine, . . .), and so on. The analogy that is defined by this classification is the relation, ‘Element X belongs to group N’, where X is the changeable element for any fixed group N. Thus helium, neon, argon, . . . , are analogous in that they are all noble gases. Fluorine, chlorine, bromine, . . . , are analogous by all belonging to the halogen group. And so on for the other groups. The symmetry here is this [5]:

1. Every group N contains more than a single element, so the element X to which the relation ‘Element X belongs to group N’ is applied can be changed.
2. The validity of the relation is immune to switching among elements X that belong to group N.

This example serves also as the example of Ernest Rutherford’s ‘butterfly collecting’ that was discussed in the introduction.

And one more example. For the purposes of blood donation and transfusion, people are classified by their blood type (A, B, AB, O) and their Rh group (+, −), giving eight classes (A+, A−, B+, . . . , O−). The analogy here is among people belonging to the same blood class, with the relation, ‘Individual X possesses blood of type/group Y’, for changeable X and fixed Y (A+, . . . , or O−). The symmetry here is:

1. More than a single person have blood of any of the eight combinations of blood type and group, so X in the relation can be changed.
2. The validity of the relation is immune to switching among people of the same blood type/group.

Thus, analogy and classification, which imply each other, are both symmetry.

1.4 Summary

Symmetry is immunity to a possible change, i.e., we have symmetry when it is possible to perform some change in the situation that nevertheless leaves some aspect of the situation unaffected. Then we have symmetry under the change with respect to that aspect. If some aspect of the situation is not immune to the change, then the situation is asymmetric under the change with respect to this aspect. Since a change requires the existence of a reference frame that is affected by the change, such a reference frame is necessarily asymmetric under the change. Thus, *for every symmetry there exists an asymmetry*. That was the gist of Sects. 1.1 and 1.2.

In Sect. 1.3 we saw that analogy is symmetry and discussed how analogy implies and is implied by classification, which is also symmetry.

2

Science Is Founded on Symmetry

In this chapter we briefly review what science is about, and we see that it strongly involves reduction, which is shown to be symmetry. We consider three ways reduction is used in science – observer and observed, quasi-isolated system and environment, and initial state and evolution – and see in detail the symmetry implied by each.

Reproducibility and predictability, which are both essential components of science, are shown to be symmetries as well. Since science rests firmly on the triple foundation of reproducibility, predictability, and reduction, science is solidly based on symmetry. Indeed, science can be said to *be* symmetry, at least to the extent that it is our view of nature through symmetry spectacles.

In addition, analogy, shown earlier to be symmetry, is seen to be essential for the operation of science.

2.1 Science

For the purpose of our discussion we take this definition of *nature*:

> *Nature is the material universe with which we can, or can conceivably, interact.*

The *material universe* is everything of a purely material character. Here I mean 'material' in the broad sense of anything related to matter, including such as energy, momentum, electric charge, fields, waves, and so on. To *interact* with something is to act upon it and be acted upon by it. That implies the possibility of performing observations and measurements on it and of receiving data from it, which is what we are actually interested in. To be able *conceivably* to interact with something means that, although we might not be able to interact with it at present, interaction is not precluded by any principle known to us and is considered attainable through further technological research and development. Thus nature, as the material universe with which we can, or can conceivably, interact, is everything of purely material character (in the broad sense) that we can, or can conceivably, observe and measure.

We live in nature, observe it, and are intrigued. We try to understand nature in order both to improve our lives by better satisfying our material needs and desires and to satisfy our curiosity. And what we observe in nature is a complex of phenomena, including ourselves, where *we are related to all of nature*, as is implied by our definition of nature as the material universe with which we can, or can conceivably, interact. The possibility of interaction is what relates us to all of nature and, due to the mutuality of interaction and of the relation it brings about, relates all of nature to us. It then follows that all aspects and phenomena of nature are actually interrelated, whether they appear to be so or not. Whether they are interrelated independently of us or not, they are certainly interrelated through our mediation. Thus all of nature, including *Homo sapiens*, is interrelated and integrated.

Now we come to *science*:

> *Science is our attempt to understand rationally and objectively the reproducible and predictable aspects of nature.*

This, as we will see, is essentially the same as

> *Science is our attempt to understand rationally and objectively the lawful aspects of nature.*

And I repeat, nature is the material universe with which we can, or can conceivably, interact. By 'our' in the above definition I mean that science is a human endeavor and is shaped by our modes of perception and our mental makeup. It is the endeavor of all humanity, not of any particular individual, so it must be as objective as possible. 'Attempt' means that we try but might not always succeed. By 'understand rationally and objectively' I mean be able to explain in a logical way that is valid for everybody. That excludes explanations based on intuition, feeling, or religious considerations, among others. We explain logically and objectively by finding *order* among the reproducible and predictable aspects of nature, formulating *laws*, and devising *theo-*