David Kollosche · Renato Marcone
Michel Knigge · Miriam Godoy Penteado
Ole Skovsmose *Editors*

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State-of-the-Art Research from Brazil and Germany

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Preface

How This Book Emerged

During the preparations for the ninth conference on Mathematics Education and Society, held in Volos, Greece, in April 2017, the first editor of this book became aware of the richness of research in the field of inclusive mathematics education in Brazil. While Germany, with its more urban geo-social organisation, has a long tradition of special needs schools and started only recently with the inclusion of students with learning impairment and with problems in their socio-emotional development into regular schools, Brazil has experience of organising schools with a large variety of special needs students in the same classroom. Germany started to extend the inclusion of students with special educational needs into regular schools only recently. Consequently, the German-speaking research community has laid its main foci on teaching concepts that allow for different paces in the learning process as well as on teacher education for inclusive mathematics education, whereas researchers in Brazil have documented the situation and investigated ways of inclusion of students with very specific special needs. It goes without saying that both research traditions complement each other and, together, have the potential to provide an overview of inclusive mathematics education that might be of mutual interest and of interest for an international audience.

We are especially grateful for the funding we received from the KoUP programme of the Universität Potsdam, Germany, on the basis of the project application of the two German editors of this volume. The resources allowed the editors and many authors to meet in person in Brazil – a meeting that contributed a lot to the supportive and inclusive spirit which accompanied the development of this book. The resources also allowed us to organise a process of professional language editing in order to present the interested reader with a pleasant reading experience. It is fair to say that without the support from the Universität Potsdam, the volume would not exist in the current form.

After we sent out invitations to submit proposals for chapters to the research communities in Brazil and in the German-speaking countries in June 2017, we were

able to accept all proposals and organised a process of peer-reviewing of the incoming manuscripts among the authors of the volume. Out of the initial 35 proposals, 2 have been withdrawn and 1 has been rejected in the reviewing process, leaving the 32 chapters present in this book.

The Content

Part I serves as an introduction that comprises not only this chapter but also explanations of the educational systems and status quo of inclusive education and inclusive mathematics education in Brazil and the German-speaking countries. As an understanding of specific studies might be intensified by a deeper knowledge of the national situations, the readers are cordially invited to acquaint themselves with the different socio-political backgrounds from which the studies here were conducted.

Part II problematises inclusion and connected concepts and revisits the question of inclusion as such. While the idea of inclusion has been debated controversially in general education, critical discussions of the idea of inclusion seem to be scarce in mathematics education research. This part of the book starts with the teachers' perspective in a chapter by Baraldi, Rosa, Capellini, and Miranda. Marcone continues with a chapter on what he calls "deficiencialism" – that is, a mindset of interpreting otherness in terms of deficiencies. In the next chapter, Faustino, Moura, Silva, Muzinatti, and Skovsmose explain that inclusion in the classroom does not necessarily imply inclusion in the ongoing learning processes. Skovsmose concludes the deliberations with his chapter on inclusion as a "contested concept".

Part III is dedicated to the various concepts for arranging inclusive education in the mathematics classroom. Here, the reader can find theoretical considerations and empirical findings concerning concepts such as learning environments as presented by Höveler; learning office as presented by Blumenthal, Voß, Sikora, and Hartke; dialogic learning as presented by Lutz-Westphal and Skutella; and landscapes of investigation as presented by Silva, Roncato, Souza, Giuglio, Gaviolli, and Scagion. The chapter by Krähenmann, Moser Opitz, Schnepel, and Stöckli illustrates how video studies of inclusive classrooms can be used to further conceptualise inclusive mathematics education. Eventually, the chapter by Balt, Ehlert, and Fritz discusses how numeracy skills can be assessed in an inclusive environment as a prerequisite of successful teaching.

Part IV comprises several contributions on teaching mathematics to learners with hearing impairment in Brazil, focussing especially on the role of the interpreter of Libras, the Brazilian Sign Language, in the mathematics classroom as in the chapters by Pinto and Segadas, by Borges and Nogueira, and by Moura and Penteado. In her concluding chapter, Peixoto analyses a special case in the context of problem-solving for deaf students.

Part V combines two chapters on autism and learning mathematics. In the first chapter, Souza and Silva report from a study with students with autism spectrum

disorder in a Brazilian elementary school. In the second chapter, Viana and Miarka present a case study of students with autism spectrum disorder learning mathematics.

Part VI focusses on the impact of language for the learning of mathematics. After Bednorz and Kleine present their evaluation of linguistic difficulties in mathematical tasks, Götze elaborates on the cognitive function of language for learning mathematics in primary school, illustrated with an example of multiplication and division. The last chapter in this part is a study by Werner, Berg, and Höhr, in which students with speech and language impairment, students with learning disabilities, and students without special educational needs are compared with regard to their verbalising skills.

Part VII contains those chapters which address emotional barriers towards a fruitful participation in the mathematics classroom. Carmo, Gris, and Palombarini set the scene with their overview and own original research on mathematics anxiety. Orbach, Herzog, and Fritz use their chapter to trace mathematics anxiety in the transition from primary to secondary school. Kollosche uses student data to identify reasons for the self-exclusion of learners from mathematics. Eventually, Hagelgans contributes a more productive chapter by focussing on the support of students with motivational problems in problem-solving activities.

Part VIII discusses mathematics education and inclusion in special institutional circumstances, including the hospital as studied by Cajango and Sales, the solidarity economy as investigated by Meneghetti and Gargarella, and the prison as examined by Oliveira and Nogueira.

Part IX concludes the book by looking at teacher education for inclusive mathematics education. Troll, Besser, Abels, Ahlers, Greve, Leiss, and Süßenbach set the scene by presenting their study of interventions in teacher education aimed at prospective teachers' abilities to manage inclusive education. Wagner and Ehlert use their chapter to go into more detail and present an analysis of prospective teachers' diagnostic competences. Bock, Siegemund, Nolte, and Ricken report from a bachelor studies seminar held at the Universität Hamburg, Germany, where prospective primary school teachers and prospective teachers for special education studied aspects of inclusive education collaboratively. Bitterlich and Jung contribute with a chapter in which they analyse the experiences of prospective teachers when facing heterogeneity in school. Eventually, Scherer provides an outlook by discussing the challenges that inclusion poses for subject-specific teacher education in mathematics.

Feldkirch, Austria — David Kollosche
Diadema, Brazil — Renato Marcone
Potsdam, Germany — Michel Knigge
Rio Claro, Brazil — Miriam Godoy Penteado
Aalborg, Denmark — Ole Skovsmose

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About the Editors

Michel Knigge is a full professor for inclusion and organisational development at the Universität Potsdam, Germany. Before that, he was a professor for educational psychology at the Universität Hamburg and at the Martin-Luther-Universität Halle-Wittenberg, Germany. His research focus lies on inclusive education in schools with a special focus on structures and related psychological processes of all stakeholders involved that hinder or enhance participation and discrimination. During the editorship for this book, he developed a strong interest in inclusive and critical mathematics education.

David Kollosche has been a professor for mathematics education at the Pädagogische Hochschule Vorarlberg, a teacher training university in Feldkirch, Austria, since 2017. After completing his studies of mathematics, English, and education at the Universität Potsdam, Germany, he worked as a high school teacher for mathematics for 2 years and obtained a doctorate from the Universität Potsdam for his socio-critical dissertation on "Society, mathematics and education". Apart from socio-critical perspectives on mathematics education, his research interests include students' relations to mathematics and mathematics education, processes of self-exclusion from mathematics, educational goals of mathematics education, and the potential and limitations of inquiry-based learning.

Renato Marcone has been an associate professor for mathematics and mathematics education at the Diadema Campus of the Universidade Federal de São Paulo, Brazil, since 2015. His research interest is related to inclusion, difference, and mathematics education. He was active as a co-founder of the central Accessibility and Inclusion Centre of the Universidade Federal de São Paulo in 2017 and as the coordinator of the local Accessibility and Inclusion Centre of the Diadema Campus, being responsible for the organisation of inclusive action on the campus. Before that, he finished his dissertation "Deficiencialism: The invention of deficiency by normality" where he discusses the concept of disability and its consequences for the teaching and the learning of mathematics.

Miriam Godoy Penteado is a researcher and lecturer at Universidade Estadual Paulista in Rio Claro, Brazil, where she teaches at the graduate programme in mathematics education. Her research addresses teacher education, collaboration between universities and schools, and students with special needs and rights. She is directing a group including researchers, doctoral students, master students, and teachers, investigating a range of issues related to problems of social inclusion and exclusion.

Ole Skovsmose has a special interest in critical mathematics education. He has investigated the notions of landscape of investigation, mathematics in action, and students' foreground. He was a full professor at the Department of Learning and Philosophy of Aalborg Universitet, Denmark, but is now retired and spends much of his time in Brazil. He has published several books including *Towards a Philosophy of Critical Mathematics Education*, *Dialogue and Learning in Mathematics Education* (together with Helle Alrø), *Travelling Through Education: Uncertainty, Mathematics, Responsibility*, *In Doubt: About Language, Mathematics, Knowledge and Life-Worlds*, *An Invitation to Critical Mathematics Education*, *Foregrounds: Opaque Stories About Learning*, and *Critique as Uncertainty*.

About the Authors

Simone Abels is a university professor for science education at the Institute of Sustainable and Environmental Chemistry at the Leuphana-Universität Lüneburg, Germany. Before that, she finished her dissertation on "Teachers as 'Reflective Practitioner': Reflection competence for science education that promotes democracy" at the Universität Hamburg, Germany, and worked as a research assistant at the Universität Hamburg as well as at the Universität Wien, Austria. Her research interests include inclusive science education and teacher professional development.

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Currently, his research interests are on understanding and supporting the teaching and learning of students with autism spectrum disorder for mathematical literacy.

Miriam Balt has been a research associate and doctoral student at the Department for Inclusive Education and Special Educational Needs of the Universität Potsdam, Germany, since 2014. Her research focuses on designing and evaluating learning progress assessments that are theoretically based on a developmental understanding of numeracy learning. In her work, she aims to promote an abilities-based approach to assessment that is mapped to a developmental progression to support targeted as well as evidence-based teaching.

Ivete Maria Baraldi holds a doctorate in mathematics education from the Universidade Estadual Paulista, Brazil. She works as an associate professor in the Department of Mathematics, and she is an adviser in the postgraduate programmes in science education and in mathematics education. Her research interests include the education process of teachers who teach mathematics in Brazil, both in the historiographic bias and in the issues of inclusive education.

David Bednorz is a research assistant for mathematics education at the Universität Bielefeld, Germany. He graduated in teacher training studies with the subjects mathematics and biology. In his current doctoral project, he investigates the difficulty of the language in mathematical tasks. Moreover, he is interested in the theoretical foundation of language in mathematical classrooms as well as in teacher education with a focus on linking theory and practice.

Margit Berg has been a professor for special needs education at the Pädagogische Hochschule Ludwigsburg, a teacher training university in Germany, since 2017. Before that, she worked as a lecturer and research assistant at the Pädagogische Hochschule Heidelberg, Germany. Her research interests focus on children with speech and language impairments and include mathematics education as well as programmes for improving children's abilities in producing and understanding language.

Michael Besser is a university professor for educational science at the Institute of Mathematics and Its Didactics at the Leuphana-Universität Lüneburg, Germany. Before that, he finished his dissertation on "Mathematics teachers' content knowledge and beliefs" at the Universität Kassel, Germany, and was a research assistant at the Max-Planck-Institut for human development in Berlin and assistant professor at the Pädagogische Hochschule Freiburg in Germany. His research interests include teachers' competence, teachers' professional development, evidence-based improvement of the quality of teaching, and assessing learners' competencies.

Elisa Bitterlich works in the project "Heterogeneity in teacher education from the start" which is part of the "Qualitätsoffensive Lehrerbildung", a joint initiative of the Federal Government and the federal states of Germany which aims to improve

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Yvonne Blumenthal works as a post-doc at the Universität Rostock and the Universität Greifswald. After completing her dissertation on "Predicting academic achievement in elementary school", she gained practical experience as a special school teacher in an inclusive setting. Her main areas of expertise are empirical school development research with a focus on students with learning disabilities and socio-emotional problems.

Anna-Sophia Bock has been a research assistant and doctoral student at the Universität Hamburg, Germany, since 2015. She works in the state-funded project ProfaLe which aims to improve the quality of teacher education. In this context, her research interests refer to the collaboration of teachers for special needs with teachers for mathematics and additionally their situation-specific skills in particular with regard to inclusive mathematics education.

Íria Bonfim Gaviolli is a master's degree student in mathematics education at the Universidade Estadual Paulista. Her research interests include discourses of normality and deficiencialism within the scope of mathematics education and the teaching and learning of children diagnosed with autism spectrum disorder, for which she has proposed the landscape of investigation as a possibility for teaching practice.

Fábio Alexandre Borges graduated in mathematics from the Universidade Estadual de Maringá, Brazil, in 2002; obtained a master's degree from the Programme on Science and Mathematics Education at the Universidade Estadual de Maringá, Brazil, in 2006; and completed his doctorate studies at the same programme in 2013. He has experience in mathematics education, with emphasis on inclusive mathematics education, working mainly in the field of deafness. He is editor of the research journal *Revista Paranaense de Educação Matemática* and professor at the Universidade Estadual do Paraná, Brazil. Furthermore, he is a member and a co-founder of the thematic group 13 (GT13) of the Sociedade Brasileira de Educação Matemática, focussing on "Difference, Inclusion and Mathematics Education". Currently, he is the director of Education at the Universidade Estadual do Paraná.

Eunice Cajango was born and raised in Belém, an Amazonian city, where she has been teaching mathematics in public schools since 2008. In 2016, she finished her master's thesis on "mathematics education in hospital classrooms" at the

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João dos Santos Carmo has been a university professor at the Department of Psychology of the Universidade Federal de São Carlos, Brazil, since 2008. In that university, he coordinates the Laboratory of Applied Studies to Learning and Cognition, and he has been a permanent professor in the postgraduate programme in psychology since 2010. Currently, he has also conducted research at the National Institute of Science and Technology for Behavior, Cognition and Teaching, Brazil, since 2012. He received his doctorate in education from the Universidade Federal de São Carlos, Brazil, in 2002. His research interests include mathematics anxiety, teaching, and learning of basic mathematical concepts.

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Antje Ehlert has been full professor for inclusive education and special educational needs with a focus on learning at the Universität Potsdam, Germany, since 2013. Her research focuses on designing and evaluating assessments and interventions for students with mathematical learning difficulties, as well as examining inclusive learning settings and their impact on students' competence development. She is particularly interested in the acquisition of arithmetical competencies and its influencing factors, and she develops summative as well as formative assessments and training programmes that built on a developmental understanding of mathematical learning.

Ana Carolina Faustino has a bachelor's degree in pedagogy and master's degree in education both from the Universidade Federal de São Carlos, Brazil. She worked as a teacher in the public school system for 7 years as well as in teacher training. She recently completed her doctorate in mathematics education at the Universidade Estadual Paulista, Brazil, under the supervision of Ole Skovsmose. Her study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES). Her research interests include critical pedagogy, critical mathematics education, mathematics education for social justice, language and communication in the mathematics classroom, and mathematics education in the early years of elementary school.

Annemarie Fritz(-Stratmann) is full professor for psychology at the Universität Duisburg-Essen, Germany. She graduated in psychology and habilitated in psychology of special education and rehabilitation. Since 2015, she has been distinguished visiting professor at the University of Johannesburg's Faculty for Education. At the Universität Duisburg-Essen, she runs a research ambulance for children with learning difficulties (FoBu). The main focus of her activities are children who suffer from learning difficulties. In the past 25 years, her research turned to children with mathematics learning difficulties. Here, the focus of her scientific work was the empirical validation of a development model of key numerical concepts and arithmetic skills from age 4 to 8. Based on this model, some diagnostic assessments and training programmes for pre-school and elementary school children were developed. Recently, her interest turned to assessment for older children in mathematics and to mathematics anxiety.

Bruna Camila Gargarella is an undergraduate student of mathematics education in an interunit course at the Universidade de São Paulo, Brazil. She participated in a scientific initiation project on the topic addressed in her book chapter under the guidance of Renata Cristina Geromel Meneghetti.

Renata Cristina Geromel Meneghetti has been an associate professor at the Universidade de São Paulo, Brazil, since 1996. She received her doctorate in mathematical education from the Universidade Estadual Paulista, Brazil, in 2001 and attended a post-doctoral programme at the Universidade de Campinas, Brazil, from 2009 to 2010. She has been a collaborating professor in the postgraduate programme in science education at the Universidade Estadual Paulista since 2009. Her research interests include ethnomathematics, alternative approaches and didactical recourses for the teaching and learning of mathematics, and historical and philosophical aspects of mathematical education.

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Gabriele Gris is a doctoral student at Universidade Federal de São Carlos, Brazil. Before that, she finished her master's thesis on behaviour analysis at Universidade Estadual de Londrina, Brazil. Her research interests include teaching and learning processes, mathematics education, educational games, and gamified strategies.

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Daniela Götze has been a professor for mathematics education at primary level at the Institute for Development and Research in Mathematics Education at the Technische Universität Dortmund, Germany, since October 2018. Before that, she had been a senior lecturer for mathematics education in primary school at the same institute since 2007. Her current research interests lie in the epistemic function of language for mathematical learning especially among children with mathematical difficulties and second-language learners. Furthermore, she designs (language-sensitive) teaching materials within teaching projects and within a development project of the mathematics textbook *Das Zahlenbuch* for primary school. In addition, she is a teacher trainer for in-service primary school teachers.

Heike Hagelgans has been the head of the Department of Practical School Studies at the Martin-Luther-Universität Halle-Wittenberg, Germany, since 2016. Previously, she did her doctorate on the subject of teacher professionalisation. Immediately thereafter, she habilitated with the topic "Underachievement" and became a private lecturer in Halle. She is a mathematics and physics teacher at grammar school. Her current research interests are problem-oriented mathematics lessons, giftedness, and research on the professionalism of teachers through practical school studies.

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Karina Höveler has been a university professor for mathematics education with a focus on elementary school at the Institute for Didactics of Mathematics and Computer Science at the Westfälische Wilhelms-Universität Münster, Germany, since 2017. Previously, she worked as a research assistant at the Technische Universität Dortmund in projects such as "Kinder rechnen anders" ("Children calculate differently", KIRA) and "Prozessbezogene und inhaltsbezogene Kompetenzen und Anregung von Schulentwicklung" ("Process- and content-oriented competencies and simulation of school development", PIKAS) as well as for the German Centre for Mathematics Teacher Education (DZLM). Her research interests include the handling of heterogeneity and the use of digital support in inclusive mathematics classrooms as well as learners' approaches and ideas in solving combinatorial counting problems.

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Judith Jung has been a research associate at the Professorial Chair for Primary Education/Mathematics at the Technische Universität Dresden, Germany, since 2013. Previously, she studied elementary school and secondary school education at the Goethe-Universität Frankfurt, Germany. Her research interest and working priorities are located within interpretative classroom research and address early mathematical learning processes, interaction processes in mathematics education, mathematical learning in view of diversity, and particularly the participation of pupils in mathematics education in the context of inclusive education.

Michael Kleine is a university professor for mathematics education at the Universität Bielefeld, Germany. He graduated in teacher training studies with the subjects mathematics and physics and in psychology. His doctoral dissertation addresses mental models in mathematical thinking. Moreover, he focused on test models for longitudinal studies in mathematics. His current research interests are learning processes of statistical thinking and probability on the one hand. On the other hand, he is interested in questions about language in mathematical classrooms and textbooks.

Helena Krähenmann worked at the Institute of Education of the Universität Zürich, Switzerland, and in the project Soutenir l'intégration, Integration unterstützen (SirIus), within which she has conducted a video study for her dissertation on "Teaching Quality in Inclusive Mathematics Classrooms at Primary School – A Video Study on Classroom Management, Classroom Climate, and Internal Differentiation" (in preparation). She previously worked as a primary school teacher and special education teacher for several years. Since 2016, she has been an external lecturer on inclusive education at the Fachhochschule Nordwestschweiz, a teacher training university in Switzerland.

Désirée Laubenstein has been a university professor for inclusive education and special needs education with special consideration of emotional and social disorders at the Universität Paderborn, Germany, since 2014. Her research focuses on the evaluation of successful factors for inclusive education for students with special educational needs. Further work priorities are the social integration of students with emotional and social disorders, power and violence discourses in educational relations, and inclusive school development processes.

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Christian Lindmeier is a university professor for basic knowledge about special needs education (Grundlagen sonderpädagogischer Förderung) at the Universität Koblenz-Landau, Germany. His research focuses on the theoretical and ethical basics of inclusive education for people with disabilities. Further work priorities are the transition from school to vocational training and work, the process of lifelong learning, social integration of students with emotional and social disorders, power and violence discourses in educational relations, and inclusive school development processes.

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Amanda Queiroz Moura is a PhD student in mathematics education at the Universidade Estadual Paulista, Brazil. Her research project focuses on the teaching and learning of mathematics in environments where deaf and hearing students are together. Her main theoretical framework is critical mathematics education. Lately, she has also been a visiting researcher at the University of Illinois in Chicago, USA. Apart from mathematics education and inclusion, her research interests

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Marianne Nolte has been a full professor for mathematics education at Universität Hamburg, Germany, since 1997. Before, she worked for several years as a teacher and as a therapist for students with dyscalculia. Her main research interests are mathematical giftedness, twice-exceptional students, language and mathematics learning, and learning disabilities in mathematics. She developed a foster programme for mathematically gifted students and is engaged in further training for mathematics teachers and in the education of prospective learning therapists focusing on mathematics and language.

Lars Orbach has been a research assistant at the Lehr- und Forschungsambulanz (Teaching and research ambulance, FoBu) of the Department of Psychology of the Universität Duisburg-Essen, Germany, since 2016. Before that, he worked at a hospital for childhood and adolescent psychiatry. He graduated at the Universität zu Köln and at the Universität Regensburg, Germany. Currently, he is working on his doctorate in psychology. He conducts research in clinical and educational psychology with the focus on emotional aspects of learning disabilities (with the main focus on assessment and effects of mathematics anxiety), and he is active as a lecturer in teacher training programmes. In addition to his position as research assistant, he currently works in an ambulance of childhood and adolescent psychotherapy in Cologne, Germany.

Livia dos Santos Palombarini is a master's candidate in psychology at the Universidade Federal de São Carlos, Brazil. She conducts research on mathematics abilities of students with Williams syndrome and mathematics anxiety. She graduated in physics by the same university with a thesis on physics anxiety of high school students. Accordingly, her research interests are physics education, physics anxiety, mathematics education, and mathematics anxiety.

Jurema Lindote Botelho Peixoto is an associate professor for mathematics at the Universidade Estadual de Santa Cruz, Brazil, where she has worked since 1996. In 2001, she completed her master's thesis in differential geometry at Universidade Federal da Bahia, Brazil. In 2015, she concluded her doctoral dissertation on "Analysis of schemes of deaf signaling students associated with the meanings of

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Matheus Pereira Scagion obtained a master's degree in mathematics education from the Universidade Estadual Paulista, Brazil, with a dissertation titled "Social representations of mathematics by older persons". He is a teacher at the State Education Network of the State of São Paulo, Brazil. His research interests include the use of mathematics as a link to include older people in society or as an opportunity to establish intergenerational relationships, as well as concerns associated with teaching mathematics for students with special needs.

Gisela Maria Fonseca da Pinto obtained a master's and a doctor's degree in mathematics teaching in the mathematics and physics history programme of the Universidade Federal do Rio de Janeiro, Brazil, in 2018. Her research was situated in the area of inclusive mathematics education, more specifically with respect to deafness. She now works at the Mathematics Department of the Universidade Federal Rural do Rio de Janeiro where she teaches in teacher training courses. She has an interest in the initial and continuing training of teachers who teach mathematics in inclusive settings and technologically supported classrooms.

Gabi Ricken has been a university professor for special education with a focus on diagnostic and intervention at the Universität Hamburg since 2011. She runs the Centre for Diagnostic Tests at the Faculty of Education. Her research interests are in the fields of early numerical development and cognitive aspects of learning mathematics, especially learning difficulties. In cooperation with colleagues, she published tests for diagnostic mathematical competencies and intelligence. Currently, she is concerned with studies in the field of teacher professional development.

Célia Regina Roncato has been a lecturer for pedagogy at the Universidade Estadual Paulista, Brazil, since 2016, and is conducting her doctoral studies in mathematics education there, supervised by Ole Skovsmose. Before that, she finished her master's thesis in critical mathematics education on "Landscapes of investigation for mathematical learning of a student with CHARGE Syndrome", and she is also a specialist in Brazilian sign language. Apart from critical perspectives, her research interest includes meaning-making in mathematics education with university students with disabilities.

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Petra Scherer was a full professor for mathematics education in Germany at the Universität Bielefeld from 1998 until 2011 and has worked as a full professor at the Universität Duisburg-Essen since 2017. Before that, she had finished her doctoral dissertation on learning by discovery in mathematics instruction in schools for learning-disabled students at the Universität Dortmund, Germany. Her research interests cover the analyses of learning and teaching processes in primary school as well as in special education and inclusive education, especially with respect to disabilities and difficulties in learning mathematics. Moreover, she is involved in developmental research of materials for classroom practice and for pre-service and in-service courses for teachers.

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Steffen Siegemund is an interim professor for inclusive classroom research and learning disabilities at Universität Erfurt, Germany. Before, he worked in the state-funded project ProfaLe at the Universität Hamburg, Germany, with a focus on interdisciplinary collaboration, evaluation, and research design. His research interests

include qualitative and quantitative research on attitudes and self-efficacy regarding inclusive teaching. He completed his doctoral dissertation on "Cognitive learning prerequisites and teaching basic mathematical skills to students with moderate intellectual disability" at Universität Würzburg, Germany. Steffen Siegemund has also gained several years of teaching experience in inclusive and special educational schools.

Simon Sikora has been working as a research assistant at the Institute for Special Needs Education of the Universität Rostock, Germany, since completing his studies in special education and primary education in 2011. He received his doctoral degree in 2017 with a thesis on progress monitoring in mathematics education. His research interests are in the field of diagnostics, prevention, and support in inclusive mathematics education.

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Katharina Skutella has been a lecturer for mathematics education at the Freie Universität Berlin in Germany since 2016. Before that, she taught mathematics education at the Humboldt Universität zu Berlin and worked as a content manager for Bettermarks, an adaptive learning system for mathematics. She finished her dissertation in mathematics at Technische Universität Berlin in 2003 and completed her teacher degree in Dortmund in 2009. Her research interests include inclusive mathematics classroom, modeling with mathematics, and Felix Klein's double discontinuity in mathematics teacher education.

Susanne Schnepel is a post-doc at the Institute of Education of the Universität Zürich, Switzerland. In her doctoral dissertation, she developed and tested an intervention for children with intellectual disabilities that focused on the development of mathematical skills in inclusive classrooms. Her fields of interest are the development of mathematical skills in pupils with intellectual disabilities, mathematics intervention studies, and inclusive instruction in general.

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Eloisa Jussara de Souza Giuglio has worked as a physics and mathematics teacher in public and private high schools in Brazil. Currently, she pursues a master's degree in mathematics education at the Universidade Estadual Paulista, Brazil. In her dissertation, she conducts research about the inclusion of children who have intellectual and visual disabilities through mathematical research activities. In addition, she discusses Vygotsky's contributions to mathematics education. Recently, in view of her interest in studies in critical mathematical education, she started developing socio-educative work through mathematics in a male penitentiary for the rehabilitation of adolescents.

Dirk Sponholz studied special education at the Universität Koblenz-Landau, Germany, and finished his teacher training in the German State of Rhineland-Palatinate, where he has been working as a special education teacher since 2000. First, he worked at a school with a special focus on learning impairment, where he coordinated the development of a school-internal career orientation concept. Then, he was delegated to the Rhineland-Palatinate Institute for in-service teacher education with responsibilities for inclusion in vocational schools and teaching development in lower secondary schools. At the same time, he worked as a teacher at a secondary school with a special focus on inclusion. Since 2011, he has been a research assistant at the Institute for Special Education of the Universität Koblenz-Landau, where he teaches prospective teachers and conducts research for his doctorate in the project GeSchwind-Sek I in the field of inclusive vocational orientation in school.

Meret Stöckli works as a post-doc at the Institute of Education of the Universität Zürich, Switzerland. In her doctoral dissertation, she examined the effectiveness of an in-class remedial programme in mathematics in primary schools. Her fields of interest include inclusive education, special educational needs, learning disabilities, and mathematical education in primary school. She is co-author of standardised mathematical performance tests for second and third grade with a focus on conceptual understanding, computational competences, and word problems.

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Stefan Voß has been working at the Universität Rostock, Germany, as a research assistant and lecturer since graduating as a special education teacher in 2010. He received his doctorate in 2014 with a thesis on curriculum-based measurements in mathematics instruction. His research focuses on inclusive instruction of students with and without special educational needs and on progress monitoring of learning and socio-emotional development in school. Currently, he is working as a visiting professor in the field of special education at the Freie Universität Berlin, Germany.

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Birgit Werner has been a full professor for special education at the Pädagogische Hochschule Heidelberg, a teacher training university in Germany, since 2005. Before that, she worked as a teacher at several schools for special education. Her doctoral dissertation on the development of special education in both parts of Germany after 1945 was finished at the Universität Leipzig, Germany. Her research interests now lie in the analysis of everyday, educational, and job-related literary and mathematical requirements for young people without school-leaving qualification, domain-specific concepts of diagnostics and support in mathematics teaching in the context of inclusion, and didactic concepts for transition design (from lower secondary level to vocational training and payed labour).

Part I
Introduction

Inclusive Mathematics Education: An Introduction

David Kollosche, Renato Marcone, Michel Knigge, Miriam Godoy Penteado, and Ole Skovsmose

Abstract Inclusion has become a global topic in education and educational studies. This introduction briefly recapitulates the idea of inclusion and the essential steps in this development.

In the last decade, inclusion has become a paramount topic in educational studies. Indeed, exclusive and segregating practices in education and beyond can be found in history and present in all modern societies (e.g. Arnold, Yeomans, Simpson, & Solomon, 2009). In many countries around the globe, the catalyst for the development of inclusive education has been a series of international declarations on diversity and education, which raised awareness or even laid legal foundations, such as the Salamanca Statement on Special Needs Education (United Nations Educational, Scientific and Cultural Organization, 1994). The Convention on the Rights of Persons with Disabilities (United Nations, 2006) can be considered the most influential document. In the convention, which was ratified by Brazil and Austria in 2008, by Germany in 2009 and by Switzerland in 2014, the states have agreed to

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