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Advances in Soft Computing

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Theoretical Advances and Applications of Fuzzy Logic and Soft Computing

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Library of Congress Control Number: 2007926021

ISSN print edition: 1615-3871
ISSN electronic edition: 1860-0794
ISBN-10 3-540-72433-8 Springer Berlin Heidelberg New York
ISBN-13 978-3-540-72433-9 Springer Berlin Heidelberg New York

Springer is a part of Springer Science+Business Media

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Printed in Germany

Typesetting: by the authors and SPS using a Springer LaTeX macro package

Printed on acid-free paper SPIN: 12060313 89/SPS 5 4 3 2 1 0

Preface

This book comprises a selection of papers from IFSA 2007 on theoretical advances and applications of fuzzy logic and soft computing. These papers were selected from over 400 submissions and constitute an important contribution to the theory and applications of fuzzy logic and soft computing methodologies. Soft Computing consists of several computing paradigms, including fuzzy logic, neural networks, genetic algorithms, and other techniques, which can be used to produce powerful intelligent systems for solving real-world problems. The papers of IFSA 2007 also make a contribution to this goal.

This book is intended to be a major reference for scientists and engineers interested in applying new fuzzy logic and soft computing tools to achieve intelligent solution to complex problems. We consider that this book can also be used to get novel ideas for new lines of research, or to continue the lines of research proposed by the authors of the papers contained in the book.

The book is divided in to sixteen main parts. Each part contains a set of papers on a common subject, so that the reader can find similar papers grouped together. Some of these parts are comprised from the papers of organized sessions of IFSA 2007 and we thank the session's organizers for their incredible job on forming these sessions with invited and regular paper submissions.

In Part I, we have two papers on "Intuitionistic Fuzzy Sets and their Applications" from a session organized by Eulalia Szmidt and Janusz Kacprzyk. These papers show important theoretical results, as well as novel applications of intuitionistic fuzzy logic. The area of intuitionistic fuzzy logic has also become a potential area of promissory results for the future of fuzzy logic.

In Part II, we have a collection of papers on the topic of "The application of fuzzy logic and Soft Computing in Flexible Querying" from a session organized by Guy DeTre and Slawek Zadrozny. These papers show important theoretical results and applications of fuzzy logic and soft computing in achieving flexible querying for database systems. The area of flexible querying has become an important subject for achieving intelligent interfaces with human users and for managing large databases.

In Part III, we have a collection of papers on "Philosophical and Human Scientific Aspects of Soft Computing" from a session organized by Vesa A. Niskanen. These papers show the interesting relationships between the philosophical aspects of soft computing and the formal-scientific aspects of soft computing. Papers on this subject are very important because they help in understanding the area of soft computing, and also enable proposing new theories and methods in this area.

In part IV, we have a collection of papers on "Search Engine and Information Processing and Retrieval" from a special FLINT Session organized by Masoud Nikravesh.

These papers describe important contributions on search engines for the web, summarization, computing with words and granular computing, for information processing and retrieval. Papers on these subjects are very important theoretically as well as in the applications because of the importance of web search for documents and images.

In Part V, we have a set of papers on "Perception Based Data Mining and Decision-Making" from a Session organized by Ildar Batyrshin, Janusz Kacprzyk, and Ronald R. Yager. These papers constitute an important contribution to data mining and linguistic summarization using fuzzy logic. Papers on these subjects are very important because data mining and building summaries are necessary in managing large amounts of data and information.

In Part VI, we have a set of papers on "Soft Computing in Medical Sciences" from a session organized by Rudolf Seising and Christian Schuh. These papers describe important contributions on the use of different soft computing methodologies for solving problems in medicine. Papers on this area are particularly important due to wide variety and complexity of the problems addressed in the medical sciences.

In Part VII, we have a collection of papers on "Joint Model-Based and Data-Based Learning: The Fuzzy Logic Approach" from a session organized by Joseph Aguilar-Martin and Julio Waissman Vilanova. These papers describe important contributions to solving the problems of learning in different types of models using fuzzy logic. Also, the new learning methods are applied to different applications. Learning from data and models is very important for solving real-world problems.

In Part VIII, we have a group of papers on "Fuzzy/Possibilistic Optimization" from a session organized by Weldon Lodwick. These papers describe important theoretical results and applications of fuzzy optimization. The optimization problem is considered from the point of view of fuzzy logic, which gives better results than traditional approaches.

In Part IX, we have a set of papers on "Algebraic Foundations of Soft Computing" from a session organized by Irina Perfilieva and Vilem Novak. These papers represent a significant contribution to the state of the art in the mathematical foundations of different areas in soft computing. The theoretical results will be the basis for future developments in soft computing.

In Part X, we have a group of papers on the subject of "Fuzzy Trees" from a session organized by Ziheng Huang and Masoud Nikravesh. These papers show important theoretical results and applications of fuzzy trees. The use of fuzzy trees is very important as a model of human decision making and for this reason can have many real-world applications.

In Part XI, we have a group of papers on "Soft Computing in Petroleum Applications" from a session organized by Leonid Sheremetov and Masoud Nikravesh. These papers describe important theoretical results and applications of soft computing in real-world problems in the petroleum industry. The point of view of soft computing for this type of problems gives better results than traditional approaches.

In Part XII, we have a group of papers on "Fuzzy Logic and Soft Computing in Distributed Computing" from a session organized by Lifeng Xi and Kun Gao. These papers describe important theoretical results and applications of fuzzy logic and soft computing to problems in distributed computing. The proposed methods using fuzzy logic and soft computing give better results than traditional approaches

In Part XIII, we have a collection of papers on "Fuzzy Logic Theory" describing different contributions to the theory of fuzzy logic. These papers show mainly theoretical results on fuzzy logic that can help advance the theory and/or provide fundamental tools for possible solutions to real-world problems.

In Part XIV, we have a group of papers on "Fuzzy Logic Applications" that show a wide range of applications of fuzzy logic theory. The papers describe in detail important real-world problems that have solved satisfactorily with fuzzy systems. Also, the fuzzy solutions are shown to be better than traditional solutions to these problems.

In Part XV, we have a collection of papers on "Neural Networks" that comprise theoretical contributions on neural networks and intelligent systems developed with neural models, as well as real applications of these areas. The papers represent an important contribution to the state of the art in both theory and applications of neural networks.

In Part XVI, we have a collection of papers on "Soft Computing" comprising important contributions in this field. These papers show theoretical results and important applications of soft computing methodologies. Also, there are papers on hybrid intelligent systems, that combine several soft computing techniques.

We end this preface of the book by giving thanks to all the people who have help or encourage us during the compilation of this book. We would like to thank our colleagues working in Soft Computing, which are too many to mention each by their name. Of course, we need to thank our supporting agencies in our countries for their help during this project. We have to thank our institutions, for always supporting our projects.

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IFSA 2007 is organized by the International Fuzzy Systems Association (IFSA), the Hispanic American Fuzzy Systems Association (HAFSA), and the Division of Graduate Studies of Tijuana Institute of Technology, with support of DGEST and CONACYT of Mexico.

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Part I

Intuitionistic Fuzzy Sets and Their Applications

An Intuitionistic Fuzzy Graph Method for Finding the Shortest Paths in Networks

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Abstract. The task of finding shortest paths in graphs has been studied intensively over the past five decades. Shortest paths are one of the simplest and most widely used concepts in networks. More recently, fuzzy graphs, along with generalizations of algorithms for finding optimal paths within them, have emerged as an adequate modeling tool for imprecise systems. Fuzzy shortest paths also have a variety of applications. In this paper, the authors present a model based on dynamic programming to find the shortest paths in intuitionistic fuzzy graphs.

Keywords: Index Matrix (IM), Intuitionistic Fuzzy Graphs (IFGs), Shortest path, Dynamic programming (DP).

1 Introduction

Over the past several years a great deal of attention has been paid to mathematical programs and mathematical models that can be solved through the use of networks. There has been a plenty of articles on network programming and several significant advances have been made. Through these advances efficient algorithms have been developed for even large scale programs. Posing problems on networks not only yields computational advantages, it also serves as a means for visualizing a problem and for developing a better understanding of the problem.

The aim of this paper is to concentrate on the most basic network problem, the shortest path problem. The fuzzy shortest path problem was first analyzed by Dubois and Prade [7]. However the major drawback to this problem is the lack of interpretation. To overcome this situation, a new model based on shortest paths in intuitionistic fuzzy graphs is presented. An algorithm for this model based on DP recursive equation approach is also developed and checked with the local telecommunication department map.

O. Castillo et al. (Eds.): Theor. Adv. and Appl. of Fuzzy Logic, ASC 42, pp. 3–10, 2007.
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In this paper, Section 2 provides preliminary concepts required for analysis. In Section 3 and 4, the shortest path problem in intuitionistic fuzzy graphs is introduced and new model has been developed. A case study work is carried out in Section 5 considering a local city's telecommunications department map as an IFG. Section 6 concludes the paper.

2 Preliminaries

Definition 2.1 [6]**.** A *graph* consists of a structure $G = (V, E)$, where V is a set of *vertices*, and the predicate $E \subseteq V \times V$ is a set of *edges*.

Definition 2.2 [4]**.** Let E be a non-empty set. An *Intuitionistic Fuzzy Set* (IFS) A in E is defined as an object of the form $A=\{(x, \mu_A(x), \gamma_A(x)) : x \in E\}$ where the fuzzy sets $\mu_A : E \to [0,1]$ and $\gamma_A : E \to [0,1]$ denote the membership and non-membership functions of A respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in E$.

Definition 2.3. Let I be a fixed set of indices and R be the set of all real numbers. By an *IM* with index sets K and L (K, L $\subset$ I), we mean the object [2]

$$[\,K, L, \{a_{k_i,l_j}\}] = \begin{array}{c|cccc} & l_1 & l_2 & \cdots & l_n \\ \hline k_1 & a_{k_1,l_1} & a_{k_1,l_2} & \cdots & a_{k_1,l_n} \\ k_2 & a_{k_2,l_1} & a_{k_2,l_2} & \cdots & a_{k_2,l_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ k_m & a_{k_m,l_1} & a_{k_m,l_2} & \cdots & a_{k_m,l_n} \end{array}$$

where K = $\{k_1, k_2, \cdots, k_m\}$, L = $\{l_1, l_2, \cdots, l_n\}$, for $1 \leq i \leq m$, and for $1 \leq j \leq n : a_{k_i,l_j} \in R$. For the IMs A = [K, L, $\{a_{k_i,l_j}\}$] and B = [P, Q, $\{a_{k_i,l_j}\}$] the usual matrix operations "addition" and "multiplication" are defined[4].

Definition 2.4. Let E_1 and E_2 be two universes and let $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in E_1\}$, $B = \{\langle x, \mu_B(x), \gamma_B(x)\rangle : x \in E_2\}$ be two IFSs ; A – over E_1 and B – over E_2. Now define [3]

$$A \times_1 B = \{\langle\langle x, y\rangle, \mu_A(x).\mu_B(y), \gamma_A(x).\gamma_B(y)\rangle : \langle x, y\rangle \in E_1 \times E_2\}$$

$$A \times_2 B = \left\{ \begin{array}{l} \langle\langle x, y\rangle, \mu_A(x) + \mu_B(y) - \mu_A(x).\mu_B(y), \gamma_A(x).\gamma_B(y)\rangle : \\ \langle x, y\rangle \in E_1 \times E_2 \end{array} \right\}$$

$$A \times_3 B = \left\{ \begin{array}{l} \langle\langle x, y\rangle, \mu_A(x).\mu_B(y), \gamma_A(x) + \gamma_B(y) - \gamma_A(x).\gamma_B(y)\rangle : \\ \langle x, y\rangle \in E_1 \times E_2 \end{array} \right\}$$

$$A \times_4 B = \left\{ \begin{matrix} \langle \langle x, y \rangle, \min(\mu_A(x), \mu_B(y)), \max(\gamma_A(x), \gamma_B(y)) \rangle : \\ \langle x, y \rangle \in E_1 \times E_2 \end{matrix} \right\}$$

$$A \times_5 B = \left\{ \begin{matrix} \langle \langle x, y \rangle, \max(\mu_A(x), \mu_B(y)), \min(\gamma_A(x), \gamma_B(y)) \rangle : \\ \langle x, y \rangle \in E_1 \times E_2 \end{matrix} \right\}.$$

It must be noted that $A \times_i B$ is an IFS, but it is an IFS over the universe $E_1 \times E_2$, where "$\times_i$" is one of the five Cartesian products above and "$\times$" is the classical Cartesian product on ordinary sets (E_1 and E_2).

Definition 2.5 [5]**.** Let X and Y are arbitrary finite non-empty sets. Intuitionistic Fuzzy Relation (IFR) is an IFS R $\subset$ $X \times Y$ of the form : R=$\{\langle \langle x, y \rangle, \mu_R(x, y), \gamma_R(x, y) \rangle : x \in X, y \in Y\}$, $\mu_R : X \times Y \to [0,1]$ and $\gamma_R : X \times Y \to [0,1]$ are the degrees of membership and non-membership as the ordinary IFSs or degrees of validity and non-validity of the relation R; and for every $\langle x, y \rangle \in X \times Y :\ 0 \le \mu_R(x, y) + \gamma_R(x, y) \le 1$.

3 Main Results

Let the oriented graph G = (V,A) be given, where V is a set of vertices and A is a set of arcs. Every graph arc connects two graph vertices.

In [5] an approach for introducing of an IFG is given. Here we will modify it in two directions on the basis of some ideas generated from IFS-theoretical and from IFS-decision making points of view. We shall start with the oldest version of the concept.

Let operation $\times$ denote the standard Cartesian product operation, while operation $\circ \in \{\times_1, \times_2, \times_3, \times_4, \times_5\}$.

Following [3] we shall note that the set $G^* = \{\langle \langle x, y \rangle, \mu_G(x, y), \gamma_G(x, y) \rangle : \langle x, y \rangle \in V \times V\}$ is called an o-IFG (or briefy, an IFG) if the functions $\mu_G : V \times V \to [0,1]$ and $\gamma_G : V \times V \to [0,1]$ denote the respective degrees of membership and non-membership of the element $\langle x, y \rangle \in V \times V$. These functions have the forms of the corresponding components of the o-Cartesian product over IFSs, and for all $\langle x, y \rangle \in V \times V :\ 0 \le \mu_G(x, y) + \gamma_G(x, y) \le 1$.

This approach supposes that the given set V and the operation o are choises and fixed previously and they will be used without changes.

On the other hand, following the IFS-interpretations in decision making, we can construct set V and values of functions μ_G and γ_G in the current time, for example on the basis of expert knowledge and we can change their forms on the next steps of the process of IFG's use.

Now, we shall introduce a definition of a new type of an IFG.[2]

Let E be a universe, containing fixed graph-vertices and let V $\subset$ E be a fixed set. Construct the IFS $V = \{(x, \mu_V(x), \gamma_V(x)) : x \in E\}$ where the fuzzy sets $\mu_V : E \to [0,1]$

and $\gamma_V : E \rightarrow [0,1]$ determine the degree of membership and the degree of non-membership to set V of the element (vertex) $x \in E$, respectively, and for every $x \in E$ such that $0 \le \mu_V(x) + \gamma_V(x) \le 1$.

Now, we shall use the idea for an IFS over universe that is an IFS over another universe [2] and will define the set $G^* = \{\langle\langle x, y\rangle, \mu_G(x, y), \gamma_G(x, y)\rangle : \langle x, y\rangle \in G \times G\}$ is called an o-Generalized IFG (or briefly, an GIFG) if the functions $\mu_G : V \times V \rightarrow [0,1]$ and $\gamma_G : V \times V \rightarrow [0,1]$ denote the respective degrees of membership and non-membership of the element (the graph arc) $\langle x, y\rangle \in V \times V$. As above, these functions have the forms of the corresponding components of the o-Cartesian product over IFSs, and for all $\langle x, y\rangle \in V \times V$ such that $0 \le \mu_G(x, y) + \gamma_G(x, y) \le 1$.

Definition 3.1. An i*ntuitionistic fuzzy path* in an IFG is a sequence of vertices and edges such that either one of the following conditions are satisfied:

$$\text{(i) } \mu_G(x, y) > 0 \text{ or (ii) } \mu_G(x, y) = 0 \text{ and } \gamma_G(x, y) < 1 \text{ , for all } \langle x, y\rangle \in P .$$

Example 3.2. Let $V = \{v_1, v_2, v_3, v_4\}$. Consider the following index matrix for the IFG G.

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} \langle 0,0\rangle & \langle 0.2,0.5\rangle & \langle 0.5,0.2\rangle & \langle 0,0\rangle \\ \langle 0.3,0.6\rangle & \langle 0,0\rangle & \langle 0.4,0.5\rangle & \langle 0.7,0\rangle \\ \langle 0,0\rangle & \langle 0,0\rangle & \langle 0.2,0.7\rangle & \langle 0.8,0.1\rangle \\ \langle 0,0\rangle & \langle 0.7,0\rangle & \langle 0,0\rangle & \langle 0,0\rangle \end{bmatrix} \end{array}$$

The graph corresponding to this IM is given in Fig. 1.

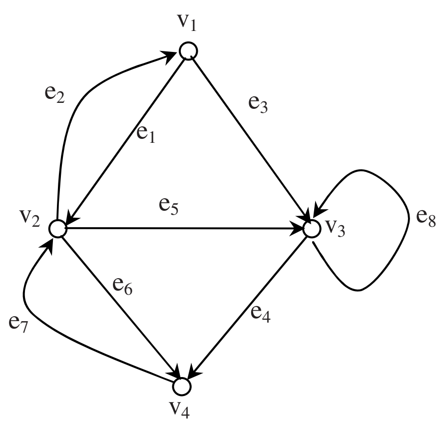

Fig. 1. Path in an IFG G

Here $v_1e_1v_2e_6v_4$, $v_1e_3v_3e_4v_4$, $v_1e_1v_2e_5v_3e_4v_4$ are intuitionistic fuzzy paths from v_1 to v_4.

4 Background

The models to be considered are an extension of a general network. A network is generally depicted by a graph and the terms will be used interchangeably. Let a graph, denoted by $G(V, E)$, be a set of points V, and a set of pairs of these points E. The set V refers to the vertices of the graph and the set E refers to the edges of the graph. An edge is denoted by a pair of vertices {i, j}.

If E is changed to a set of ordered pairs of distinct elements of V, then $G(V, E)$ is a directed graph and E is the set of ordered pairs (i, j). The ordered pairs (i, j) are referred to as arcs or edges and an arc goes from vertex i to vertex j. An arc (i, i) is referred to as a loop.

A path from a vertex s to a vertex t is a sequence of arcs of the form $(s, i_1), (i_1, i_2), \ldots (i_k, t)$. In other words, vertex t can be reached from vertex s. A path from s to t is denoted as an (s, t) path. An (s, t) path is open if $s \neq t$ and is closed if s = t. A cycle is a closed path (s, s) in which no vertices are repeated except s and there exists at least one arc. A graph that contains no cycles is called acyclic.

In a given graph both a source vertex and a sink vertex can be designated. These are interpreted as terminal vertices at which some activity begins and ends. In an acyclic directed graph with N vertices, the source can be labeled as vertex 1 and the sink as vertex N and all other vertices can be labeled such that for any arc (i,j), $i < j$.

A special type of an acyclic directed graph is a layered graph. This is an acyclic directed graph in which the vertex set V can be partitioned into M subsets, $V_1, \ldots, V_M$, such that if $|V_k| > 1$, the vertices in V_k are sequentially numbered and there does not exist an arc (i, j) for $i, j \in V_k$. For a layered graph, generally V_1 is the source vertex and V_M is the sink vertex.

If each arc (i, j) has an associated weight or length c_{ij}, then an (s, t) path has an associated weight or length equal to the sum of the weights of the arcs in the path. This in turn gives rise to the shortest path problem, which is to find the path with minimal weight between two vertices s and t. There are a variety of ways to find the shortest path for a network [8]. Some of them are general methods such as the labeling algorithm follow from DP. It is assumed that the graphs for the models to be presented are directed acyclic graphs. As any graph that has no cycles of negative weight can easily be converted to a directed acyclic graph [8], this is not a major restriction.

Assume the following in order to use the hybrid dynamic programming method for the shortest route problems and other network problems: (i) The network is directed and acyclic; (ii) the network is layered.

It should be noted that many applications naturally take on a layered network form [1]. For example, activity networks such as PERT are always directed and acyclic. It is also generally true that edges have positive lengths. This is especially true in terms of transportation related problems.

The multi-criteria DP recursion that will be used is

$$f(N) = (1, 1, 1 \ldots\ldots 1),$$
$$f(i) = \operatorname{dom}(e_{ij} \tilde{+} f(i)), \tag{1}$$

where e_{ij} is an R-tuple associated with each arc or edge (i, j) or the path from i to j. This R-tuple consists of the membership and non-membership grades of arc (i, j) or the membership and non-membership grades of the paths from i to j in the respective intuitionistic fuzzy sets associated with the possible lengths, 1 through R. Hence,

$$e_{ij} = (\mu_{e_{ij}}, \gamma_{e_{ij}})$$

where $\mu_{e_{ij}} = (\mu_1(i,j), \mu_2(i,j), \ldots \mu_R(i,j))$ and $\gamma_{e_{ij}} = (\gamma_1(i,j), \gamma_2(i,j), \ldots \gamma_R(i,j))$. The operator $\tilde{+}$ represents the combinatorial sum and dom is the domination operator.

The combinatorial sum for fuzzy shortest paths is defined as follows. Recall that we assume there are M layers and the possible lengths of an edge are 1 through R. Therefore, the shortest a path could be in length is M - 1 and the longest a path could be in length is (M - 1). R. To find the paths of possible lengths, combinations of the possible lengths must be considered. To find the possible length I of a path, the lengths that can be used in combination are 1 , 2 , . . . ,I - M + 1. The combinatorial sum of two tuples is then defined as follows. Let $Z = \min\{\mu_x(j,k), \mu_y(k,q)\}$ for membership values and $Z = \max\{\gamma_x(j,k), \gamma_y(k,q)\}$. Then $e_{j,q} = e_{j,k} \tilde{+} e_{k,q}$ where the membership and non-membership grades of i-th element of the R-tuple $e_{j,q}$ is given by $m(e_{j,q})^i = \max\limits_{x+y=i} (Z) = \mu_i(j,q)$ and

$$nm(e_{j,q})^i = \min_{x+y=i} (Z) = \gamma_i(j,q).$$

The recursive equation (1) will yield the set of non-dominated paths from source 1 to N. We then define

$$m(\tilde{P}_{1,N}) = \left\{1/\max_i(\mu_1^i(1.N), \cdots, K/\max_i(\mu_K^i(1.N), \cdots, R/\max_i(\mu_R^i(1.N))\right\}$$

and

$$nm(\tilde{P}_{1,N}) = \left\{1/\min_i(\gamma_1^i(1.N), \cdots, K/\min_i(\gamma_K^i(1.N), \cdots, R/\min_i(\gamma_R^i(1.N))\right\}$$

where $\mu_K^i(s,t)$ and $\gamma_K^i(s,t)$ represent the membership and non- membership grades in the intuitionistic fuzzy set K of the path from vertex s to vertex t given by the i-th non-dominated R-tuple. Now, intuitionistic fuzzy shortest path length, denoted by $P_{1,N}$, is equal to nondom $\{m(\tilde{P}_{1,N}), nm(\tilde{P}_{1,N})\}$.

5 Case Study

Consider the map of telecommunication department of Erode city in India. An IFG is formed with 5 layers (Fig. 2).

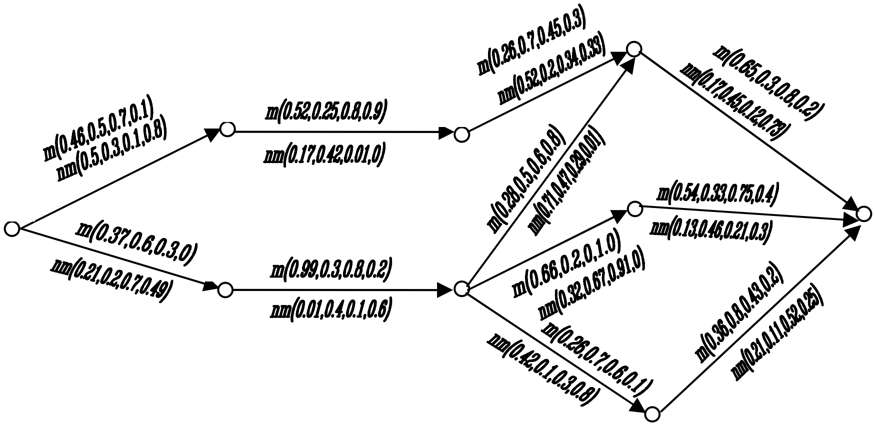

Fig. 2. A directed acyclic IFG

Assume that each edge can take a length of 1, 2, 3 or 4. The four tuple associated with each edge gives the membership value of the edge in each of the fuzzy sets 1, 2, 3and 4. In the graph there are 5 layers. Hence, M - 1 = 4 and the shortest possible path is 4 units and the most a path could be is 16 units since R = 4. If we apply the recursion (membership) given by equation (1), we obtain

f(9) = (1, 1, 1 . . . . . 1),
f(6) = (0.65, 0.3, 0.8, 0.2, 0, 0, …,0),
f(7) = (0.54, 0.33, 0.75, 0.4, 0, 0, …, 0),
f(8) = (0.36, 0.8, 0.43, 0.2, 0, 0,…, 0),
f(4) = $e_{46} \tilde{\mp} f(6)$ = {(0, 0.26, 0.65, 0.45, 0.7, 0.45, 0.3, 0.2, 0, 0,…,0),
f(5) = {0, 0.54, 0.5, 0.7, 0.65, 0.6, 0.8, 0.4, 0, 0,…,0}.
f(2) = {0, 0, 0.26, 0.52, 0.45, 0.65, 0.65, 0.7, 0.7, 0.45, 0.3, 0.2, 0, 0,…,0}.
f(1) = {0, 0, 0, 0.37, 0.54, 0.5, 0.6, 0.6, 0.65, 0.65, 0.7, 0.7, 0.45, 0.3, 0.2, 0.1}.

Similarly, for the non-membership function, we have

f(9) = (0, 0, 0,…, 0),
f(6) = (0.17, 0.45, 0.12, 0.73, 0, 0, …,0),
f(7) = (0.13, 0.46, 0.21, 0.3, 0, 0, …, 0),
f(8) = (0.21, 0.11, 0.52, 0.25, 0, 0,…, 0),
f(4) = {(0, 0.52, 0.2, 0.34, 0.2, 0.34, 0.33, 0.73,0, 0,…,0),
f(5) = {0, 0.32, 0.21, 0.11, 0.3, 0.25, 0.3, 0.8, 0, 0,…,0},
f(2) = {0, 0.17, 0.42, 0.01, 0, 0.2, 0.2, 0.2, 0.2, 0.17, 0.17, 0.01, 0.01, 0.01,0.01, 0.01},

f(3) ={0, 0.01, 0.32, 0.1, 0.11, 0.13, 0.11, 0.12,0.25, 0.01, 0.01, 0.01, 0.01,0.01, 0.01,0.01},
f(1) = {0, 0.21, 0.2, 0.2, 0.21, 0.2, 0.2, 0.2, 0.2, 0.2, 0.21, 0.2, 0.2, 0.2, 0.2, 0.2}.

The intuitionistic fuzzy shortest path length is then given by
$P_{1,9}$ = {1/0, 2/0, 3/0.2, 4/0.2, 5/0.21, 6/0.2, 7/0.2, 8/0.2, 9/0.2, 10/0.2, 11/0.21, 12/0.2, 13/0.2, 14/0.2, 15/0.2, 16/0.1}.

Therefore it is determined that the shortest possible path has length 4. This path corresponds to the path 1-2-4-6-9 in the original network and is formed by the general backtracking techniques of DP. Note that this model represents intuitionistic fuzzy shortest path. Therefore, it can be used as a decision tool due to the maintenance of the underlying structure. Also note that the procedure presented is a naive DP approach and can be easily made more efficient.

6 Conclusion

In this paper we have presented new formulation for intuitionistic fuzzy shortest path problems. This method is developed because the classical fuzzy shortest path problem yields a fuzzy length with no actual path associated with it [8]. The formulation presented circumvents this problem. Dynamic programming based method is developed to solve the new problems and a numerical example is given for better understanding. This type of analysis may have potential in developing new formulations for general fuzzy mathematical programming, or for analyzing current formulations.

Acknowledgments

The author Parvathi Rangasamy would like to thank University Grants Commission, NewDelhi, India for its financial support to the Minor Research Project MRP 686/05(UGC-SERO) dated Feb2005.

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On Imprecision Intuitionistic Fuzzy Sets & OLAP – The Case for KNOLAP

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Abstract. Traditional data repositories are typically focused on the storage and querying of crisp-precise domains of data. As a result, current commercial data repositories have no facilities for either storing or querying imprecise-approximate data. However, when considering scientific data (i.e. medical data, sensor data etc) value uncertainty is inherited to scientific measurements. In this paper we revise the context of "value uncertainty", and examine common models related to value uncertainty as part of the OLAP model. We present our approach for extending the OLAP model to include treatment of value uncertainty as part of a multidimensional model inhabited by flexible date and non-rigid hierarchical structures of organisation.

1 Introduction

In this paper we introduce the semantics of the Intuitionistic Fuzzy cubic representation in contrast to the basic multidimensional-cubic structures. The basic cubic operators are extended and enhanced with the aid of Intuitionistic Fuzzy Logic [1], [2].

Since the emergence of the OLAP technology [3] different proposals have been made to give support to different types of data and application purposes. One of this is to extend the relational model (ROLAP) to support the structures and operations typical of OLAP. Further approaches [4], [5] are based on extended relational systems to represent data-cubes and operate over them. The other approach is to develop new models using a multidimensional view of the data [6].

Nowadays, information and knowledge-based systems need to manage imprecision in the data and more flexible structures are needed to represent the analysis domain. New models have appeared to manage incomplete datacube [7], imprecision in the facts and the definition of fact using different levels in the dimensions [8].

Nevertheless, these models continue to use inflexible hierarchies thus making it difficult to merge reconcilable data from different sources with some incompatibilities in their schemata. These incompatibilities arise due to different perceptions-views about a particular modelling reality.

In addressing the problem of representing flexible hierarchies we propose a new multidimensional model that is able to treat with imprecision over conceptual hierarchies based on Intuitionistic Fuzzy logic. The use of conceptual hierarchies enables us to:

O. Castillo et al. (Eds.): Theor. Adv. and Appl. of Fuzzy Logic, ASC 42, pp. 11–20, 2007.
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