

LEARNING MADE EASY



Algebra I Essentials

for
dummies[®]
A Wiley Brand



The “must-know”
formulas and equations

Exactly what you need
to know to ace Algebra I

Concise coverage
of key topics

Mary Jane Sterling
Mathematics Instructor

Algebra I Essentials

**for
dummies**[®]
A Wiley Brand



Algebra I Essentials

by Mary Jane Sterling

for
dummies[®]
A Wiley Brand

Algebra I Essentials For Dummies®

Published by: **John Wiley & Sons, Inc.**, 111 River Street, Hoboken, NJ 07030-5774, www.wiley.com

Copyright © 2019 by John Wiley & Sons, Inc., Hoboken, New Jersey

Published simultaneously in Canada

No part of this publication may be reproduced, stored in a retrieval system or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, scanning or otherwise, except as permitted under Sections 107 or 108 of the 1976 United States Copyright Act, without the prior written permission of the Publisher. Requests to the Publisher for permission should be addressed to the Permissions Department, John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030, (201) 748-6011, fax (201) 748-6008, or online at <http://www.wiley.com/go/permissions>.

Trademarks: Wiley, For Dummies, the Dummies Man logo, Dummies.com, Making Everything Easier, and related trade dress are trademarks or registered trademarks of John Wiley & Sons, Inc. and may not be used without written permission. John Wiley & Sons, Inc. is not associated with any product or vendor mentioned in this book.

LIMIT OF LIABILITY/DISCLAIMER OF WARRANTY: THE PUBLISHER AND THE AUTHOR MAKE NO REPRESENTATIONS OR WARRANTIES WITH RESPECT TO THE ACCURACY OR COMPLETENESS OF THE CONTENTS OF THIS WORK AND SPECIFICALLY DISCLAIM ALL WARRANTIES, INCLUDING WITHOUT LIMITATION WARRANTIES OF FITNESS FOR A PARTICULAR PURPOSE. NO WARRANTY MAY BE CREATED OR EXTENDED BY SALES OR PROMOTIONAL MATERIALS. THE ADVICE AND STRATEGIES CONTAINED HEREIN MAY NOT BE SUITABLE FOR EVERY SITUATION. THIS WORK IS SOLD WITH THE UNDERSTANDING THAT THE PUBLISHER IS NOT ENGAGED IN RENDERING LEGAL, ACCOUNTING, OR OTHER PROFESSIONAL SERVICES. IF PROFESSIONAL ASSISTANCE IS REQUIRED, THE SERVICES OF A COMPETENT PROFESSIONAL PERSON SHOULD BE SOUGHT. NEITHER THE PUBLISHER NOR THE AUTHOR SHALL BE LIABLE FOR DAMAGES ARISING HEREFROM. THE FACT THAT AN ORGANIZATION OR WEBSITE IS REFERRED TO IN THIS WORK AS A CITATION AND/OR A POTENTIAL SOURCE OF FURTHER INFORMATION DOES NOT MEAN THAT THE AUTHOR OR THE PUBLISHER ENDORSES THE INFORMATION THE ORGANIZATION OR WEBSITE MAY PROVIDE OR RECOMMENDATIONS IT MAY MAKE. FURTHER, READERS SHOULD BE AWARE THAT INTERNET WEBSITES LISTED IN THIS WORK MAY HAVE CHANGED OR DISAPPEARED BETWEEN WHEN THIS WORK WAS WRITTEN AND WHEN IT IS READ.

For general information on our other products and services, please contact our Customer Care Department within the U.S. at 877-762-2974, outside the U.S. at 317-572-3993, or fax 317-572-4002. For technical support, please visit <https://hub.wiley.com/community/support/dummies>.

Wiley publishes in a variety of print and electronic formats and by print-on-demand. Some material included with standard print versions of this book may not be included in e-books or in print-on-demand. If this book refers to media such as a CD or DVD that is not included in the version you purchased, you may download this material at <http://booksupport.wiley.com>. For more information about Wiley products, visit www.wiley.com.

Library of Congress Control Number: 2019932871

ISBN: 978-1-119-59096-5 (pbk); ISBN: 978-1-119-59098-9 (ePDF); ISBN: 978-1-119-59095-8 (ePub)

Manufactured in the United States of America

10 9 8 7 6 5 4 3 2 1

Contents at a Glance

Introduction	1
CHAPTER 1: Setting the Scene for Actions in Algebra	5
CHAPTER 2: Examining Powers and Roots	17
CHAPTER 3: Ordering and Distributing: The Business of Algebra	25
CHAPTER 4: Factoring in the First and Second Degrees	33
CHAPTER 5: Broadening the Factoring Horizon.....	45
CHAPTER 6: Solving Linear Equations	57
CHAPTER 7: Tackling Second-Degree Quadratic Equations.....	71
CHAPTER 8: Expanding the Equation Horizon	87
CHAPTER 9: Reconciling Inequalities	103
CHAPTER 10: Absolute-Value Equations and Inequalities.....	115
CHAPTER 11: Making Algebra Tell a Story.....	121
CHAPTER 12: Putting Geometry into Story Problems	133
CHAPTER 13: Grappling with Graphing	143
CHAPTER 14: Ten Warning Signs of Algebraic Pitfalls	157
Index	161

Table of Contents

INTRODUCTION	1
About This Book	1
Conventions Used in This Book	2
Foolish Assumptions	2
Icons Used in This Book	2
Where to Go from Here	3
Beyond the Book	4
CHAPTER 1: Setting the Scene for Actions in Algebra	5
Making Numbers Count	5
Facing reality with reals	6
Going green with naturals	6
Wholesome whole numbers	6
Integrating integers	6
Behaving with rationals	6
Reacting to irrationals	7
Picking out primes and composites	7
Giving Meaning to Words and Symbols	7
Valuing vocabulary	7
Signing up for symbols	8
Going for grouping	9
Operating with Signed Numbers	10
Adding signed numbers	10
Subtracting signed numbers	12
Multiplying and dividing signed numbers	12
Dealing with Decimals and Fractions	14
Changing fractions to decimals	14
Changing decimals to fractions	15
CHAPTER 2: Examining Powers and Roots	17
Expanding and Contracting with Exponents	17
Exhibiting Exponent Products	18
Taking Division to Exponents	19
Taking on the Power of Zero	20
Taking on the Negativity of Exponents	20
Putting Powers to Work	21
Circling around Square Roots	22

CHAPTER 3:	Ordering and Distributing: The Business of Algebra	25
	Taking Orders for Operations	25
	Dealing with Distributing	27
	Making Numbers and Variables Cooperate	28
	Relating negative exponents to fractions	29
	Creating powers with fractions	30
	Making Distributions over More than One Term	31
CHAPTER 4:	Factoring in the First and Second Degrees	33
	Making Factoring Work	33
	Facing the factoring method	34
	Factoring out numbers and variables	35
	Getting at the Basic Quadratic Expression	36
	Following Up on FOIL and unFOIL	37
	Making UnFOIL and the GCF Work Together	40
	Getting the Best of Binomials	41
	Facing up to the difference of two perfect squares	42
	Creating factors for the difference of perfect cubes	43
	Finishing with the sum of perfect cubes	43
CHAPTER 5:	Broadening the Factoring Horizon	45
	Grabbing onto Grouping	45
	Getting the groups together	45
	Grouping and unFOILing in the same package	46
	Tackling Multiple Factoring Methods	47
	Beginning with binomials	48
	Finishing with binomials	49
	Recognizing when you have a quadratic-like expression	49
	Knowing When Enough Is Enough	50
	Recruiting the Remainder Theorem	51
	Getting real with synthetic division	52
	Making good choices for synthetic division	53
	Factoring Rational Expressions	54
CHAPTER 6:	Solving Linear Equations	57
	Playing by the Rules	57
	Solving Equations with Two Terms	58
	Depending on division	59
	Making use of multiplication	60
	Reciprocating the invitation	62

	Taking on Three Terms.....	62
	Eliminating a constant term	63
	Vanquishing the extra variable term.....	63
	Breaking Up the Groups.....	64
	Nesting isn't for the birds.....	64
	Distributing first	64
	Multiplying before distributing.....	65
	Focusing on Fractions	67
	Promoting proportions	67
	Taking advantage of proportions.....	68
	Changing Formulas by Solving for Variables	69
CHAPTER 7:	Tackling Second-Degree Quadratic Equations	71
	Recognizing Quadratic Equations	71
	Finding Solutions for Quadratic Equations	72
	Applying Factorizations	73
	Zeroing in on the multiplication property of zero	74
	Solving quadratics by factoring and applying the multiplication property of zero	75
	Solving Three-Term Quadratics.....	76
	Applying Quadratic Solutions	80
	Calling on the Quadratic Formula	81
	Ignoring Reality with Imaginary Numbers.....	84
CHAPTER 8:	Expanding the Equation Horizon	87
	Queuing Up to Cubic Equations	87
	Solving perfectly cubed equations.....	88
	Going for the greatest common factor	89
	Grouping cubes.....	90
	Solving cubics with integers.....	91
	Using Synthetic Division	93
	Working Quadratic-Like Equations.....	96
	Rooting Out Radicals.....	99
CHAPTER 9:	Reconciling Inequalities	103
	Introducing Interval Notation	104
	Comparing inequality and interval notation	104
	Graphing inequalities	105
	Performing Operations on Inequalities.....	106
	Adding and subtracting numbers to inequalities	107
	Multiplying and dividing inequalities.....	107

	Finding Solutions for Linear Inequalities.....	109
	Expanding to More than Two Expressions.....	109
	Taking on Quadratic and Rational Inequalities.....	111
	Using a similar process with more than two factors.....	112
	Identifying the factors in fractional inequalities	113
CHAPTER 10:	Absolute-Value Equations and Inequalities	115
	Acting on Absolute-Value Equations.....	115
	Working Absolute-Value Inequalities.....	117
CHAPTER 11:	Making Algebra Tell a Story	121
	Making Plans to Solve Story Problems	121
	Finding Money and Interest Interesting	123
	Investigating investments and interest.....	123
	Greening up with money	125
	Formulating Distance Problems.....	126
	Making the distance formula work for you	126
	Figuring distance plus distance.....	127
	Figuring distance and fuel.....	128
	Stirring Things Up with Mixtures.....	129
CHAPTER 12:	Putting Geometry into Story Problems	133
	Triangulating a Problem with the Pythagorean Theorem.....	133
	Being Particular about Perimeter.....	134
	Triangulating triangles	134
	Squaring up to squares and rectangles	135
	Recycling circles.....	136
	Making Room for Area Problems.....	137
	Ruminating about rectangles and squares.....	137
	Taking on triangles	138
	Rounding up circles	139
	Validating with Volume.....	140
	Prodding prisms and boxing boxes.....	140
	Cycling cylinders.....	141
	Pointing to pyramids and cones	141

CHAPTER 13: Grappling with Graphing	143
Preparing to Graph a Line.....	143
Incorporating Intercepts.....	145
Sliding the Slippery Slope.....	146
Computing slope.....	146
Combining slope and intercept.....	149
Creating the slope-intercept form.....	150
Graphing with slope-intercept.....	150
Making Parallel and Perpendicular Lines Toe the Line.....	151
Criss-Crossing Lines.....	152
Turning the Curve with Curves.....	153
Going around in circles with a circular graph.....	154
Putting up with parabolas.....	154
CHAPTER 14: Ten Warning Signs of Algebraic Pitfalls	157
Including the Middle Term.....	157
Keeping Distributions Fair.....	158
Creating Two Fractions from One.....	158
Restructuring Radicals.....	158
Including the Negative (or Not).....	159
Making Exponents Fractional.....	159
Keeping Bases the Same.....	159
Powering Up a Power.....	160
Making Reasonable Reductions.....	160
Catching All the Negative Exponents.....	160
INDEX	161

Introduction

One of the most commonly asked questions in a mathematics classroom is, “What will I ever use this for?” Some teachers can give a good, convincing answer. Others hem and haw and stare at the floor. My favorite answer is, “Algebra gives you power.” Algebra gives you the power to move on to bigger and better things in mathematics. Algebra gives you the power of knowing that you know something that your neighbor doesn’t know. Algebra gives you the power to be able to help someone else with an algebra task or to explain to your child these logical mathematical processes.

Algebra is a system of symbols and rules that is universally understood, no matter what the spoken language. Algebra provides a clear, methodical process that can be followed from beginning to end. What *power!*

About This Book

What could be more *essential* than *Algebra I Essentials For Dummies*? In this book, you find the main points, the nitty-gritty (made spiffy-jiffy), and a format that lets you find what you need about an algebraic topic as you need it. I keep the same type of organization that you find in *Algebra I For Dummies*, 2nd Edition, but I keep the details neat, sweet, and don’t repeat. The fundamentals are here for your quick reference or, if you prefer, a more thorough perusal. The choice is yours.

This book isn’t like a mystery novel; you don’t have to read it from beginning to end. I divide the book into some general topics — from the beginning vocabulary and processes and operations to the important tool of factoring to equations and applications. So you can dip into the book wherever you want, to find the information you need.

Conventions Used in This Book

I don't use many conventions in this book, but you should be aware of the following:

- » When I introduce a new term, I put that term in *italics* and define it nearby (often in parentheses).
- » I express numbers or numerals either with the actual symbol, such as 8, or the written-out word: *eight*. Operations, such as + are either shown as this symbol or written as *plus*. The choice of expression all depends on the situation — and on making it perfectly clear for you.

Foolish Assumptions

I don't assume that you're as crazy about math as I am — and you may be even *more* excited about it than I am! I do assume, though, that you have a mission here — to brush up on your skills, improve your mind, or just have some fun. I also assume that you have some experience with algebra — full exposure for a year or so, maybe a class you took a long time ago, or even just some preliminary concepts.

You may be delving into the world of algebra again to refresh those long-ago lessons. Is your kid coming home with assignments that are beyond your memory? Are you finally going to take that calculus class that you've been putting off? Never fear. Help is here!

Icons Used in This Book

The little drawings in the margin of the book are there to draw your attention to specific text. Here are the icons I use in this book:



ALGEBRA
RULES

To make everything work out right, you have to follow the basic rules of algebra (or mathematics in general). You can't change or ignore them and arrive at the right answer. Whenever I give you an algebra rule, I mark it with this icon.



EXAMPLE

An explanation of an algebraic process is fine, but an example of how the process works is even better. When you see the Example icon, you'll find one or more problems using the topic at hand.



REMEMBER

Paragraphs marked with the Remember icon help clarify a symbol or process. I may discuss the topic in another section of the book, or I may just remind you of a basic algebra rule that I discuss earlier.



TIP

The Tip icon isn't life-or-death important, but it generally can help make your life easier — at least your life in algebra.



WARNING

The Warning icon alerts you to something that can be particularly tricky. Errors crop up frequently when working with the processes or topics next to this icon, so I call special attention to the situation so you won't fall into the trap.

Where to Go from Here

If you want to refresh your basic skills or boost your confidence, start with the fractions, decimals, and signed numbers in the first chapter. Other essential concepts are the exponents in Chapter 2 and order of operations in Chapter 3. If you're ready for some factoring practice and need to pinpoint which method to use with what, go to Chapters 4 and 5. Chapters 6, 7, and 8 are for you if you're ready to solve equations; you can find just about any type you're ready to attack. Chapters 9 and 10 get you back into inequalities and absolute value. And Chapters 11 and 12 are where the good stuff is: applications — things you can do with all those good solutions. I finish with some graphing in Chapter 13 and then give you a list of pitfalls to avoid in Chapter 14.

Studying algebra can give you some logical exercises. As you get older, the more you exercise your brain cells, the more alert and “with it” you remain. “Use it or lose it” means a lot in terms of the brain. What a good place to use it, right here!

The best *why* for studying algebra is just that it's beautiful. Yes, you read that right. Algebra is poetry, deep meaning, and artistic expression. Just look, and you'll find it. Also, don't forget that it gives you *power*.

Welcome to algebra! Enjoy the adventure!

Beyond the Book

In addition to what you're reading right now, this book comes with a free access-anywhere Cheat Sheet. To get this Cheat Sheet, go to www.dummies.com and search for "Algebra I Essentials For Dummies Cheat Sheet" by using the Search box.

IN THIS CHAPTER

- » Enumerating the various number systems
- » Becoming acquainted with “algebra-speak”
- » Operating on and simplifying expressions
- » Converting fractions to decimals and decimals to fractions

Chapter 1

Setting the Scene for Actions in Algebra

What exactly is algebra? What is it *really* used for? In a nutshell, *algebra* is a systematic study of numbers and their relationships, using specific rules. You use *variables* (letters representing numbers), and formulas or equations involving those variables, to solve problems. The problems may be practical applications, or they may be puzzles for the pure pleasure of solving them!

In this chapter, I acquaint you with the various number systems. You’ve seen the numbers before, but I give you some specific names used to refer to them properly. I also tell you how I describe the different processes performed in algebra — I want to use the correct language, so I give you the vocabulary. And, finally, I get very specific about fractions and decimals and show you how to move from one type to the other with ease.

Making Numbers Count

Algebra uses different types of numbers, in different circumstances. The types of numbers are important because what they look like and how they behave can set the scene for particular

situations or help to solve particular problems. Sometimes it's really convenient to declare, "I'm only going to look at whole-number answers," because whole numbers do not include fractions or negatives. You could easily end up with a fraction if you're working through a problem that involves a number of cars or people. Who wants half a car or, heaven forbid, a third of a person?

I describe the different types of numbers in the following sections.

Facing reality with reals

Real numbers are just what the name implies: real. Real numbers represent real values — no pretend or make-believe. They cover the gamut and can take on any form — fractions or whole numbers, decimal numbers that go on forever and ever without end, positives and negatives.

Going green with naturals

A *natural number* (also called a *counting number*) is a number that comes naturally. The natural numbers are the numbers starting with 1 and going up by ones: 1, 2, 3, 4, 5, and so on into infinity.

Wholesome whole numbers

Whole numbers aren't a whole lot different from natural numbers (see the preceding section). Whole numbers are just all the natural numbers plus a 0: 0, 1, 2, 3, 4, 5, and so on into infinity.

Integrating integers

Integers are positive and negative whole numbers: . . . -3, -2, -1, 0, 1, 2, 3,

Integers are popular in algebra. When you solve a long, complicated problem and come up with an integer, you can be joyous because your answer is probably right. After all, most teachers like answers without fractions.

Behaving with rationals

Rational numbers act rationally because their decimal equivalents behave. The decimal ends somewhere, or it has a repeating pattern to it. That's what constitutes "behaving."