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Taiki Haga

Renormalization Group Analysis of Nonequilibrium Phase Transitions in Driven Disordered Systems

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Renormalization Group Analysis of Nonequilibrium Phase Transitions in Driven Disordered Systems

Doctoral Thesis accepted by
the Kyoto University, Kyoto, Japan

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Author
Dr. Taiki Haga
Department of Physics
Kyoto University
Kyoto, Japan

Supervisor
Prof. Shin-ichi Sasa
Department of Physics
Kyoto University
Kyoto, Japan

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Supervisor's Foreword

Phase transitions as a result of symmetry breaking occurs are classified into several classes (universality class) depending on the type of symmetry and the spatial dimension. This classification has been done for many systems in equilibrium, and various textbooks are published. Now, when a symmetry spontaneously breaks in systems out of equilibrium, there are not so many examples studied concretely. Under such circumstances, Springer Thesis by Taiki Haga provides us a new phase transition in a disordered system driven by an external force.

The most striking result is that the quasi-long-range order appears in a three-dimensional system. This phenomenon is not clearly observed for equilibrium systems, and may be specific to systems out of equilibrium. The idea is simple. Some disorder destroys the phase order of the XY model in three dimensions, while an external driving restores the order. As a result, the system behavior is expected to be equivalent to that of pure XY model in two dimensions. This may be a sort of dimensional reduction for disordered nonequilibrium systems, which is his proposal. Of course, sufficient evidence is needed to assert qualitatively new properties of quasi-long-range order in three dimensions. He succeeded in raising its credibility by conducting numerical experiments and by performing the intensive renormalization group analysis.

This theoretical analysis is quite tough. Indeed, for stochastic dynamics out of equilibrium, the renormalization group for the disordered system are formulated in a non-perturbative way. In order to complete it, it is necessary to deeply understand the contemporary development of the renormalization group, because the calculation requires advanced skills everywhere. Here, in this Springer Thesis, the renormalization group analysis for the disordered system and non-perturbative formulation are reviewed. These explanations and their concrete calculations are quite instructive. They also compile complicated calculations intelligently and give useful topics for future research.

Readers of this Springer Thesis can learn the most advanced knowledge on the phase transition out of equilibrium. I am sure that graduate students in theoretical physics as well as researchers can enjoy reading this Springer Thesis.

Kyoto, Japan
December 2018

Prof. Shin-ichi Sasa

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1. T. Haga, *Nonequilibrium quasi-long-range order of a driven random-field O(N) model*, Physical Review E **92**, 062113 (2015).
2. T. Haga, *Dimensional reduction and its breakdown in the driven random-field O(N) model*, Physical Review B **96**, 184202 (2017).
3. T. Haga, *Nonequilibrium Kosterlitz-Thouless transition in a three-dimensional driven disordered system*, Physical Review E **98**, 032122 (2018).

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Contents

Chapter 1
Introduction

1.1 Physics of Phase Transitions

Our naive intuition says that systems composed of simple constituents are also simple. In other words, the behavior of complex systems is trivially predictable from the property of their elements. This reasonable belief had been widely accepted by ancient people before the era of Descartes and Newton. Therefore, they relied on supernatural principles to explain the origin of "irreducible complexity" in our world, such as life. However, we today recognize that this intuition is incorrect and the whole system can have properties which its parts do not have. No matter how complicated living things seem to be, they are ultimately composed of atoms which obey a simple dynamical rule. This is one of the most remarkable and profound facts in the world. Then, we are naturally led to a question *how rich macroscopic physics emerges from a simple microscopic principle*, or more naively, *why our universe is so complicated despite of the simpleness of the fundamental law*. To answer this challenging question is the goal for those who are working in condensed matter physics, statistical physics, and biological physics. Unfortunately, our current understanding concerning this problem is very limited.

As an elementary step toward clarifying the mechanism underlying the emergence of macroscopic physics, we restrict our attention to the simplest phenomena which are easy for mathematical modeling and analysis. It is *phase transition* [1, 2]. When one continuously changes external parameters, such as temperature and pressure, the macroscopic state of many-body systems can discontinuously change at some critical point. Examples include liquids to solids transitions in interacting particle systems, and para-to ferromagnetic transitions in magnetic systems. Although these phenomena are ubiquitous in our daily life, it is highly nontrivial task to understand their mechanism from a microscopic dynamical rule. Note that the property of the interactions among microscopic constituents, such as molecules or spins, do not change at the transition point. Therefore, phase transition is one of the simplest emergent phenomena that cannot be explained only from the nature of the individual parts composing the systems.

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Phase transition is a consequence of the competition between *interaction* and *fluctuation*. Systems with large degrees of freedom tend to be "ordered" in the ground state, where the interaction energy is minimized. For example, interacting particles form into a crystal and spin systems settle in a completely ordered ferromagnetic or anti-ferromagnetic state at zero temperature. If the interactions between the constituents are simple, the ground states of the systems composed of them are also trivial. There is no interesting structure in the ground state. We now ask what happens when such interacting systems are stirred by *fluctuations*? There are two distinct types of fluctuations, time-dependent and time-independent fluctuations. The typical example of the former type is thermal fluctuation, which originates from the dynamics of microscopic degrees of freedom ignored in the effective description of the system. One can also consider nonthermal time-dependent fluctuation. For example, suppose that a macroscopic dissipative system, such as fluids and granular systems, is randomly agitated by an external agent. As a more subtle type of fluctuation, there is one resulting from the uncertainty principle in quantum systems, i.e., quantum fluctuations. In this section, let us concentrate on systems driven by thermal fluctuations. Note that fluctuations do not have any remarkable features themselves, and they are often described by Gaussian white noise, which is the simplest stochastic process. What is surprising in many-body physics is that the competition between simple interactions and random fluctuations yields a wide variety of nontrivial structures and complicated dynamics.

The intuitive explanation of phase transitions is simple. When the fluctuation is quite small, the structure of the ground state is hardly affected and the system remains ordered. However, if the fluctuation prevail over the interaction, the order is destroyed and a disordered phase (high-temperature phase, paramagnetic phase) is realized. To clarify the mathematical structure underlying these phenomena, we introduce an analytically tractable model; an N-component spin system with ferromagnetic interaction. Let $\boldsymbol{S}_i = (S_i^1, \ldots, S_i^N)$ be a spin variable at site i in the D-dimensional hyper cubic lattice. The norm of each spin is fixed at unity: $|\boldsymbol{S}_i|^2 = 1$. The Hamiltonian is given by

$$\mathcal{H}(\{\boldsymbol{S}_i\}) = -J \sum_{\langle ij \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j - \sum_i \boldsymbol{h} \cdot \boldsymbol{S}_i, \tag{1.1}$$

where $\langle ij \rangle$ indicates the nearest-neighbor sites and J is a positive constant. $\boldsymbol{h}$ is a uniform external field. The Hamiltonian is invariant with respect to the global rotation of the spins in the absence of the external field. Obviously, this Hamiltonian attains its minimum when all spins are completely ordered. It is convenient to introduce a continuous version of this spin model. Let $\boldsymbol{\phi}(\boldsymbol{r}) = (\phi^1(\boldsymbol{r}), \ldots, \phi^N(\boldsymbol{r}))$ be an N-component real vector field. The simplest Hamiltonian with the $O(N)$ rotational symmetry is given by

$$\mathcal{H}[\boldsymbol{\phi}] = \int d^D\boldsymbol{r} \left[\sum_\alpha \frac{1}{2} K |\nabla \phi^\alpha(\boldsymbol{r})|^2 + U(\rho(\boldsymbol{r})) - \boldsymbol{h} \cdot \boldsymbol{\phi}(\boldsymbol{r}) \right], \tag{1.2}$$

where K is a positive constant and $\rho(\boldsymbol{r}) = |\boldsymbol{\phi}(\boldsymbol{r})|^2/2$ is the field amplitude. The choice of the local potential $U(\rho)$ is arbitrary as long as it is lower bounded, for example, we have

$$U(\rho) = r\rho + g\rho^2, \tag{1.3}$$

with a positive constant g. The first term in Eq. (1.2) denotes an elastic energy, which favors uniform configurations, and the second term determines the amplitude of the field. If $U(\rho)$ has its minimum at a nonzero value, the ground state of this model is a completely ordered ferromagnetic state. We call the model defined by Eq. (1.1) or (1.2) the $O(N)$ model. It is worth to note that the continuous model Eq. (1.2) can be considered as a long-distance description of the lattice model Eq. (1.1). Namely, the field $\boldsymbol{\phi}(\boldsymbol{r})$ corresponds to a coarse-grained spin variable:

$$\boldsymbol{\phi}(\boldsymbol{r}) = l^{-D} \sum_{i \in C_r} \boldsymbol{S}_i, \tag{1.4}$$

where C_r is a hyper-cube with length l centered at $\boldsymbol{r}$. The coarse-graining scale l is chosen such that it is much smaller than the correlation length, but C_r still contains many spins. For $N = 1$ this model is called the Ising model, for $N = 2$ the XY model and for $N = 3$ the Heisenberg model.

The equilibrium dynamics are described by the following equation of motion:

$$\partial_t \phi^\alpha = -\frac{\delta \mathcal{H}[\boldsymbol{\phi}]}{\delta \phi^\alpha} + \xi^\alpha. \tag{1.5}$$

The time-dependent Gaussian random noise $\xi(\boldsymbol{r}, t)$ satisfies

$$\begin{aligned} \langle \xi^\alpha(\boldsymbol{r}, t) \rangle &= 0, \\ \langle \xi^\alpha(\boldsymbol{r}, t) \xi^\beta(\boldsymbol{r}', t') \rangle &= 2T \delta^{\alpha\beta} \delta(\boldsymbol{r} - \boldsymbol{r}') \delta(t - t'), \end{aligned} \tag{1.6}$$

where T is a temperature. The physical meaning of Eq. (1.5) is as follows; the first term of the right-hand side represents the force that reduces the energy and the second term is the thermal fluctuation. At zero temperature, the system relaxes toward its ground state, while at finite temperature, it reaches a statistically steady state characterized by the Boltzmann-Gibbs distribution,

$$P_{\mathrm{G}}[\boldsymbol{\phi}] = \frac{e^{-\beta \mathcal{H}[\phi]}}{Z}, \tag{1.7}$$

where $\beta = 1/T$ is the inverse temperature and

$$Z = \int \mathcal{D}\boldsymbol{\phi} e^{-\beta \mathcal{H}[\phi]} \tag{1.8}$$

is the partition function. The average of an arbitrary physical quantity $A[\boldsymbol{\phi}]$ in the steady state is given by

$$\langle A[\boldsymbol{\phi}]\rangle = \int \mathcal{D}\boldsymbol{\phi} A[\boldsymbol{\phi}] P_{\mathrm{G}}[\boldsymbol{\phi}]. \tag{1.9}$$

To calculate the average Eq. (1.9) is highly nontrivial and there are only a few exactly solvable examples. Various theoretical methods to approximately evaluate Eq. (1.9) have been developed, such as mean-field theory, high temperature expansion, spin-wave approximation, and renormalization group (RG) theory.

The ground state of the $O(N)$ model is a completely ordered state. When the spatial dimension of the system is higher than the so-called lower critical dimension D_{lc}, the ordered phase is stable against small thermal fluctuations. For the Ising model ($N = 1$), $D_{\mathrm{lc}} = 1$, and for $N \geq 2$, $D_{\mathrm{lc}} = 2$. The important difference between the cases that $N = 1$ and $N \geq 2$ is the absence or presence of massless modes. For $N \geq 2$, there are infinitely many ground states which are continuously connected by the global rotation of the spins. Due to this degeneration, the system has infinitesimally low energy excitations, which are called massless or Goldstone modes. At two dimensions, such excitations destroy the ordered phase at any finite temperatures (Mermin-Wagner theorem). Above the lower critical dimension, there is a long-range ordered (LRO) phase at low temperatures, wherein the equal-time correlation function $C(\boldsymbol{r}' - \boldsymbol{r}) = \langle \boldsymbol{\phi}(\boldsymbol{r}') \cdot \boldsymbol{\phi}(\boldsymbol{r})\rangle$ attains a nonzero constant M^2 in the long-distance limit $|\boldsymbol{r}' - \boldsymbol{r}| \to \infty$. The magnetization M decreases with temperature, and eventually it vanishes at some critical temperature T_{c}. In the high-temperature phase, the correlation function decays exponentially, $C(\boldsymbol{r}) \sim e^{-|r|/\xi_{\mathrm{c}}}$, where ξ_{c} is the correlation length. Near the critical point between two phases, the system exhibits anomalous behaviors, such as the divergence of the correlation length and the characteristic time-scale. These critical phenomena are characterized by the critical exponents and scaling functions, which are known to be independent of the microscopic detail of the system. This remarkable universality can be explained by the RG theory.

Let us briefly sketch the concept of the RG theory [1–4]. The partition function Eq. (1.8) contains all fluctuations with the momentum (wavenumber) $0 \leq |\mathbf{q}| \leq \Lambda$, where Λ is an ultra-violet cutoff corresponding to the inverse of the lattice constant. In the RG theory, one follows the evolution of the effective Hamiltonian when high energy fluctuations are successively integrated out. We split the field $\boldsymbol{\phi}$ into slowly and rapidly varying contributions, $\boldsymbol{\phi} = \boldsymbol{\phi}^L + \boldsymbol{\phi}^S$, where $\boldsymbol{\phi}^L$ contains long wavelength modes with $|\mathbf{q}| < k$ and $\boldsymbol{\phi}^S$ contains short wavelength modes with $k < |\mathbf{q}| < \Lambda$. The effective Hamiltonian $\tilde{\mathcal{H}}_k$ for the slowly varying component is defined by

$$e^{-\tilde{\mathcal{H}}_k[\phi^L]} = \int \mathcal{D}\boldsymbol{\phi}^S \, e^{-\mathcal{H}[\phi^L + \phi^S]}, \tag{1.10}$$

where the inverse temperature β is absorbed into the Hamiltonian. While in the original model the length is measured in units of the lattice constant Λ^{-1}, in the coarse-grained model described by $\tilde{\mathcal{H}}_k$ the length should be measured in units of k^{-1}. This redefinition of the cutoff $\Lambda \to k$ leads to the rescaling of the parameter in $\tilde{\mathcal{H}}_k$. When k is chosen to be close to Λ, the successive application of the above transformation yields a continuous flow in the parameter space of the Hamiltonian. The thermodynamic phases (long-range ordered or disordered phase) and the critical point are characterized by a fixed point, at which the flow of the effective Hamiltonian vanishes. For example, the long-range ordered and disordered phases correspond to the zero and infinite temperature fixed points, respectively. Between these fixed points, there is a nontrivial fixed point that controls the critical behavior. The critical exponents and scaling functions can be obtained from the structure of the RG flow near the fixed point. The important point is that the nature of the fixed point is independent of the microscopic detail of the system. Therefore, the critical behaviors in a wide variety of systems can be classified into a few universality classes which depend only on symmetry and spatial dimension.

In general, the RG transformation Eq. (1.10) cannot be performed exactly, thus we need some approximations. In many cases, the perturbative method would be useful, where the nonlinear term in the effective Hamiltonian is treated as a perturbation. It is justified when the coefficient of the nonlinear term evaluated at the fixed point is quite small. This condition is satisfied near the upper critical dimension, above which the nonlinear term becomes irrelevant. Although the perturbative RG approach has enjoyed considerable success, one should keep in mind that it fails for some types of systems. Interacting systems with quenched disorder are remarkable examples that the perturbative RG fails, as we will discuss later.

1.2 Phase Transitions in Disordered Systems

In the previous section, we discussed phase transitions in interacting systems driven by time-dependent fluctuations. Next, let us consider what happens when time-independent and spatially random fluctuations are exerted on the systems. This type of fluctuations is called *quenched disorder*. The precise meaning of "time-independent" is that the disorder does not change on the typical time scales in which we are interested. For example, suppose ferromagnetic materials with defects or impurities, which do not move on the time scale of the flip of individual spins. Obviously, the effect of the quenched disorder to destroy the ordered state is much stronger than that of the thermal fluctuations. Since all specimens in experiments inevitably contain impurities or defects, it is important from technical viewpoint to investigate the effect of the quenched disorder. In addition, it is also an intriguing problem from theoretical perspectives to consider what type of phase transitions and critical phenomena can emerge from the competition between the interaction and the quenched disorder.

In this section, we introduce some well-studied models of disordered systems and discuss their properties. From a simple phenomenological argument, we first determine the lower critical dimensions for these models. We next show that the standard perturbative approach leads to a beautiful conclusion; *dimensional reduction*, which states that the critical behavior of disordered systems in spatial dimension D is the