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Satya R. Chakravarty *Editor*

Poverty, Social Exclusion and Stochastic Dominance

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Poverty, Social Exclusion and Stochastic Dominance

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Introduction

Satya R. Chakravarty

Tony Atkinson has been and forever will remain an eminent figure in the field of economics. He had devoted his entire life to rigorous study of income inequality, poverty, and redistribution, with major contributions in every possible dimension like models, data, policies, etc. Every single work of his is marked with unparalleled clarity and depth leaving an impression on economists from around the globe. This collection of 13 articles has been influenced heavily by some of Atkinson's innovative ideas directly or indirectly.

The first essay, published in the Canadian Journal of Economics, 1983, considers general ethically flexible relative and absolute indicators of poverty. The framework we consider is axiomatic, which relies on Sen (1976). The relative index is a reasonably natural change-over of the Atkinson (1970)-Kolm (1969)-Sen (1973) relative inequality index of a censored income distribution, a distribution obtained by replacing all incomes above the poverty threshold limit by the threshold limit itself, into a relative poverty index. The poverty threshold limit or poverty line represents an income level necessary to maintain a subsistence standard of living. The Atkinson–Kolm–Sen index is the relative shortfall of the equally distributed equivalent or representative income of the society from its mean income. The representative income associated with a distribution of income is that level of income which, if enjoyed by everybody, makes the existing distribution socially welfare indifferent. It is a relative index in the sense that it remains invariant under equi-proportionate changes in all incomes. From policy perspective, it determines the fraction of total income that could be saved if society distributed incomes equally without any welfare loss. It also indicates size of proportionate welfare loss resulting from presence of inequality. Analogously, the relative poverty index, which remains unaltered under equi-proportionate variations in all incomes and the threshold limit, gives the magnitude of similar welfare loss because of existence of poverty. Pyatt (1987) studied this relative ethical poverty aggregator using the notion of affluence and basic incomes

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and examined the implications when the population representative income is the sum of representative income of affluence and the representative basic income.

The absolute poverty index proposed in the essay turns out to be a fairly natural conversion of the Blackorby–Donaldson (1980a) absolute inequality index, when applied to a censored income profile, into an absolute poverty index. While an absolute inequality index is insensitive to an equal absolute addition to all incomes, its poverty sister fulfils this insensitivity property when along with incomes we also translate the threshold limit by the same absolute amount. From policy point of view the absolute poverty index ascertains the per capita cost of poverty. If in a censored income distribution each individual were given the amount of money, as measured by the value of the poverty index, then the index would be zero.

The ethical approach we adopt in the next essay of the collection, published in Econometrica, has a similarity with the Blackorby-Donaldson (1980b) proposal. Blackorby and Donaldson (1980b) showed that if social evaluation is done with respect to the Gini welfare function of the poor, then their general ethical relative poverty index, defined on the income distributions of the poor, coincides with the Sen (1976) index of poverty. Unfortunately, as shown in the first essay, the Blackorby–Donaldson (and hence the Sen) indices violate continuity and the transfer axiom, a postulate that requires poverty to increase from a poor person to any one richer, in a specific situation. Shorrocks (1995) suggested a modification of the Sen index that avoids this shortcoming. It is rigorously demonstrated in the second essay that this modification coincides with the general functional form for relative indices proposed in Essay 1, if social evaluation is done with respect to the Gini social welfare function defined on the censored income distributions.

The third essay, published in Mathematical Social Sciences, characterizes a subgroup decomposable index of poverty, using the symmetric utilitarian social welfare function. The symmetric utilitarian social welfare function is given by the sum of identical individual utility functions. In his pioneering contribution, Atkinson (1970) made use of this form of welfare function for characterizing his inequality index. He has also demonstrated that of two income distributions of a given total if one has higher welfare value than the other for the symmetric utilitarian form welfare function where the utility function is strictly concave, then the Lorenz curve of the former lies nowhere below that of the latter and strictly inside at some places. The converse is true as well. These two statements are equivalent to the condition that the former can be obtained from the latter by a sequence of rank-preserving transfers transferring incomes from persons with high incomes to persons with lower incomes. These results have been generalized substantially from different perspectives, among others, by Dasgupta et al. (1973) and Rothschild and Stiglitz (1970, 1973).

The subgroup decomposable index characterized in the third essay can be regarded as a normalized sum of the utility shortfalls of the incomes of the poor from the income situation when all the poor persons are at the poverty line. In the words of Zheng (1997, p. 150), "following Sen's axiomatic approach, Chakravarty (1983) proposed a decomposable measure. This was among the first distribution-sensitive measures possessing this property. Unlike the approach of Sen (1976), Chakravarty derived his measure by solving a functional equation, which directly takes the three basic axioms into account". The three basic axioms employed in the characterization are transfer, monotonicity and normalization. While the monotonicity axiom demands that a reduction in the income of a poor should increase poverty, the normalization axiom requires the poverty index to take on the value one when all the incomes are zero. According to subgroup decomposability, for any partitioning of the population into two or more subgroups with respect to some homogenous characteristic, such as age, sex, region, and ethnic classes; overall poverty is given by the population share weighted average of subgroup poverty levels. This property enables us to identify those subgroups that are characterized by low population proportions but high poverty values. Evidently, from poverty reduction policy perspective such subgroups should be given priority by policy analysts. The index, thus, has a straightforward policy application. Foster and Jin (1998) developed a poverty ordering using this index. Several multidimensional extensions of the index have been suggested in the literature from different perspectives (see Chapter "Multidimensional Indicators of Inequality and Poverty" of this volume).

Apart from its extensive applications for poverty evaluation, this decomposable index has been applied to many other situations, including vulnerability to poverty and dynamic and forward-plooking poverty analysis because of its simple analytical formulation. In a situation of vulnerability, it has been noted that "Calvo and Dercon's measure is the expected Chakravarty index" (Dutta et al. 2011, p. 645) (see also Calvo and Dercon 2013). In their analysis of forward-looking and dynamic metric of poverty, Calvo and Dercon (2009, p. 57) argued that "one of the contributions of this paper is to identify the Chakravarty poverty index as the best choice if the poverty analysis moves from static poverty on to vulnerability".

In many recent contributions, measurement of richness at the top of an income distribution as a complement to poverty at the bottom of the profile has become a cornerstone of analysis (see Piketty and Saez 2006; Atkinson 2007; Atkinson et al. 2017). The poverty measure presented in the third essay has been suitably transformed into an index of richness for investigation of the extent of affluence in a society (see Peichl et al. 2010).

A recent trend in the poverty literature is to consider an endogenous poverty threshold, a poverty cut-off limit that is responsive to the actual distribution of income. In fact, this is a well-accepted phenomenon in many developed countries. In many OECD countries, there is a practice of using a constant fraction of the mean or median income of a country as the poverty threshold of the respective country. Consequently, any change in the income distribution of the country will change its poverty cut-off limit also. In contrast, an absolute poverty threshold is independent of the actual distribution of income and given exogenously. Chapter 4 of the volume, written jointly with Nachiketa Cattopadhyay, Joseph Deutsch, Zoya Nissanov and Jacques Silber, published in Research in Economic Inequality, 2016, develops an axiomatic characterization of an amalgam poverty line, a poverty line expressed as a mixture of an absolute poverty line and a reference income (e.g., the mean or median income). Some of the existing suggestion, for instance, those put forward by Atkinson and Bourguignon (2001) and EU standard for basing the poverty threshold on the actual distributions of income, become polar cases of our general formulation.

Analysis of poverty based on a single period data does not give us a true picture of the extent of deprivation of the poor people of a society. There are many reasons for not regarding poverty as a timeless concept, instead to regard it as an inter-temporal issue. There are studies that demonstrate that consecutive periods of poverty are worse than scattered poverty occurrences over time (Rodgers and Rodgers 1993; Jenkins 2000). A person afflicted by long duration of poverty may become deprived from attainment of "minimally acceptable levels" of one or more basic dimensions of human well-being (Sen 1992, p. 139). The fifth chapter of the volume, written jointly with Bossert and D'Ambrosio, published in Journal of Economic Inequality, 2012, addresses this issue in an axiomatic framework. It examines the measurement of individual and global poverty in an inter-temporal context using subgroup decomposability. Consequently, duration of poverty spells plays a significant role in the analysis, which involves counting of the number of periods in poverty and number of periods out of poverty. (See Atkinson 2003, for a counting based approach in a static framework.) Importance of persistence of poverty in a state of poverty is focused.

Multidimensionality of human welfare is a well-accepted phenomenon now. As a result, in welfare economics research there has been a shift of emphasis from single dimensional to multidimensional framework in recent years. This is because often income alone cannot be sufficient to represent the well-being of a population. For instance, insufficient supply of a public good, say, inadequacy of a malaria prevention program in a society cannot be traded off by a rich person's high income. Prices of many dimensions of well-being, such as pollution control program may not exist. Consequently, an attempt to use prices as weights for dimensions to derive a single indicator of well-being may not be worthwhile. It is, therefore, quite reasonable to use dimension-by-dimension achievements of different individuals to design an overall indicator of well-being of a population. An achievement matrix gives us achievements of the persons in a society in different dimensions, when represented in a matrix form. It is also referred to as a social distribution, or a social/distribution matrix. (See Chakravarty 2018, for a detailed discussion.)

The sixth chapter of the collection presents the well-known Bourguignon–Chakravarty family of multidimensional relative poverty indices (Journal of Economic Inequality, 2003). This 'is an early seminal conceptual paper on this topic' (Klasen 2018, p. xiv). For each dimension, a threshold limit indicating minimally acceptable level of the dimension, required for a subsistence standard of living, is specified. A multidimensional relative poverty index is an indicator of global deprivation that satisfies ratio scale invariance, that is, it is one whose value does not change when achievements in a dimension and the associated poverty cutoff limit are scaled up/down by the same positive quantity, where the scaling factor may change from dimension to dimension. For instance, we can change income unit from euro to dollar and calorie intake unit from calorie to joule. (The value of a multidimensional absolute poverty index is insensitive to equal absolute changes in dimensional achievements and threshold quantities. Equivalently, it fulfills the translation invariance property.)

In this chapter, implications of postulates for a general multidimensional poverty index are investigated, particularly, in terms of tradeoffs between dimensional achievements above and below threshold limits, and shapes of iso-poverty contours are studied. Shapes of iso-poverty contours when the elasticity of substitution between dimensional deprivations, gaps between threshold limits and corresponding dimensional achievements, depends on the poverty levels are examined as well. The paper directly employs the notion of inter-dimensional association, suggested in Atkinson and Bourguignon (Review of Economic Studies, 1982), to judge how poverty changes under a correlation increasing switch between two individuals' achievements. While most of the axioms for a single dimensional poverty index can be generalized to a multidimensional framework, axioms relating to inter-dimensional association do not have any single dimensional counterpart. Detailed investigations are made on the analytical properties of the Bourguignon–Chakravarty complements, substitutes and Leontief indices (Vélez and Robles 2008). [For additional discussions and excellent characterizations of the Bourguignon-Chakravarty family see Lasso de la Vega et al. (2009) and Lasso de la Vega and Urrutia (2011).]

The seventh chapter, prepared jointly with Conchita D'Ambrosio, published in a Springer Volume edited by Berenger and Bresson (2012), deals with a family of unit consistent multidimensional poverty indices. Unit consistency of a multidimensional poverty index demands that when the individual achievements in a dimension and the corresponding threshold limit are equi-proportionally changed, where the proportionality factor need not be the same across dimensions, then the poverty ranking of two social distributions should not alter. Evidently, all relative indices are unit consistent, but the converse is not true. No member of the family can be regarded as an absolute index. Axioms relating to the Atkinson–Bourguignon (1982) notion of inter-dimensional association play a significant role in making clear distinction between members of the family in terms of parametric restrictions.

When complete information on dimensional achievements of different individuals are available, it is possible to dichotomize their positions with respect to deprivations, that is, whether a person is deprived or non-deprived in a dimension. However, often complete information on dimensional achievements becomes unavailable. For instance, some people may be reluctant to reveal correct information on income or expenditure data. Often it becomes difficult to judge exact literacy position of a person. There may exist a wide range of cutoff limits that becomes acceptable to a social planner. Consequently, it may become difficult to judge whether a person is deprived or not in a dimension. An appropriate tool to measure poverty in such a situation is fuzzy set theory. Essential to the notion of fuzzy set theory is a membership function. A membership function is used to map the position of a person with respect to achievements of the person in different dimensions. The values of a membership function are regarded as membership grades or values and these values are limited between 0 and 1. If a person is fully deprived in a dimension, that is, his achievement is at the minimum level, then the function takes on the value 1. In contrast, if the person's achievement is not below the dimensional threshold limit, the membership grade is 0. When a person/surveyor is unclear about the status of achievement in a dimension then the membership function assigns a grade lying between 0 and 1. It decreases as the achievement level increases from the minimum level to the threshold cutoff. A membership function can be linear or non-linear, but it must possess the property that its values are bounded between 0 and 1.

Chapter 8 of the volume presents a rigorous discussion on the axioms, including the one involving the Atkinson–Bouruignon type inter-dimensional association, for a multidimensional poverty index in a fuzzy set up. It is clearly indicated how standard multidimensional poverty indices can be reformulated in a fuzzy framework. A characterization of a fuzzy membership function is also presented. Often the choice of a threshold limit for a dimension in a multidimensional poverty measurement analysis may involve ambiguity. It may vary in a certain range. This in turn raises the possibility that the poverty ranking of two alternative social matrices for alternative choices of threshold limits may not be the same. The ninth chapter of the volume, a joint contribution of Bourguignon and Chakravarty, published in a volume edited by Basu and Kanbur (Oxford University Press, 2008), looks for necessary and sufficient conditions under which poverty ordering of two social matrices will be the same when poverty threshold limits are allowed to vary within a broad range. The ranking conditions depend explicitly on the nature of inter-dimensional association. This notion of poverty ordering is known as poverty-threshold ordering, which contrasts with poverty-measure ordering that establishes conditions for ranking of social distributions when threshold limits are assumed to be fixed but variability of functional forms of poverty indicators is allowed. (For parallel single dimensional rankings, see Atkinson 1987; Foster and Shorrocks 1988.)

Often it becomes necessary to dichotomize individual achievements in a dimension using a binary variable that takes on the values 0 and 1, where a value of 1 indicates that the person under consideration is deprived in the dimension, and a value 0 means that the person has no feeling of deprivation with respect to the dimension. For instance, sometimes it becomes necessary to know whether a person has a desired level of literacy, his income exceeds a certain limit, he likes the environment in his workplace; and so on. If the person is found to possess the desired level of literacy, his income exceeds the given limit and likes the environment in the workplace; then he is non-deprived in each of the three dimensions and the value 0 can be assigned to indicate his non-deprivation in each case. On the other hand, if he is deprived in a dimension, for instance, if he does not possess the desired level of literacy, then the value 1 can be assigned to indicate this deprivation. Note that such dichotomizations of dimensional achievements apply to both ordinal and non-ordinal dimensions. Examples of dimensions of the latter category can be income, wealth, etc. and the former category include dimensions like self-reported health status, environment in workplace, etc. We refer to the total number of dimensions c in which a person is deprived as his deprivation count. If d stands for the number of dimensions of well-being, then the person's functioning score, the number of dimensions in which he is non-deprived, is given by $(d-c)$.

The next three chapters of the volume, based on the counting approach (Atkinson 2003), rely on binary representation of dimensions of well-being. Of these the tenth chapter, written jointly with Conchita D'Ambrosio, published in Review of Income and Wealth, 2006, presents a formal treatment of the notion of social exclusion, an area to which Atkinson contributed significantly. Social exclusion means relegation of one or more population subgroups and individuals to socially disadvantageous positions, where disadvantage may arise with respect to one or more dimensions of well-being. Examples of such dimensions can be health, literacy, income, and social rights (e.g. communing with friends, access to banking facilities, labor market participation), etc. In other words, social exclusion is a denial of human rights and it segregates people from social relations, thus, blocking them from full participation in normal activities of the society. It is a combined result of personal deprivations in terms of individuals' exclusion from regular participation in society functionings. It is a failure of the society to provide basic rights of human living conditions. Thus, social exclusion arises from the absence of consumption of the individuals due to inability to afford, not due to their preferences. The affected individuals become socially isolated and unimportant. Gender, caste, ethnic discrimination may translate into social exclusion. From general perspective, it may be defined as the process that excludes people from complete participation in the society in which they live. Hence socially excluded individuals are unable to enjoy the minimal standard of well-being. (For a recent treatment, see Atkinson et al. 2017.)

The chapter presents a formal treatment of social exclusion in an axiomatic framework. It also investigates the implications of a social exclusion dominance relation in terms of aggregate exclusion measures and a T-transformation, a transformation reflecting egalitarianism. The essay clearly argues that social exclusion should not be equated with multidimensional inequality or poverty.

Material deprivation is concerned with economic tightness arising from inability of an individual to reach minimal consumption in dimensions representing material living conditions. Qualitative dimensions such as whether a person is sick or not do not come under the purview of analysis of material deprivation. While multidimensional poverty, in addition to, material dimensions takes into non-material dimensions like communing with friends also, material deprivations looks into living conditions in former dimensions only. In their report prepared for the Commission of Economic Performance and Social Progress, formed at the initiative of the French Government, Stiglitz et al. (2009) suggested the inclusion of dimensions indicating material comfort for evaluation of well-being of a population from a multidimensional perspective.

In the eleventh essay of the volume, prepared jointly with Walter Bossert and Conchita D'Ambrosio, published in Review of Income and Wealth, 2013, an analytical discussion on material deprivation is presented. A person is assumed to be materially deprived if he is found to be deprived in at least one dimension. This contrasts with the intersection method of identification of material deprivation which requires deprivation in all the dimensions. The material deprivation score of a person is defined as the number of dimensions in which he happens to be deprived. The essay characterizes a social material deprivation index as a weighted sum of individual material deprivation scores and investigates its properties. The index characterized is quite general in the sense that it includes an arbitrary number of dimensions and no specific definition of materialistic dimensions is used. It is also employed to evaluate material deprivation in the European Union. Various combinations of materialistic dimensions can be used to illustrate our general index. However, in the essay, for illustrative purpose, the set of materialistic dimensions proposed by the European Union are considered.

Stochastic dominance is a standard tool for ordering of social situations, for instance, welfare ranking of income distributions (Atkinson 1970), ranking risky prospects on the basis of rate of returns (Levy 2006; Chakravarty 2013) etc. The variables considered are generally assumed to be of continuous type. But as we have argued earlier, often dichotomization becomes necessary to indicate a person's achievements in dimensions like health, literacy, etc. There can be a clear division of the total number of dimensions into functioning score, the number of dimensions in which the person's achievements are at the respective desired levels, and the deprivation count, the number of dimensions in which he is deprived. Consequently, the functioning score of a person is a non-negative integer varying between 0 and the total number of dimensions of well-being. The twelfth chapter of the volume, written jointly with Claudio Zoli, published in Journal of Economic Theory, 2012, identifies analytically the necessary and sufficient conditions under which one vector of functioning scores integer generalized Lorenz dominates that of another. The integer generalized Lorenz curve of a vector of functioning scores is the plot of cumulative functioning scores, divided by the population size, against cumulative population shares, when the scores are ranked from the lowest to the highest. If the generalized integer Lorenz curve of a vector of functioning scores lies nowhere below that of another, we say that the former integer generalized Lorenz dominates the latter. This is also same as the stipulation that the former second-order integer dominates the latter. It is rigorously shown that if the vector of functioning scores of one population dominates that of another by the above criterion, then the former can be obtained from the latter by a sequence of transformations satisfying monotonicity and non-increasingness of marginal social evaluation, where a social evaluation function is a real valued function defined on the set of vectors of functioning scores. The converse is also true. According to monotonicity, if the functioning score of a person increases by 1, then social evaluation of the profile of functioning scores cannot reduce. Non-increasingness of marginal social evaluation demands that an increase in the functioning score of a person by 1 has higher impact on social evaluation the lower is the score of the person. These two conditions are also equivalent to the requirement that the generalized Gini social evaluation function cannot assume a lower value for the former profile than for the latter one. If the total number of functioning scores of the two profiles are the same, then our result can be regarded as integer counterpart to a well-known result on welfare ranking in income distribution literature (see Atkinson 1970; Rothschild and Stiglitz 1970).

The final chapter of the volume, prepared jointly with Maria Ana Lugo, for Oxford Handbook of Well-Being and Policy (edited by Adler and Fleurbaey 2016), presents a survey of multidimensional indicators of welfare, inequality, and poverty. Given that well-being of a population is a multidimensional phenomenon, multidimensional economic inequality summarizes the level of dispersion arising from the distribution of achievements in different dimensions of well-being among individuals in a society. For both inequality and poverty, two different approaches are analyzed. The first is a direct approach which begins by specifying a set of desirable postulates for a general indicator and the indices under consideration are scrutinized on the basis of these postulates. The second approach defines a measure of well-being at the outset and an underlying index is defined at the next step.

Aggregation of dimension-by-dimension indicators of inequality or poverty does not give us a true picture of the desired objective since this dashboard-based approach ignores a noteworthy feature of analysis of multidimensional well-being, possible correlation, a measure of inter-dimensional association.

Atkinson's multidimensional inequality index, which can be regarded as a multidimensional translation of the single dimensional Atkinson index, determines the proportion of total achievements in each dimension that could be saved if the society distributed the totals for different dimensions equally among persons without any loss of welfare. It also gives the fraction of welfare lost through unequal distribution of totals of different dimensional achievements. The chapter presents a conscientious discussion on this quantifier of multidimensional inequality.

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Ethically Flexible Measures of Poverty

Satya R. Chakravarty

Abstract This paper introduces new measures of both relative and absolute poverty using the notion of representative income of a community corresponding to the censored income distribution. These new measures satisfy the monotonicity and transfer axioms proposed by Sen (1976) in all cases.
Abstract *Des mesures de pauvrete′ e′thiquement flexibles. Ce me′moire pre′sente des measures nouvelles de lapauvrete′ relative et absolute`a partir de la notion de revenu representatif d'une communaute′ correspondant `a la re′partition du revenu censure′ au sens de Takayama. Ces mesures nouvelles satisfont aux axiomes de monotonicite′ et de transfert propose′par* Sen (1976) *dans tous les cas.*

1 Introduction

This study proposes new indices for the measurement of poverty through a social welfare approach, building on the papers by Takayama (1979), Blackorby and Donaldson (1980a), and Clark et al. (1981). Blackorby and Donaldson constructed their poverty index employing a social evaluation function defined on the incomes of the poor. With their poverty index it is possible that a transfer of income from a poor person to the richest poor person may actually reduce the value of the index if the transfer enables its recipient to cross the poverty line, thus violating what is known as the transfer axiom (see the following section). Takayama defined the censored income distribution as one where all incomes above the poverty line are set equal to the poverty line, and he then used the Gini index of the censored income distribu-

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tion as an index of poverty. But this index violates the monotonicity axiom (see the next section). The same difficulty arises with the other indices based on the same approach proposed by Hamada and Takayama (1977). Clark, Hemming, and Ulph defined the social evaluation function over the censored income distribution and used a Blackorby–Donaldson type approach to construct their poverty index. However, they did so only for the symmetric mean of order $\alpha(< 1)$ social evaluation function.

In this paper, we recognize that the Clark, Hemming, and Ulph approach does not depend upon the use of such a restricted social evaluation function and we present the general approach implicit in the Clark, Hemming, and Ulph example. All the indices introduced here will satisfy both the monotonicity and transfer axioms. We propose measures of the relative as well as the absolute variety. The relative measures are related to the Atkinson–Kolm–Sen (AKS) relative inequality indices (see Atkinson 1970; Kolm 1969; Sen 1973) and the absolute measures to the Blackorby–Donaldson (BD) absolute inequality indices (see Blackorby and Donaldson 1980b), if applied to the censored income distribution. The social evaluation functions that we employ here are *strictly S-concave*.[1] That is, if two censored income distributions have the same mean and if one is more unequal than the other (by the Lorenz criterion), then the former is ranked as worse than the latter by the social evaluation function. For a given poverty line, this property is preserved in the indices we suggest for poverty measurement.

2 Properties for a Measure of Poverty

With a population of size n, the distribution of incomes is represented by a vector $y = (y_1, y_2, \ldots, y_n)$, where $y_i \geq 0\, \forall\, i = 1, 2, \ldots, n$. Let us assume that the incomes are arranged in nonincreasing order, that is, $y_1 \geq y_2 \geq \cdots \geq y_n$. Let $q(\leq n)$ be the number of the poor who have incomes not above the poverty line z (given exogenously).

A poverty index which is assumed to be a nonnegative scalar function of y, and z is said to be a relative poverty index or an absolute poverty index according as it satisfies (a) or (b).

(a) The value of the poverty index remains unchanged when all the incomes and the poverty line itself are multiplied by the same positive scalar.
(b) The value of the poverty index remains unchanged when the same amount of income is added to or subtracted from all the incomes and the poverty line itself.

A poverty index $P(y, z)$—whether a relative index or an absolute index—is required to satisfy the following properties:

[1] A numerical function f defined on R^n (the n-dimensional Euclidean space) is said to be S-concave if $f(yB) \geq f(y)$ for all $y \in R^n$ and for all bistochastic matrices B of order n. f is strictly S-concave if the inequality is strict whenever yB is not a permutation of y. It can be shown that (Berge 1963) S-concavity implies symmetry; and that symmetry and quasiconcavity imply S-concavity, but the reverse is not true.

1. $P(y, z)$ is independent of the incomes of the rich. (This is a strong justification for basing the poverty index on the censored income distribution.)
2. $P(y, z)$ is increasing in z.
3. Given other things, a reduction in income of a person below the poverty line must increase the poverty index (Monotonicity Axiom).
4. Given other things, a pure transfer of income from a person below the poverty line to anyone who is richer must increase the poverty index, unless the number of persons below the poverty line is strictly reduced by the transfer (*Weak Transfer Axiom*).
5. Given other things, a pure transfer of income from a person below the poverty line to anyone who is richer must increase the poverty index (*Strong Transfer Axiom*).
6. $P(y, z)$ is left unchanged by a permutation of the incomes (*Impartiality*).
7. $P(y, z)$ is jointly continuous in (y, z).

Property 1 states that income variations of any individual above the poverty line do not change the poverty index unless the individual falls below the poverty line. Property 2 demands that the index of poverty should increase as the poverty line representing the subsistence income level is raised. Properties 3, 4, and 5 have been discussed in the literature (see, e.g., Sen 1976, 1977, 1979; Chakravarty 1980, 1981; Kakwani 1980) and need no fresh discussion. Property 6 is unavoidable as long as income recipients are not distinguished by anything other than income. In addition to the above properties, the poverty index may be required to satisfy the population symmetry axiom, which is stated as follows: "If the same population is replicated several times, then the poverty index should be the same for the original income distribution and for the distribution obtained through replication." This axiom is parallel to Dalton's (1920) principle of population for inequality indices.

3 Relative Measures of Poverty

Before we propose the new index, we shall briefly discuss the AKS relative inequality indices, the Takayama and Hamada–Takayama indices, and the Blackorby–Donaldson index.

3.1 The AKS Relative Inequality Indices

Throughout the third section, we shall assume that W, the social evaluation function, is *continuous, increasing, strictly S-concave and homothetic*. Homotheticity means W should be of the form

$$W = \phi\big(\bar{W}(y)\big), \tag{1}$$

where ϕ is increasing in its argument and $\bar{W}$ is positively linearly homogeneous.

The AKS representative income (ξ) of the population is that income that, if distributed equally, is ethically indifferent (indifferent as measured by the social evaluation function) to y and is implicitly defined by

$$W(\xi \, 1_n) = W(y), \tag{2}$$

where 1_n is the n-co-ordinated vector of ones. Solving (2) (uniquely) for ξ, we get

$$\xi = E(y), \tag{3}$$

where E is a particular numerical representation of W. E is homogeneous of degree one.

Letting $\lambda > 0$ be the mean of the distribution y, the AKS inequality index is defined as

$$I(y) = 1 - E(y)/\lambda. \tag{4}$$

Clearly, I is homogeneous of degree zero; that is, it is a relative index. Further, I is strictly S-convex (it agrees with Lorenz quasi-ordering) if W is strictly S-concave and, in this case, it is symmetric and ranges between zero and one, attaining the value of zero at equality. Given a functional form for I, we can find E and W from (4), (3), and (1).

3.2 The Takayama and Hamada–Takayama Indices

Takayama (1979) defined the censored income distribution y^* corresponding to y as

$$\begin{aligned} y^* &= \left(y_1^*, y_2^*, \ldots, y_n^*\right) \\ &= \left(z, z, \ldots, z, y_{n-q+1}, \ldots, y_n\right). \end{aligned} \tag{5}$$

He defined the Gini coefficient I_G^* of the censored income distribution y^* as the Gini coefficient of poverty of distribution y.

$$\begin{aligned} I_G^*(y) &= \frac{1}{2\,n^2\,\lambda^*} \sum_{i=1}^{n} \sum_{j=1}^{n} \left| y_i^* - y_j^* \right| \\ &= 1 - \frac{1}{n^2\,\lambda^*} \sum_{i=1}^{n} \sum_{j=1}^{n} (2i-1) y_i^*, \end{aligned} \tag{6}$$

where $\lambda^* = \frac{1}{n} \sum\limits_{i=1}^{n} y_i^*$.

Other measures of relative inequality have been applied to the censored income profile to derive corresponding measures of poverty in Hamada and Takayama (1977).

To show that all such indices violate the monotonicity axiom[2] we assume, for simplicity, that all the incomes are below the poverty line.[3] Now suppose all the incomes are multiplied by some positive scalar c such that all of them remain below the poverty line. Then a poverty measure should increase or decrease according as c is less than or greater than unity. But the Takayama and Hamada–Takayama measures remain invariant under such circumstances. Hence we have the following theorem.

Theorem 1 If a relative inequality index defined on the censored income distribution is used as a relative poverty index (the Takayama and Hamada–Takayama approach), the poverty index will violate the monotonicity axiom.

3.3 The Blackorby–Donaldson Index

In the case of the Blackorby–Donaldson poverty index we shall additionally assume that the social evaluation function W is completely strictly recursive (any group of poorer people is strictly separable from the richer ones).[4] This particular requirement allows the "representative income of the poor" to be defined for each possible q and to be independent of the incomes of the rich. The representative income of the poor ξ_p is given by

$$W\left(y_1, y_2, \ldots, y_{n-q}, \xi_p, \xi_p, \ldots, \xi_p\right) = W(y). \tag{7}$$

Since W is completely strictly recursive, ξ_p is independent of $\left(y_1, y_2, \ldots, y_{n-q}\right)$, and we may write

$$\xi_p = E^q\left(y_{n-q+1}, \ldots, y_n\right), \tag{8}$$

where E^q is homogeneous of degree one.

Blackorby and Donaldson (1980a) considered

$$B(y, z) = q/n\left[1 - \xi_p/z\right], \tag{9}$$

as a general relative poverty index. B (y, z) is Sen's (1976) index if the evaluation is done with the Gini social evaluation function of the poor (see Blackorby and

[2]Kakwani (1981) demonstrates that Takayama's index violates the monotonicity axiom only when the poverty line strictly exceeds the median of the income distribution.

[3]In such a case, if all the incomes assume a common value, then the Takayama and Hamada Takayama indices take the value zero, irrespective of the common income value. This result is clearly undesirable.

[4]For a detailed discussion of the notion of recursivity, see Blackorby et al. (1978, ch. 6).

Donaldson 1980a, b). The Gini function[5] is not completely strictly recursive, but complete strict recursivity is only a minimal requirement for $B\,(y, z)$.

The index $B\,(y, z)$ suffers from a number of defects; apart from the requirement that the social evaluation function is completely strictly recursive. First, $B\,(y, z)$ is not continuous. To prove this, let $y_{n-q+1} = z$ and $y_{n-q+2} < z$. The income profile is $y^0 = \left(y_1, \ldots, y_{n-q}, z, y_{n-q+2}, \ldots y_n\right)$. We now raise y_{n-q+1} by $\tau > 0$ lowering the number of poor to $(q - 1)$. The new income profile is $y^{\tau} = \left(y_1, \ldots, y_{n-q}, z + \tau\ y_{n-q+2}, \ldots y_n\right)$. Let ξ_p^0 and ς_p^{τ} denote the representative incomes of the poor corresponding to the income profiles y^0 and y^{τ}, respectively. Since W is strictly recursive, we have

$$W(y^{\tau}) = W\left(y_1, \ldots, y_{n-q}, z + \tau,\ \xi_p^{\tau}, \ldots, \xi_p^{\tau}\right) \tag{10}$$

and

$$\begin{aligned} W\left(y^0\right) &= W\left(y_1, \ldots, y_{n-q}, z, \xi_p^{\tau}, \ldots, \xi_p^{\tau}\right) \\ &= W\left(y_1, \ldots, y_{n-q}, \xi_p^0, \ldots, \xi_p^0\right). \end{aligned} \tag{11}$$

By strict S-concavity, $\xi_p^{\tau} < z\left(\text{since } y_{n-q+2} < z\right)$ and

$$\xi_p^0 < \left((q - 1)\,\xi_p^{\tau} + z\right)/q. \tag{12}$$

Now

$$\begin{aligned} B\left(y^0, z\right) &= (q/n)\left[\left((z - \xi_p^0\right)/z)\right] \\ &> (q/n)\left[1 - \left((q - 1)\xi_p^{\tau} + z\right)/qz\right] \\ &= (q/n)\left[\left(qz - (q - 1)\xi_p^{\tau} - z\right)/qz\right] \\ &= ((q - 1)/n)\left[\left(z - \xi_p^{\tau}\right)/z\right] \\ &= B(y^{\tau}, z). \end{aligned} \tag{13}$$

Equation (13) must hold for all $\tau > 0$. But $B\left(y^0, z\right)$ is independent of τ, and, because of independence of the incomes of the rich (guaranteed by strict recursivity), so is $B(y^{\tau}, z)$. Therefore, continuity of B requires $B\left(y^0, z\right) > B\left(y^0, z\right)$, a contradiction.[6]

We now show that $B\,(y, z)$ violates the strong transfer axiom. Consider the income profile $\bar{y} = \left(y_1, \ldots, y_{n-q}, z - \upsilon/q, z - 2\upsilon/q, \ldots, z - ((q - 1)\upsilon)/q, z - \upsilon\right)$ where $\upsilon > 0$ is small. Transfer υ amount of income from the poorest person to the richest poor person. The new income profile is $y^{\upsilon} = \left(y_1, \ldots, y_{n-q}, z + ((q - 1)\upsilon)/q, z - 2\upsilon/q, \ldots, z - ((q - 1)\upsilon)/q, z - 2\upsilon\right)$.

[5] For a discussion, see Blackorby and Donaldson (1978).

[6] The author thanks one of the referees for pointing to the discontinuity aspect of $B\,(y, z)$

Denote the representative incomes of the poor corresponding to the distributions $\bar{y}$ and y^{υ} by $\bar{\xi}_p$, and ξ_p^{υ}, respectively. Since the social evaluation function is continuous and $\upsilon > 0$ is arbitrary, we can make $\left(z - \bar{\xi}_p\right)$ very close to $\left(z - \xi_p^{\upsilon}\right)$ so that the inequality

$$(q/n)\left[\left(z - \bar{\xi}_p\right)/z\right] < ((q-1)/n)\left[\left(z - \xi_p^{\upsilon}\right)/z\right] \tag{14}$$

does not hold. This shows that $B\ (y, z)$ violates the strong transfer axiom.

We summarize the above results in the following theorem.

Theorem 2 The Blackorby–Donaldson relative poverty index violates (i) continuity and (ii) the strong transfer axiom.

3.4 The New Index

Let ξ^* denote the representative income corresponding to the censored income distribution $y^* = \left(y_1^*, \ldots, y_n^*\right)$. So

$$\begin{aligned} \xi^* &= E\left(y_1^*, y_2^*, \ldots, y_n^*\right) \\ &= E\left(z, z, \ldots, z, y_{n-q+1}, \ldots, y_n\right). \end{aligned} \tag{15}$$

Our new relative poverty index Q is the proportionate gap between the poverty line z and the representative income ξ^* corresponding to y^*; that is,

$$Q = (z, y) = \left(z - \xi^*\right)/z. \tag{16}$$

The index Q lies in the interval [0, 1], the lower and upper limits being attained, respectively, when $y_i \geq z\,\forall = 1, 2, \ldots, n$ and when $y_i = 0\,\forall\, i = 1, 2, \ldots, n$. Since E is homogeneous of degree one, we can rewrite $Q\ (y, z)$ as

$$Q(y, z) = 1 - E\left(1, 1, \ldots, 1, y_{n-q+1}/z, \ldots, y_n/z\right). \tag{17}$$

Since E is increasing, Q is increasing in z. It is obvious that Q satisfies continuity.

The claim that Q satisfies the monotonicity axiom follows from increasingness of E. Since $E(y^*)$ is strictly S-concave, it ranks any Lorenz superior censored income distribution with the same mean as y^* as better than y^*. This is equivalent to the condition that y^* is obtained from the Lorenz superior distribution by a finite sequence of transformations transferring income from the worse off persons to the better off persons (see Dasgupta et al. 1973, Theorem 1, 181–3). This shows that Q satisfies the weak and strong versions of the transfer axiom. Impartiality of Q follows from symmetry of W, which is a consequence of S-concavity. We therefore have proved the following.