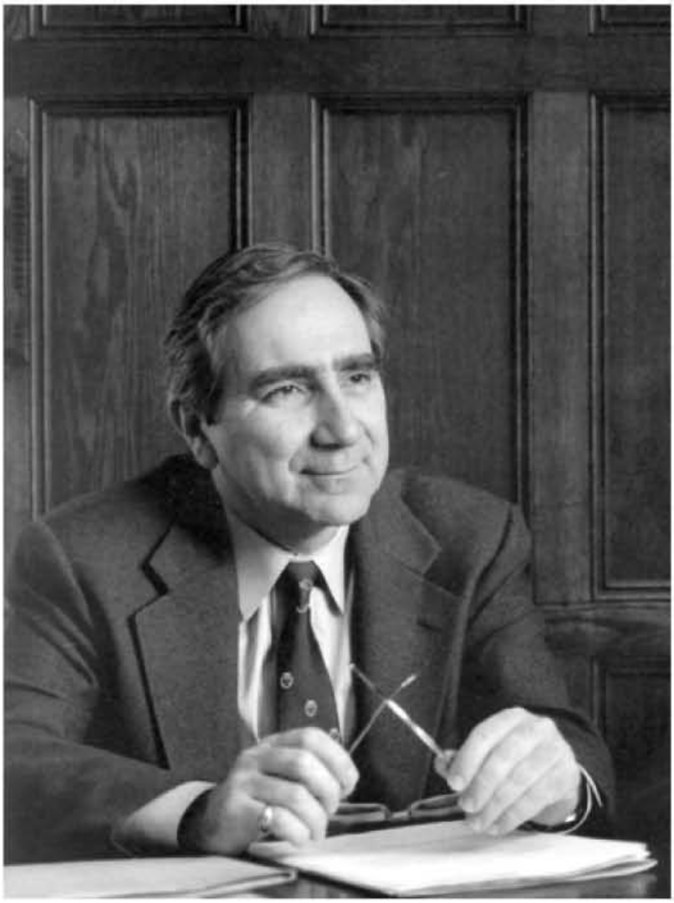
Matthew O. Jackson • Andrew McLennan (Eds.)
Foundations in Microeconomic Theory



Hugo F. Sonnenschein

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Foundations in Microeconomic Theory

A Volume in Honor of Hugo F. Sonnenschein

Springer

Professor Matthew O. Jackson
Stanford University
Department of Economics
Stanford, CA 94305-6072
USA
jacksonm@stanford.edu

Professor Andrew McLennan
School of Economics
University of Queensland
Rm. 520, Colin Clark Building
The University of Queensland
NSW 4072 Australia
mclennan@socsci.umn.edu

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Introduction

What a wonderful occasion it is to be celebrating 65 years of Hugo Sonnenschein! Given his many contributions to economic research and academia more broadly, there is much to celebrate. This volume, presented to Hugo at a conference in his honor at the University of Chicago in October 2005, highlights one of his deepest contributions. It is perhaps the hardest to detect from reading his bios and vita; but something that he is famous for among economists in general and economic theorists in particular. It is his incredible record as a mentor and advisor of students.

In putting this volume together, we have collected papers from Hugo's students with the aim of demonstrating his tremendous impact as an advisor. The papers span decades, with the earliest coming from his advisees in the first years of his career and the most recent coming in the last two years after his return to research and advising that followed his adventures as a university administrator. The contributors include not only his graduate advisees, but also some of his undergraduate advisees and still others who did not have him as an advisor, but nonetheless consider him a primary mentor in their training as economic theorists. Each paper is accompanied with a brief preface by the student that provides background on the paper and indicates Hugo's influence on its genesis. The impressive quality of the contributions is a fitting tribute to the overall impact that Hugo has had through his advice and mentoring. Moreover, the papers highlight the variety of ways in which Hugo has had an influence. Some are papers that came out of theses and show Hugo's central hand as advisor, and some were authored jointly with Hugo years after graduation. In other cases the influence was less direct, being related to Hugo's work or exhibiting some other personal touch that Hugo had, such as introducing the student to someone of similar interests, or posing questions that seemed to lead to nothing but puzzles at the time, but which later blossomed into profound insights. Together, we hope that this volume makes obvious the scope and depth of Hugo's impact and influence in his role as advisor.

An enormous amount has been written about instruction, and about the craft of teaching. University administrations worry about it, and devote time and resources to training their faculty to excel at it. In contrast, the role of the advisor still remains much more elusive and has received much less study, even though the preponderance of scholars, particularly at the highest levels of academia, would include their thesis supervisors in any list of the most important people in their lives.

final steps on the path to intellectual and scholarly independence. Whether it was helping a student who lacked confidence or experience by guiding them towards questions to cut their teeth on, or having the patience to sit down with a student and paper and go through each word and sentence asking what its purpose was and whether it properly conveyed what the author intended; Hugo excelled at all aspects of guidance. What was most remarkable, was his ability to consistently get his students to perform to the very best of their abilities. As many of us have learned, such advising and mentoring can be much more difficult than it seemed when we were working with Hugo. We believe the reminiscences gathered here combine to give a useful and fascinating picture of how one scholar succeeded brilliantly in this capacity.

The Papers

Much of the work in this volume had its origins in the 1970's and 80's, which was a period of upheaval in economic theory. Hugo's reputation in research was built on important results in the theory of the consumer, general equilibrium theory, and social choice. While some of his students worked in these areas, many of his students followed, and to a large extent led, the newer currents associated with the rapid development of game theory. Often, especially in other disciplines, Ph.D. students are expected to pursue the advisor's research program, but in many cases Hugo was supervising work that was at least a certain distance from his own specializations. Even though such distance makes the advisor's role more challenging intellectually, the tendency is for the advisor to receive a smaller share of the credit than he would in connection with theses that are clearly related to the advisor's own research program. Viewing the work done by Hugo's students collectively reveals that such an assessment would be mistaken, and the common themes visible in the papers in this volume make very evident the part that Hugo played in the rapid progress of that era. Far from being a bystander at the revolution, he was one of its masterminds. The list of papers included here is impressive:

1. Kevin Sontheimer : "An Existence Theorem for the Second Best," *Journal of Economic Theory*, 3(1), 1–22, 1971.
2. John Roberts : "An Equilibrium Model with Involuntary Unemployment at Flexible, Competitive Price and Wages," *American Economic Review*, 77, 856–874, 1987.
3. Kunio Kawamata : "Price distortion and Potential Welfare," *Econometrica*, 42(3), 435–460, 1974.
4. Salvador Barberà : "The manipulation of social choice mechanisms that do not leave "too much" to chance," *Econometrica*, 45(7), 1573–1588, 1977.
5. Javier Ruiz-Castillo : "Residential Land Use. The Continuous Case," *Economic Letters*, 8: 7–12, 1981.
6. William Novshek : "Cournot Equilibrium with Free Entry," *Review of Economic Studies*, 47, 473–486, 1980.

7. Richard Peck : "Power, Majority Voting, and Linear Income Tax Schedules," *Journal of Public Economics*, 36, 53–67, 1988.
8. Andrew McLennan : "Sequential Bargaining as a Non-Cooperative Foundation for Walrasian Equilibrium," with Hugo Sonnenschein, *Econometrica*, 59(5), 1395–1424, 1991.
9. Dilip Abreu : "Virtual Implementation in Iteratively Undominated Strategies: Complete Information," with Hitoshi Matsushima, *Econometrica*, 60(5), 993–1008, 1992.
10. Vijay Krishna : "Finitely Repeated Games," with Jean-Pierre Benoît, *Econometrica*, 53, 905–922, 1985.
11. David Pearce : "Nonpaternalistic Sympathy and the Inefficiency of Consistent Intertemporal Plans." Original Paper.
12. Matthew Jackson : "Strategy-Proof Exchange," with Salvador Barberà, *Econometrica*, 63(1), 51–88, 1995.
13. Marc Dudey : "Dynamic Monopoly with Nondurable Goods," *Journal of Economic Theory*, 70, 470–488, 1996.
14. In-Koo Cho : "Signaling Games and Stable Equilibria," with David Kreps, *Quarterly Journal of Economics*, 102, 179–221, 1987.
15. Faruk Gul : "Unobservable Investment and the Hold-Up Problem," *Econometrica*, 69(2), 343–376, 2001.
16. Arunava Sen : "The Implementation of Social Choice Functions via Social Choice Correspondences: A General Formulation and a Limit Result", *Social Choice and Welfare*, 12, 277–292, 1995.
17. Philip Reny : "On the Existence of Pure and Mixed Strategy Nash Equilibria in Discontinuous Games." *Econometrica*, 67, 1029–1056, 1999.
18. James Dow : "Nash Equilibrium under Knightian Uncertainty: Breaking Down Backward Induction," with Sergio Werlang, *Journal of Economic Theory*, 64, 305–324, 1994.
19. George Mailath and Jeroen Swinkels : " Extensive Form Reasoning in Normal Form Games," with Larry Samuelson, *Econometrica*, 61, 273–302, 1993.
20. James Bergin : "Player type distributions as state variables and information revelation in zero sum repeated games with discounting," *Mathematics of Operations Research*, 17(3), 640–656, 1992.
21. Daniel Vincent : "Repeated Signaling Games and Dynamic Trading Relationships," *International Economic Review*, 39(2), 275–293, 1998.
22. Lin Zhou : "Impossibility of Strategy-Proof Mechanisms in Economies with Pure Public Goods," *Review of Economic Studies*, 58, 107–119, 1991.
23. Zachary Cohn : "On Linked Bargaining," *forthcoming*.

We thank the publishers of these articles for granting us permission to reprint them here.

The papers are remarkably coherent in subject matter and style, without being overly narrow in scope. The papers all lie in microeconomic theory, and moreover all make contributions to the foundations of the theory. That is, they are not descriptive in nature, nor positive models of some particular observed stylized fact. They are foundational in nature, building our understanding of the fundamentals of strategic interaction and the behavior of markets. The contributions by Barbera, Abreu, Jackson, Sen, and Zhou, all lie in social choice theory and in particular in the area of strategy-proofness and implementation. There the issues addressed concern which social decision rules can be achieved, when individuals act in their own self interest in potentially manipulating their private information about their preferences and the state. The papers by Pearce, Krishna, Cho, Reny, Mailath, Swinkels, and Bergin are all in the area of game theory, and all deal with modeling and understanding equilibrium. In a sense, these deal with modeling behavior at an even more fundamental level. Each of these papers is concerned with either defining, proving existence of or analyzing the structure of the set of equilibria; and some of the papers do all three. The remaining papers all deal with market behavior in one way or another. The papers by Sontheimer, Roberts, Kawamata, Ruiz-Castillo, Novshek, and Peck, are all related to understanding whether certain imperfections in markets lead to competitive or non-competitive behavior, and what the resulting welfare implications are. The papers by McLennan, Dudey, Gul, Vincent, and Cohn, in one way or another deal with issues of repeated or linked market interactions and the competitiveness or efficiency of the resulting allocations. Together, the papers give us an impression of the magnitude and scope of Hugo's contributions as a mentor, advisor, friend and scholar.

A Brief Biographical Sketch of Hugo F. Sonnenschein

Hugo Sonnenschein was born in 1940. He received his Bachelor's Degree from the University of Rochester in 1961, where he majored in mathematics. He completed his Ph.D. in economics at Purdue University in 1964, writing his thesis under the supervision of Stanley Reiter.

Hugo's first academic appointment was at the University of Minnesota. Subsequently he held positions at the University of Massachusetts at Amherst and Northwestern University, before moving to Princeton University in 1976. Hugo's role as advisor began as soon as his career did. He was an important intellectual influence on students at the University of Minnesota before going to Northwestern University, where he began advising dissertations in earnest. The bulk of the thesis supervision that this volume celebrates took place at Princeton. Remarkably, in addition to his teaching, advising, and research during this period, he was the editor of *Econometrica* from 1977 to 1984.

In 1988 Hugo began a second distinguished career as an academic administrator, serving as Dean of the School of Arts and Sciences at the University of Pennsylvania from 1988 to 1991, and as Provost of Princeton University from 1991 to 1993. He was appointed to the presidency of the University of Chicago in 1993 and held that position until he resigned in 2000. As president he significantly improved the university's finances, and did not shy away from initiating much needed, although controversial changes, to the university's core curriculum and size of the undergraduate body. Supporters and skeptics agree that he succeeded in his main objectives, with important long run benefits for the university and city of Chicago. Since 2000 Hugo has been the Adam Smith Professor and President Emeritus of the University of Chicago.

Hugo is a member of the National Academy of Sciences and the American Philosophical Society and a fellow of the American Academy of Arts and Sciences. He was President of the Econometric Society in 1988, has received honorary doctoral degrees from six colleges and universities including Tel Aviv University and the Universitat Autonoma de Barcelona, and is a trustee and on the executive council of the Board of Trustees of the University of Rochester and an honorary member of the Board of Trustees of the University of Chicago. He is a former chairman of the Board of Governors of Argonne National Laboratory, and has served as a Member of the Civic Committee of the City of Chicago and on the Boards of Directors of the National Merit Scholarship Corporation, the Van Kampen Mutual Funds, Winston Laboratories, and various non-profit organizations.

Hugo is married to Elizabeth 'Beth' Gunn Sonnenschein, whom he met in 1957 as a freshman at the University of Rochester. She has a Ph.D. in cancer epidemiology and has held appointments on the medical faculties of the University of Illinois and New York University, and has served as President of the Board of the Chicago Child Care Society. They have three daughters and five grandchildren.

Selected Works of Hugo Sonnenschein

"The Relationship Between Transitive Preferences and the Structure of Choice Space", *Econometrica*, 1965.

"The Terms of Trade, the Gains from Trade, and Price Divergence," with Anne O. Krueger, *International Economic Review*, 1967.

"The Dual of Duopoly Is Complementary Monopoly: or, Two of Cournot's Theories Are One," The *Journal of Political Economy*, 1968.

"Price Distortion and Economic Welfare," with Edward Foster, *Econometrica*, 1970.

"Demand Theory without Transitive Preferences," in Chipman et al., editors, *Preferences, Utility and Demand*, 1971.

"Market Excess Demand Functions," *Econometrica*, 1972.

"General possibility theorems for group decisions," with Andreu Mas-Colell, *Review of Economic Studies*, 1972

"Do Walras' Identity and Continuity Characterize a Class of Community Excess Demand Functions?," *Journal of Economic Theory*, 1973.

"The Utility Hypothesis and Market Demand Theory," *Western Economics Journal*, 1973.

"An axiomatic characterization of the price mechanism," *Econometrica*, 1974.

"Equilibrium in Abstract Economies without Ordered Preferences," with Wayne Shafer, *Journal of Mathematical Economics*, 1975.

"On the existence of Cournot equilibrium without concave profit functions," with John Roberts, *Journal of Economic Theory*, 1976.

"The Demand Theory of the Weak Axiom of Revealed Preference," with Richard Kihlstrom and Andreu Mas-Colell, *Econometrica*, 1976.

"Equilibrium with Externalities, Commodity Taxation and Lump-Sum Transfers," with Wayne Shafer, 1976, *International Economic Review*, 1976.

"On the Foundations of the Theory of Monopolistic Competition," with John Roberts, *Econometrica*, 1977.

"Preference Aggregation with Randomized Social Orderings," with Salvador Barberà, *Journal of Economic Theory*, 1978.

"Cournot and Walras Equilibrium," with William Novshek, *Journal of Economic Theory*, 1978.

"Two Proofs of the Gibbard-Satterthwaite Theorem on the Possibility of a Strategy-Proof Social Choice Function," with David Schmeidler, in Gottinger and Leinfellner, editors, *Decision Theory and Social Ethics*, 1978.

"Small Efficient Scale as a Foundation for Walrasian Equilibrium," with William Novshek, *Journal of Economic Theory*, 1980.

"Strategy-proof allocation mechanisms at differentiable points," with Mark Satterthwaite, *Review of Economic Studies*, 1981.

"Existence of rational expectations equilibrium." with Robert Anderson, *Journal of Economic Theory*, 1982.

"Market Demand and Excess Demand Functions," with Wayne Shafer, in Arrow and Intriligator, *Handbook of Mathematical Economics, Vol. II*, 1982.

"Foundation of Dynamic Monopoly and the Coase Conjecture," with Faruk Gul and Robert E. Wilson – *Journal of Economic Theory*, 1986.

"On Delay in Bargaining with One-Sided Uncertainty," with Faruk Gul, *Econometrica*, 1988.

"Sequential Bargaining as a Non-Cooperative Foundation for Walrasian Equilibrium," with Andrew McLennan, *Econometrica*, 1991.

"Voting by Committees," with Salvador Barberà and Lin Zhou, *Econometrica*, 1991.

Editor, *Handbook of Mathematical Economics, Vol. IV*, with Werner Hildenbrand, 1991.

"Understanding When Agents are Fairmen or Gamesmen," with Matthew Spiegel, Janet Currie, and Arunava Sen – *Games and Economic Behavior*, 1994.

"Overcoming Incentives by Linking Decisions," with Matthew O. Jackson, *Econometrica*, 2007.

1 Kevin C. Sontheimer on Hugo F. Sonnenschein

I had my first encounter with Hugo in the fall of 1966. Hugo had joined the Department of Economics after I had finished the microtheory and other courses I needed for the PhD, and so I never had the benefit and pleasure of having him as an instructor. In fact I had not worked for or with him in any capacity, or even had a one-on-one meeting with him before the fall of 1966.

I had spent the summer of 1966 away from the University of Minnesota working on an initial effort to develop a dissertation topic and research plan. I had devoted about three months trying on my own to lay out an analytical method and framework for developing a general model of custom unions. My goal was to develop a model that would allow for the investigation of the existence of equilibrium and potentially some of the welfare properties of equilibrium in a world of multiple custom unions. When I returned to Minnesota I showed the product of my efforts to an appropriate faculty member with whom I had taken several courses. He read the write-up of my efforts and proposal. His response was that he did not think my proposed approach would work, and that he did not have any ideas as to how the problem(s) could be successfully attacked. I then went to a second faculty member, and she offered me some good advice. She suggested that, given the technical complexity of the proposed problem(s), I might try talking with Hugo Sonnenschein. It was excellent advice.

I then met with Hugo. I described what I wanted to try to do, the results of my summer's work, and my meetings with the other two faculty members. Hugo's immediate reaction was to suggest that I take on a less complex problem. Instead of trying to deal with a world of custom unions, why not deal with a trade model in which individual (small) countries can employ tariff-subsidy distortions? In particular, since the existence of a competitive international market equilibrium had not been proven in the presence of tariff-subsidy distortions, why not just try to do that? Hugo erased the overly ambitious vision of the inexperienced researcher in two succinct and gently put sentences. His wisdom was obvious. I accepted it immediately. The end result was that, with his subsequent guidance and supervision, my dissertation was completed in reasonable time and was published in *Econometrica*. The paper I have offered for inclusion in this festschrift is an outgrowth of my dissertation. It is on the existence of competitive equilibrium in a closed economy with tax-subsidy distortions and lump sum transfers. The existence problem in the latter case differs significantly from that in the neoclassical trade model. The latter paper, like my dissertation, would never have been written (by me) had it not been for Hugo's earlier counsel and guidance. I selected it not just because it reflects Hugo's influence, but also for a second reason. The second reason is that Hugo and Wayne Shafer subsequently used my 1971 *Journal of Economic Theory* paper in their paper on "Equilibrium With Externalities, Commodity Taxation, and Lump Sum Transfers" that appeared in the 1976 volume of the *International Economic Review*. So I owe Hugo heartfelt thanks for not only providing invaluable counsel, guidance, and supervision, but also for the great pleasure of having a link to a piece of his work. Hugo is an outstanding scholar, teacher, and advisor. I wish I also had had the benefit of his famously wonderful classroom instruction. Then I could be even more deeply in his debt. Thank you Hugo, and happy birthday!

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An Existence Theorem for the Second Best

KEVIN C. SONTHEIMER

Assistant Professor, Department of Economics,
State University of New York at Buffalo, New York 14226

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1. INTRODUCTION

There have been a considerable number of papers dealing with the existence problem for various perfectly competitive market models. This paper is an addition to the relatively short list of investigations into the existence problem for market models that deviate from the perfectly competitive model [4, 5, 10, 13, 17, 18]. Specifically, we establish the existence of a market clearing equilibrium in which producers face a vector of prices ρ and each i-th consumer faces a distorted price vector $p^i(\rho)$, and may receive a lump-sum tax or subsidy in addition to factor earnings and his share of profits. The sufficient conditions on consumption sets and preferences, production sets, distortion functions $p^i(\rho)$, and the tax-subsidy arrangements preserve the Arrow–Debreu [1, Theorem 1] and Debreu [2] models as special cases.[1]

Existence theorems for economic market models serve four functions for the economist. Such a theorem provides for·

(1) a set of conditions that are sufficient to explain the formation of prices in the market which the model represents,

(2) an indication of the "richness" of his model, i.e., how wide a class of empirical phenomena his model encompasses,

(3) assurance that normative theorems relating equilibrium and optimal allocations are not vacuous,

(4) assurance that the assumption of the existence of equilibrium in comparative static exercises is unnecessary. (It does not make unnecessary, however, consideration of the dependence of comparative static theorems on underlying dynamic adjustment processes and the stability of equilibria.)

[1] The more general model thereby retains all of the interpretations of the special models, with corresponding interpretations of the distortion factors (such as excise tax and subsidy schemes).

The third function is hardly the least important. For example, the "theory of the second best" is concerned with the optimality of equilibrium where the market models considered are constrained to deviate from the perfectly competitive model. A second best optimization problem in a general equilibrium setting requires a specification of (a) the economic environment, (b) decision rules or behavioral rules for some or all of the participants, and (c) a set of admissible policy functions with specified domains. The "second best" problem is then to determine those policies, if any, which yield an optimal (Pareto or Bergsonian) allocation. A typical theorem may be loosely paraphrased as follows: If an equilibrium for the given environment, decision rules or behavioral functions, and policy functions *exists*, then it is optimal relative to the specified class of equilibria. Under conditions sufficient to guarantee the existence of the market equilibrium, the theorem is nonvacuous.

In this sense, the problem considered here takes within its compass a class of markets that are the general setting for many "second best" problems. In the model considered here the decision rule of each producer is the competitive price-taking profit maximization rule, and for each consumer it is the competitive price-taking preference maximization rule. The deviation from perfect competition is due to the admissibility of non-equalities between the relative prices facing producers and those facing consumers. This model is then a member of that class of models where piecemeal policy control can be had by adjusting an instrument which is an argument of the behavior functions of some of the market participants.[2] Optimization problems within such a context are well known in the public finance literature and similar problems are readily found in the context of international trade.

The technique of proof used here is, in fact, based on the technique used to prove the existence of equilibrium for the neoclassical trade model with tariff-subsidy distortions [17, 18]. However, the model considered here differs considerably from the neoclassical trade model, especially in its characterization of consumption and production possibilities. These distinctions are exemplified by two examples of nonexistence which have no direct analogs in the neoclassical trade model (Section 3). It is the force of one of the examples that, unlike in the trade model, it is necessary to impose a restriction on the size of the distortions in the closed private ownership economy (Section 4). If the distortions are discriminatory, the size restriction depends on the distribution of wealth.

[2] The importance of this latter class of second best models has been advocated by McManus [11].

2. The Economy

Following Arrow–Debreu [1] and Debreu [2], define an economy E as a triple

$$E = [(X^i, \precsim_i), (Y^j), \omega],$$

where

(a) $X^i \subset R^n$, $X^i \neq \varnothing$, and $\precsim_i$ is a complete preordering of X^i, $i = 1, 2,..., m$,

(b) $Y^j \subset R^n$, $Y^j \neq \varnothing$, $j = 1, 2,..., r$, and

(c) $\omega \in R^n$.

The *distribution of ownership* $D(E)$ for an economy E is a couple

$$D(E) = (W, \theta),$$

where

(a) W is a $(n \times m)$ matrix whose ki-th element ω_{ki} is the amount of the k-th good owned by the i-th consumer, $\omega_{ki} \geqq 0$, and $W \cdot 1 = \omega$,

(b) θ is an $(r \times m)$ matrix whose ji-th element θ_{ji} is the fractional share of the j-th firm owned by the i-th consumer, $\theta_{ji} \geqq 0$, and $\theta \cdot 1 = 1$.[3]

The private ownership economy $\mathscr{E}$ defined by $D(E)$ is the quintuple

$$\mathscr{E} = [(X^i, \precsim_i), (Y^j), \omega, W, \theta].$$

Two distinctive features of the perfectly competitive economy are that each participant is a price-taking maximizer, and that all participants are guided by the same prices. In the private ownership economy with price distortions, it is also assumed that each participant is a price-taking maximizer. However, it is no longer assumed that all participants are guided by the same prices. Rather, it is assumed that the class of participants called consumers is guided by a set of prices that may be different from the prices that guide the producers.

Let $I = \{1, 2,..., m\}$ be the index set of the set of consumers and $N = \{1, 2,..., n\}$ the index set for goods, and let μ be any function sending I into N. The price of the $\mu(i)$-th good will be distortion free for the i-th consumer to insure that distortions are not self-cancelling. Let $\rho \in R^n$ be a vector of producers prices, and let p^i be a function from R^n into R^n such that $p^i_{\mu(i)}(\rho) = \rho_{\mu(i)}$, $i \in I$. Given any $\rho \in R^n$, $p^i(\rho)$ denotes the vector of accounting prices facing the i-th consumer, $i \in I$.

[3] The symbol 1 denotes a vector of appropriate order all of whose coordinates are identically unity.

The *private ownership economy with price distortions* $\mathscr{E}_d$ is the sextuple

$$\mathscr{E}_d = [(X^i, \precsim_i), (Y^j), \omega, W, \theta, (p^i)].$$

An *equilibrium allocation* for $\mathscr{E}_d$ is an $(m + r + 1)$-tuple $\langle(\hat{x}^i), (\hat{y}^j), \hat{p}\rangle$ of points of R^n such that

(a) $\langle(\hat{x}^i), (\hat{y}^j)\rangle \in A$,

(b) $\hat{x}^i \succsim_i x$, for $\hat{x}^i, x \in \{x' \in X^i \mid p^i(\hat{p}) \cdot x' \leqq p^i(\hat{p}) \cdot \omega^i + \sum_j \theta_{ji}\hat{p} \cdot \hat{y}^j + \hat{\tau}^i\}, i \in I$,

(c) $\hat{\tau}^i = [p^i(\hat{p}) - \hat{p}] \cdot (\hat{x}^i - \omega^i), i \in I$,

(d) $\hat{p} \cdot \hat{y}^j \geqq \hat{p} \cdot y$ for all $y \in Y^j$, $j = 1, 2, \ldots, r$,

(e) $\hat{x} \leqq \omega + \hat{y}$ and $\hat{x}_k < \omega_k + \hat{y}_k \Rightarrow \hat{p}_k = 0$,

where ω^i is the i-th column of W, $\hat{x} = \sum_i \hat{x}^i, \hat{y} = \sum_j \hat{y}^j$, and A is the *attainable set*

$$A = \left\{\langle(x^i), (y^j)\rangle \in \left(\prod_{i=1}^{m} X^i\right) \times \left(\prod_{j=1}^{r} Y^j\right) \middle| \sum_i x^i \leqq \omega + \sum_j y^j\right\}.$$

If $p^i(\hat{p}) = \hat{p}$ for all $i \in I$, the equilibrium allocation is a perfectly competitive market equilibrium allocation.

3. Two Counterexamples

A reasonable question to ask is whether the conditions of Debreu [2] are sufficient to assure the existence of an equilibrium allocation for $\mathscr{E}_d$.[4] The answer is negative. The two examples below demonstrate the insufficiency of the Debreu [2] conditions and motivate the additional restrictions given in Section 4.

Consider the problem of allocation in the world of Robinson Crusoe. Robinson's consumption set X is convex, closed, and bounded from below (Fig. 1). The endowment point ω is such that there are points interior to X that are strictly smaller than ω, i.e., Robinson could dispose of some labor and food and still survive. The dashed line through the points C and ω is the upper boundary of the translated production set $(Y + \omega)$. The production set Y is assumed to be convex, closed, $0 \in Y$, $-\Omega \subset Y$, and $Y \cap (-Y) = \{0\}$. The thin lines drawn through X are

[4] The elegant investigations of McKenzie [9] and Debreu [3] have established the sufficiency of conditions weaker than those of Debreu [2] for the perfectly competitive case.

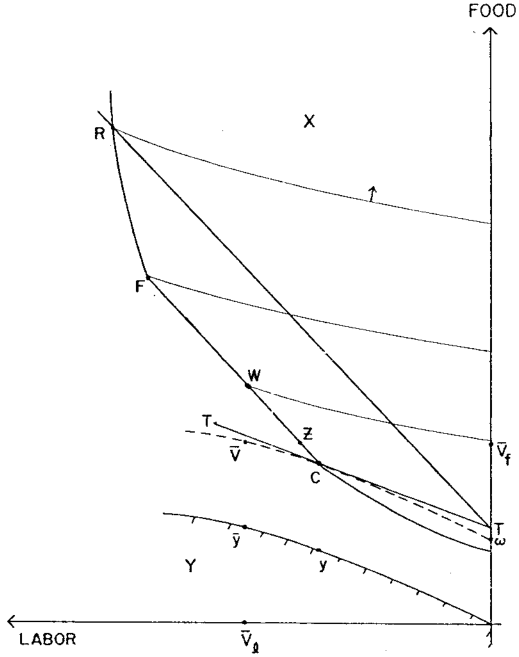

FIG. 1

Robinson's indifference curves. Note that each indifference curve intersects both the left and right hand boundaries of X.[5]

Assume that the profits of Robinson the producer are transferred to Robinson the consumer in a lump sum fashion. The wealth of Robinson the consumer is then the sum of the profits from production, his factor earnings, the value of his initial commodity holdings, and a lump sum budget adjustment τ. Robinson the consumer is assumed to choose a preference maximizing bundle x given his wealth and prices $p(\rho)$ while Robinson the producer chooses a profit maximizing bundle y at prices ρ. We seek to determine or not we are assured that there exist ρ and τ that yield choices x and y satisfying conditions (a)–(e) above for a given distortion mapping p and the given X, Y, ω, and preferences.

Let the price distortion be due to a fixed wage supplement (tax on leisure) or a subsidy on food. The value of the wage supplement is $s = (\eta - \xi)$ (for the specific case) or $\delta = (\eta - \xi/\xi)$(for the ad valorem case) where ξ

[5] Professor Koopmans [8] has noted that there may be objections to admitting preferences which yield indifference curves that terminate in the boundary of the consumption set. There does not seem to be, however, any criteria by which one can clearly rule out such preferences.

is the marginal productivity of labor at C and η is the slope of the linear segment of the boundary of X between F and C (Fig. 1). Then for a given price vector $\rho = (\rho_L, \rho_F)$, the distorted price vector is $p(\rho) = (\rho_L + \rho_F s, \rho_F)$ for the specific case, or $p(\rho) = [\rho_L(1 + \delta), \rho_F]$ for the ad valorem case [18]. If the producer's price vector $\rho = (\rho_L, \rho_F)$ is such that $\rho_L/\rho_F = \xi$ the relative price facing Robinson the consumer is η.

Assume the relative price facing Robinson the producer is $\rho_L/\rho_F = \xi$ so that his profit maximizing choice is $y \in Y$ such that $y + \omega = C$. The normalized value of the profits at y plus ω corresponds to the intercept of the line TT on the food axis. Therefore for $\tau = 0$ the budget line for Robinson the consumer passes through the intercept of TT on the food axis, and has slope equal to η. The preference maximizing choice for Robinson the consumer is at R for $\tau = 0$, $\rho_L/\rho_F = \xi$. Clearly, the consumption choice R and the production choice y corresponding to C do not correspond to an equilibrium allocation. The excess demand for food is positive, the excess demand for labor is negative, and the zero lump sum budget adjustment does not offset the wage supplements or food subsidies when $\rho_L/\rho_F = \xi$. It is also clear that positive lump sum income adjustments cannot obtain an equilibrium allocation when $\rho_L/\rho_F = \xi$.

For lump sum transfers $\tau < 0$ the optimal consumption bundle will move down along the boundary of X between R and C. Consider the lump sum transfer τ that would make the budget line

$$\{x \in R^2 \mid p(\rho) \cdot x = p(\rho) \cdot \omega + \pi(\rho) + \tau\}$$

coincident with the linear segment $\overline{CF}$, where $\pi(\rho)$ is the profit from production. Then F is the preference maximizing consumption choice for Robinson. The excess demand for food is still positive, the excess demand for labor is negative, and the negative transfer is not offset by the wage supplement or food subsidies. For a larger (in absolute value) negative transfer the budget line would have an empty intersection with X, and Robinson could not survive.[6]

Now consider the same distortion [either $s = (\eta - \xi)$ or $\delta = (\eta - \xi)/\xi$] with a new price listing. If the price of labor rises relative to that of food, the optimal production vector will be to the right of y, profits will be lower, and Robinson's budget line will have a slope greater than η. Therefore, all optimal consumption bundles will be along the segment CR, or above. The excess demand for food will always be positive; no lump sum budget adjustment τ will make the excess demand for food nonpositive.

[6] Formally, his demand is not well-defined. One of the purposes of an existence proof, in this formal sense, is to determine conditions under which the market relations are well defined.

Nor will any lump sum adjustment be offset by the wage supplements or food subsidies.

Now consider price vectors such that the relative producer price is less than ξ, and the corresponding relative consumer price is less than η. For such prices the profit maximizing productions will lie to the left of y (outside the attainable set A). Let the producer prices be such that production takes place at $\bar{y}$ (Fig. 1). The quantity of labor demanded is therefore $\bar{V}_l$, and the available supply of food is $\bar{V}_f$. But Robinson cannot supply $\bar{V}_l$ units of labor without consuming a quantity of food in excess of $\bar{V}_f$. Assume that the consumer prices (corresponding to the producer prices at $\bar{y}$) were such that each point along the boundary RFC could be sustained as an optimal consumption if an appropriate lump sum budget adjustment was made. Then as lump sum adjustments were made so that the optimal consumption point moved from F towards W, the excess demand for food would be positive but decreasing, and the excess demand for labor negative but increasing. At W the excess demand for labor would be zero but the excess demand for food would still be positive. As the optimal consumption point moves from W towards Z, the excess demand for labor is positive, and the excess demand for food is

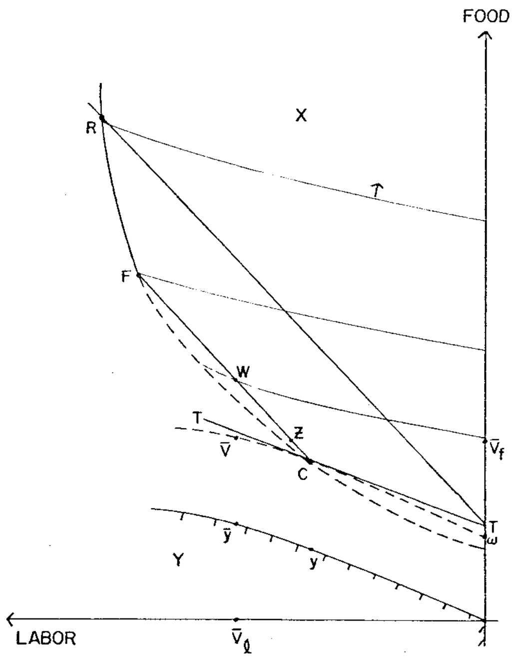

FIG. 2

positive. At Z the excess demand for food would be zero but the excess demand for labor positive.

In summary, there does not exist a vector of producer prices, and a lump sum budget adjustment τ, that will make the excess demands for food and labor simultaneously nonpositive.[7]

Now consider Fig. 2, which is a modification of Fig. 1. The linear segment $\overline{CF}$ of the boundary of X is replaced by the dashed curve between C and F, and the indifference curves are correspondingly extended out to the new boundary. But if the wage supplement or food subsidy were still chosen to be as big as above [$s = (\eta - \xi)$ or $\delta = (\eta - \xi/\xi)$], there still would not exist an equilibrium.[8]

The force of these examples is that some restrictions on the geometry of the consumption sets, or preferences, and/or the size of distortions $[p(\rho) - \rho]$ will be required in addition to the Debreu [2] conditions to assure the existence of an equilibrium allocation for $\mathscr{E}_d$ (Section 3).

4. Equilibrium Conditions

Debreu has shown that the following conditions on the consumption sets, preferences, distribution of ownership, and production sets are sufficient to guarantee the existence of a perfectly competitive equilibrium for $\mathscr{E}$:

1. Consumption Sets: X^i is closed, convex, and has a lower bound for $\leqq$[9]:
2. Preferences:
   (a) There is no satiation consumption in X^i, $i \in I$,
   (b) For every $\bar{x}^i \in X^i$, the sets $\{x^i \in X^i \mid x^i \gtrsim_i \bar{x}^i\}$ and $\{x^i \in X^i \mid x^i \lesssim_i \bar{x}^i\}$ are closed in X^i, $i \in I$,
   (c) If $\bar{x}^i$ and $\tilde{x}^i$ are two points in X^i and $1 > t > 0$, then $\bar{x}^i >_i \tilde{x}^i$ implies $t\bar{x}^i + (1 - t)\,\tilde{x}^i >_i \tilde{x}^i$, $i \in I$;
3. Distribution of Ownership: $\omega^i > x_0^i$ for some $x_0^i \in X^i$, $i \in I$;
4. Production Sets:
   (a) $0 \in Y^j, j = 1, 2,..., r$,
   (b) $Y = \sum_j Y^j$ is closed and convex,
   (c) $Y \cap (-Y) \subset \{0\}$
   (d) $Y \supset (-\Omega)$.

[7] Regardless of whether or not the lump sum transfers are offset by wage supplements or food subsidies.

[8] I am indebted to my former colleague, Professor Charles J. Goetz, Virginia Polytechnic Institute, for this observation.

[9] $\leqq$ denotes the usual partial ordering of elements of R^n.

The first example above demonstrated the insufficiency of the Debreu conditions if price distortions and lump sum income adjustments are admissible. The particular discontinuity in demand behavior in the first example is overcome if the following condition holds:

I. If $x \in bdX^i$ and there does not exist an $x' \in \text{int } X^i$ such that $x' \leqslant x$, then x is an extreme point of X^i,

where bdX^i, int X^i denote the boundary and interior of X^i, respectively, and $x' \leqslant x$ means $x' \leqq x$ and $x' \neq x$. An alternative assumption that removes the particular discontinuity in demand behavior in the first example is

II. If $x', x'' \in \{x \in bdX^i \mid \nexists x''' \in \text{int } X^i \ni x''' \leqslant x\}$, then $x' \sim_i x''$ where $\sim_i$ denotes the "indifference" relation ($x' \gtrsim_i x''$ and $x'' \gtrsim_i x'$).

Condition I allows indifference surfaces to terminate in the "lower boundary" of X^i, but rules out flat segments. Condition II says nothing about the geometry of the "lower boundary," but states that the "lower boundary" of X^i is an indifference surface.

In the second example above, it was seen that even if the Debreu conditions (1–4) and our Condition I (or II) holds, an equilibrium may not exist in the presence of price distortions and lump sum budget adjustments. Example 2 demonstrates that a restriction on the size of the distortions $[p^i(\rho) - \rho]$ is required. A suitable restriction is given below as Condition III after introducing some notation.

For any hyperplane H in R^n, denote the closed upper and lower half-spaces by H^+ and H_-, respectively. Given $\rho \in R^n$, $x \in R^n$, and p^i, $H(\rho, x)$ and $H[p^i(\rho), x]$ denote the hyperplanes through x with normals ρ and $p^i(\rho)$, respectively.

Given $\rho \in R^n$, $\omega^i \in R^n$, p^i, and $b^i \in R$, let

$$H(\rho, \omega^i; b^i) = \{x \in R^n \mid \rho \cdot x = \rho \cdot \omega^i - b^i\}$$

and $V(\rho, \omega^i; b^i) = H(\rho, \omega^i; b^i) \cap bdX^i$, where b^i is a fixed lump sum subsidy ($b^i \leqq 0$) or tax ($b^i \geqq 0$).

III. For any $\rho \in R^n$, $v \in V(\rho, \omega^i; b^i)$, if

$$\phi \neq H_-(\rho, \omega^i; b^i) \cap X^i \subset H^+[p^i(\rho), v],$$

then $X^i \subset H^+[p^i(\rho), v]$ for given b^i, $i \in I$.

Condition III states that if $v \in X^i$ is a minimum cost consumption (evaluated at $p^i(\rho)$) for all $x \in X^i$ such that $\rho \cdot x \leqq \rho \cdot \omega^i - b^i$ then v is a minimum cost consumption for all $x \in X^i$.[10] Condition III has the defect that it is not immediately obvious that III is a statement about the permissible size of distortions. It would be preferable to have a simple statement of the form $S \leqq S^*$ for specific distortions, and similarly for ad valorem distortions. While such a statement could be obtained, it suffers from the lack of a unique S^*. The difficulty arises because the relation ($\geqq$) is only a partial ordering on R^n, $n > 1$. The construction of the maximal class is omitted here because it yields no further insight into the role and effect of distortions on the market mechanism.

No conditions such as I, II, or III are necessary to establish the existence of equilibrium in the neoclassical trade model with trade tax-subsidy distortions [18]. In the neoclassical trade model the consumption space is assumed to be Ω, the nonnegative orthant, the geometry of which obviates the need for conditions such as I, II, or III. The special assump-

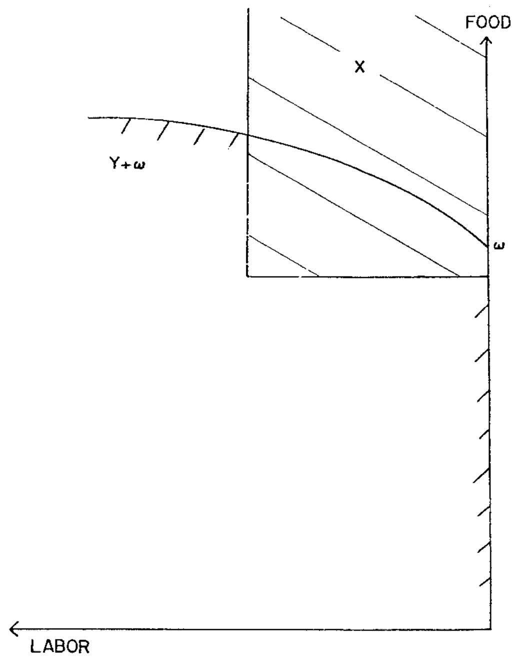

FIG. 3

[10] In fact, if the i-th consumer receives a lump sum subsidy ($b^i \leqq 0$) the subsidy can be ignored. For such a consumer it is sufficient that $H_{-}(\rho, \omega^i) \cap X^i \subset H^{+}[p^i(\rho), v]$ implies $X^i \subset H^{+}[p^i(\rho), v]$, $v \in V(\rho, \omega^i)$.

tion of $X^i = \Omega$ thereby obscures the possibility of the size of the distortion factors being too large for the "invisible hand" to overcome. Consider the situation of Fig. 3. Since distortions are finite, examples of nonexistence similar to the two above cannot be generated. An existence theorem could be established without conditions I and III (or II and III) if each X^i was restricted to be a cylinder in R^n and bounded from below.[11] However, the interest and relevance of such a theorem is questionable and, therefore, not formally considered here.

5. Supply, Demand, and the Distortion Mappings

The production and supply side of the market is identical to that of Debreu [2]. The supply correspondence of the j-th firm, $j = 1, 2,..., r$, is homogeneous of degree zero in accounting prices. Because of the free disposal assumption (4d), the supply correspondences are only well-defined for nonnegative producer's prices. We may summarize as follows:

Let $M^j \subset \Omega$ such that $\rho \in M^j$ if and only if $\rho \cdot y$ has a maximum on Y^j. Then the supply mapping T^j of the j-th firm is a correspondence that sends $\rho \in M^j$ into $\mathscr{P}(Y^j)$ where $y \in T^j(\rho)$ if and only if $\rho \cdot y = \max \rho \cdot Y^j$.[12] For any $t > 0$, $T^j(t\rho) = T^j(\rho)$, $j = 1, 2,..., r$. The profit mapping π^j of the j-th firm is a function that sends M^j into R^+ and $\pi^j(\rho) = \max \rho \cdot Y^j$. The total supply correspondence is defined as $T(\rho) = \sum_{j=1}^{r} T^j(\rho)$, and the total profit mapping is defined as $\pi(\rho) = \sum_{j=1}^{r} \pi^j(\rho)$, $\rho \in \bigcap_{j=1}^{r} M^j$.

The consumer's demand depends on the prices he confronts and his wealth, given $(X^i, \succsim_i)$. In the perfectly competitive model, the consumer's wealth is determined by the market prices ρ and the distribution of ownership $D(E)$. But the presence of lump sum budget adjustments τ means that the consumer's wealth is no longer uniquely determined by market prices and $D(E)$. Rather the consumer's wealth is now given by the value of his initial holdings, factor earnings, share of profits, and lump sum tax or subsidy levied on him. The consumer is assumed to compute his wealth at the prices $p^i(\rho)$ rather than the base prices ρ.

The domain of the price distortion mappings can be restricted to Ω, the nonnegative orthant, because of condition 4(d). The price distortion mappings are assumed to satisfy

5. $p^i : \Omega \to \Omega$ is continuous, homogeneous of degree one, and $p^i(0) = 0$, $p_k{}^i(\rho) = 0$ only if $\rho_k = 0$, $k = 1, 2,..., n$, for every $i \in I$.

[11] A cylinder in R^n is a set of the form $\{x \mid a_j \leqq x_j \leqq c_j, j = 1, 2,..., k \leqq n\}$.
[12] $\mathscr{P}(Y^j)$ denotes the power set of Y^j.

The budget set for the i-th consumer is:
Given $\rho \in \bigcap_{j=1}^{r} M^j$, and $\tau^i \in R$,

$$B\_^i(\rho, \tau^i) \cap X^i = \left\{ x \in X^i \mid p^i(\rho) \cdot x \leq p^i(\rho) \cdot \omega^i + \sum_j \theta_{ji}\pi^j(\rho) + \tau^i \right\}^{13}.$$

The budget set may be empty if the wealth of the consumer is "too small." The consumer cannot be "too wealthy." If the budget set is nonempty, the consumer is assumed to choose $x \in X^i$ if and only if x is a greatest element for $\succsim_i$ on $B\_^i(\rho, \tau^i) \cap X^i$.

The correspondence Q^i that sends a price, lump sum pair (ρ, τ^i) into $\mathscr{P}(X^i)$ such that $x \in Q^i(\rho, \tau^i)$ if and only if x is a greatest element for $\succsim_i$ on $B\_^i(\rho, \tau^i) \cap X^i$ is called the quasidemand mapping of the i-th consumer, $i \in I$.[14]

To summarize:

For $i \in I$, let $\mathscr{U}^i \subset (\bigcap_{j=1}^{r} M^j) \times R$ be such that $(\rho, \tau^i) \in \mathscr{U}^i$ if and only if $\succsim_i$ has a greatest element on $B\_^i(\rho, \tau^i) \cap X^i$. The quasidemand mapping Q^i sends $(\rho, \tau^i) \in \mathscr{U}^i$ into $\mathscr{P}(X^i)$ where

$$Q^i(\rho, \tau^i) = \{x \in B\_^i(\rho, \tau^i) \cap X^i \mid x \succsim_i x' \text{ for all } x' \in B\_(\rho, \tau^i) \cap X^i\}.$$

The aggregate quasidemand mapping $Q = \sum_i Q^i$ is a correspondence from $\bigcap_{i \in I} \mathscr{U}^i$ into $\mathscr{P}(X)$.

The quasidemand mapping Q^i, $i \in I$, is homogeneous of degree zero in ρ and τ^i.

6. Income Redistribution and An Existence Theorem

It is of interest to give consideration to the question of variations in the distribution of income since distortions between the prices facing consumers and those facing producers will typically alter the distribution of income away from that implied by ρ and $D(E)$. In the analysis below it is assumed that the device of lump sum taxes and subsidies is available to effect income changes. The lump sum mechanism may be used to preserve the income distribution implied by ρ and $D(E)$, or to guide the economy

[13] The notation here is somewhat ambiguous. In $B\_^i(\rho, \tau^i)$ above, τ is a scalar whereas in $H\_^i(\rho, x)$, x is an n-vector. However, in each case the meaning is made clear by the context.

[14] The expression quasidemand mapping is used to distinguish Q^i from the conventional demand mapping for which the wealth scalar is completely determined by ρ and $D(E)$.

to some other distribution.[15] The redistribution, if any, may take place within the consumption sector only (zero net outflow of funds), or funds may be withdrawn from the private consumption sector to finance the expenditure of an $(m + 1)$-th consumer (who might be called the government or public sector).

Denote a *fixed target* lump sum tax or subsidy for the i-th consumer as b^i (Section 4). A consumer may receive or lose the fixed amount b^i in addition to his factor earnings, share of profits, and adjustments τ^i. The fixed amounts b^i may differ from consumer to consumer ($b^i \neq b^h$, $i \neq h$) as may the distortion mappings p^i, $i \in I$. The equilibrium guaranteed by the existence theorem below thereby provides for the presence of discriminatory price distortions, income redistribution, and government consumption.

The definition of an equilibrium allocation given in Section 2 does not provide for income redistribution nor for the presence of the $(m + 1)$-th consumer. The definition of an equilibrium allocation with income redistribution and an $(m + 1)$-th consumer is given below after providing more analysis of the model.[16]

The difficulties posed by the possible lack of convexity and boundedness of some of the Y^j, and the lack of boundedness of the X^i, are overcome by what are now almost standard techniques. The attainable set A for the economy E is bounded [2, 16].[17] Further, if $\bar{Y}^j$ denotes closed convex hull of $Y^j, j = 1, 2,..., r$, it is known that $\sum_j \bar{Y}^j = Y$ [2, 16]. Then the modified attainable set

$$\bar{A} = \left\{ \langle (x^i), (y^j) \rangle \in \left(\prod_{i=1}^{m} X^i \right) \times \left(\prod_{j=1}^{r} \bar{Y}^j \right) \middle| \, x = \sum_i x^i \leqq \omega + \sum_j y^j \right\}$$

is convex and compact. The i-th projection $\mathrm{pr}_i \bar{A}$ is the set obtained by taking the projection of $\bar{A}$ on the space R^n containing the i-th consumer's consumption set X^i, and similarly for $\mathrm{pr}_j \bar{A}$ for the j-th producer. Since $\bar{A}$ is convex and compact, $\mathrm{pr}_i \bar{A}$, $i = 1, 2,..., m$ and $\mathrm{pr}_j \bar{A}$, $j = 1, 2,..., r$ are also convex and compact. Picking a closed cube $K \subset R^n$ with center at 0, vertex $\gamma^* > \omega + y$ for all

$$\langle (x^i), (y^j) \rangle \in \bar{A},$$

[15] Indeed, in the definition of equilibrium allocation given above (Section 2) the lump sum budget adjustment $\hat{\tau}^i = [p^i(\hat{p}) - \hat{p}]. (\hat{x}^i - \omega^i)$ implies that $\hat{p} \cdot \hat{x}^i = \hat{p} \cdot \omega^i + \sum_i \theta_{ij} \hat{p} \cdot \hat{y}^j$ subject to the conditions on preferences (2).

[16] If we are to interpret the $(m + 1)$-th consumer as the government, then it is not clear what is meant by the $(m + 1)$-th consumer's preferences. This, and the problem of government product are avoided in the analysis considered here.

[17] The argument here will be shortened by appropriate references to the concerned literature.

and

$$\text{pr}_i \bar{A} \subset \text{int } K, \qquad i = 1, 2, \ldots, m,$$

$$\text{pr}_j \bar{A} \subset \text{int } K, \qquad j = 1, 2, \ldots, r,$$

define

6. $\hat{X}^i = X^i \cap K$, $\hat{Y}^j = \bar{Y}^j \cap K$.

The consumption set for the $(m+1)$-th consumer is defined to be

$$7.\ \hat{X}^{m+1} = \left(\hat{Y} - \sum_{i=1}^{m} \hat{X}^i + \omega\right) \cap \Omega.$$

It is now necessary to consider the continuity properties of the supply and quasi-demand mappings when restricted to picking subsets of $\hat{Y}^j$, $j = 1, 2, \ldots, r$, and $\hat{X}^i$, $i = 1, 2, \ldots, m+1$, respectively.

The restricted supply mapping $\hat{T}^j$ of the j-th firm is a correspondence that sends $\rho \in \Omega$ into $\mathscr{P}(\hat{Y}^j)$, where $y \in \hat{T}^j(\rho)$ if and only if $\rho \cdot y = \max \rho \cdot \hat{Y}^j$ The restricted supply mapping $\hat{T}^j : \Omega \to \mathscr{P}(\hat{Y}^j)$ is upper semicontinuous, convex valued, and homogeneous of degree zero [2, 16].

Since $0 \in \hat{Y}^j$ and $\hat{Y}^j$ is compact the restricted profit function of the j-th producer will be nonnegative and defined for all $\rho \in \Omega$. However, it is clear that since $p^i(0) = 0$ for all $i = 1, 2, \ldots, m$, that 0 is not a potential equilibrium price vector. Letting $\Omega_0 = \Omega \sim \{0\}$, the restricted quasidemand mapping can be defined as mapping $\Omega_0 \times R$ into $\mathscr{P}(\hat{X}^i)$ for the i-th consumer, $i = 1, 2, \ldots, m$. It is also clear that the demand behavior will not be continuous over all of $\Omega_0 \times R$. This lack of continuity occurs for two reasons:

(a) $\hat{X}^i$ is bounded and $B^i(\rho, \tau^i) \cap \hat{X}^i = \phi$ can occur, and

(b) even if $B^i(\rho, \tau^i) \cap \hat{X}^i \neq \phi$, discontinuities can occur along the boundary of $\hat{X}^i$ for price vectors on the boundary of Ω_0.

A "smoothed" quasidemand mapping $\bar{Q}^i : \Omega_0 \times R \to \mathscr{P}(\hat{X}^i)$ will now be defined which will be suitable for the existence proof. First we introduce some notation and define a mapping $\hat{Q}^i$ which sends points of $\Omega_0 \times R$ into $\mathscr{P}(\hat{X}^i)$ such that $\hat{Q}^i$ is upper semicontinuous over $(\text{int } \Omega_0) \times R$. The continuity of $\hat{Q}^i$ will be established using Condition I in addition to the Debreu (Arrow–Debreu) conditions. The argument for Condition II instead of I is omitted. Condition I is used in 9(a) below.

Let $B^i_{tr}(\rho, \tau^i)$ be the translate of $B^i(\rho, \tau^i)$ that is supporting to $\hat{X}^i$ from below. Also, let $\gamma^i \in X^i$ such that $\gamma^i \geqq x$ for all $x \in \hat{X}^i = X^i \cap K$. Since X^i is not bounded such an γ^i exists and $\gamma^i \leqq \gamma^*$. Given $(\rho, \tau^i) \in \Omega_0 \times R$, we define $\hat{Q}^i$ such that