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Bo Zhang Dongyuan Qiu

m-Mode SVPWM Technique for Power Converters





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 ISSN 2520-8853
 ISSN 2520-8861
 (electronic)

 CPSS Power Electronics Series
 ISBN 978-981-13-1381-3
 ISBN 978-981-13-1382-0
 (eBook)

 https://doi.org/10.1007/978-981-13-1382-0
 ISBN 978-981-13-1382-0
 ISBN 978-981-13-1382-0
 ISBN 978-981-13-1382-0

Library of Congress Control Number: 2018966435

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Preface

In traditional SVPWM, all operating modes or voltage space vectors of the inverter need participate in modulation, which results in a large amount of calculation in the digital processor and the corresponding computational complexity. It is well known that reducing the number of voltage space vectors is a potential methodology to simplify the calculation of SVPWM.

Since 2001, the authors have studied the modulation characteristics of power electronic converters to solve the computational complexity problem of SVPWM. In terms of the system state controllability, it is found that not all voltage space vectors are required for the SVPWM process, when the inverter is regarded as a switched linear system. Further, some redundant voltage space vectors can be ignored to reduce the computational complexity of SVPWM and achieve the desired control output. Based on the above results, the rotating voltage space vector can be synthesized by fewer voltage space vectors, and a novel and simpler SVPWM can be obtained. In fact, any rotating voltage space vector has a variety of synthesis methods on the basis of the space vector graph of SVPWM, when the angle between two voltage space vectors forming the rotating voltage space vector is less than 180°, so it is possible to choose the modulation method having fewer voltage space vectors to realize SVPWM. The drawn conclusion of the system state controllability analysis in geometry can be confirmed.

Therefore, in view of the system state controllability, the authors do propose the mechanism and criterion of *m*-mode SVPWM, and develop the corresponding strategies applied for two-level inverters, dual-output inverters, multiphase inverters, three-level inverters, modular multilevel inverters, and PWM rectifiers. Theoretical and experimental results validate that *m*-mode SVPWM has simpler calculation, lower switching frequency, and higher efficiency than the existing SVPWM. The *m*-mode SVPWM, we concluded, is an innovative one.

This book consists of four parts. According to the switched linear system theory, the first part reveals the state controllability of power electronic converters and puts forward the corresponding *m*-mode state controllability criteria and the *m*-mode SVPWM mechanism. Based on the above theory, the *m*-mode SVPWM strategies of the three-phase four-wire inverter, nine-switch dual-output inverter, five-leg

dual-output inverter, and three-phase three-level inverter are proposed in the second part in detail, and the proposed SVPWMs are compared with the traditional ones to verify their superiorities. By reducing the number of inverter operating modes, the following part exhibits the application of *m*-mode state controllability to the complex modular multilevel inverter to simplify the PWM strategy. The *m*-mode SVPWM mechanism, in the last part, is popularized and applied to the PWM rectifier.

And, withal, an inspiration is given from this book: multi-interdisciplinary research can achieve novel outcomes. For example, considering power electronic converter only from the circuit theory, the factors that promote the development of power electronic technology show less vitality. Switched linear system theory, however, draws new elicitation for the study of power electronic converters. It is even possible to achieve further discoveries, tap the potentials, and take full advantage of the new characteristics of power electronic converters. Therefore, it is interdisciplinary that enlightens the future direction in research of power electronics technology.

Guangzhou, China October 2018 Bo Zhang Dongyuan Qiu

Acknowledgements

For the completion of this book, authors would like to acknowledge the efforts of our former students, i.e., Dr. Zongbo Hu, Dr. Xiangfeng Li, and Mr. Wenbin Wang. In the process of trying to apply the switched linear system theory to power electronic converters, we made joint efforts, experienced confusion and perplexity in overcoming obstacles, and finally advanced this new theory. In addition, we appreciate the support of the Team Program of Natural Science Foundation of Guangdong Province (No. 2017B030312001), the Key Program of National Natural Science Foundation of China (No. 50937001), and the National Key R&D Program of China (No. 2018YFB0905800) to our research.

Guangzhou, China October 2018 Bo Zhang Dongyuan Qiu

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Chapter 1 Introduction



1.1 Review of PWM Converter

Pulse-width modulation (PWM) is one of the core technologies of power electronic converters and it was initially proposed to allow inverters to output sinusoidal AC voltage and current. Up to now, it has been applied to the AC–AC matrix converters and PWM rectifiers. Although PWM has been proposed for nearly 60 years since 1964, due to the continuous emergence of various new power electronic converters and the increasing requirements for the quality of converters' output voltage and current, PWM is still one of the most popular research directions in the field of power electronics, and continues to attract attention and interest of researchers.

PWM methods are usually divided into sinusoidal pulse-width modulation (SPWM) and space vector pulse-width modulation (SVPWM) according to the principle of generation. For inverters, there are three main indicators to evaluate the performance of PWM method: (1) Harmonic content; (2) Utilization of the DC voltage; and (3) Switching times. Besides, with the development of PWM method, it is necessary to consider the difficulty of the control method implementation, the possibility of soft switching, and the ability to suppress common mode (CM) and differential mode (DM) interferences simultaneously.

The analysis of the inverters' harmonic content is mainly based on the Fourier series method [1]. Assuming that the output voltage v of the inverter is a function of the period T, its root mean square (RMS) value is defined as

$$V_{\rm rms} = \sqrt{\frac{1}{T} \int\limits_{0}^{T} v^2 \mathrm{d}t}$$
(1.1)

1

Since the voltage signal *v* is periodic, it can be expressed by the Fourier series as follows:

$$v = V_1 \cos \omega t + V_2 \cos 2\omega t + V_3 \cos 3\omega t + \cdots, \qquad (1.2)$$

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B. Zhang and D. Qiu, *m-Mode SVPWM Technique for Power Converters*, CPSS Power Electronics Series, https://doi.org/10.1007/978-981-13-1382-0_1

where $\omega = 2\pi/T = 2\pi f$, V_n (n = 1, 2, 3, ...) is the amplitude of the component at frequency $f_n = nf$.

Thus,

$$V_{\rm rms} = \sqrt{\frac{1}{T} \int_{0}^{T} \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} V_n V_k \cos n\omega t \cos k\omega t \,\mathrm{d}t}$$
(1.3)

The integral term of $n \neq k$ in Eq. (1.3) is zero, so there is

$$V_{\rm rms} = \sqrt{\frac{1}{T} \int_{0}^{T} \sum_{n=1}^{\infty} V_n^2 \cos^2 n\omega t \, dt}$$
$$= \sqrt{\frac{1}{T} \int_{0}^{T} \sum_{n=1}^{\infty} \frac{V_n^2}{2} (1 + \cos 2n\omega t) \, dt}$$
(1.4)

The integral result of the double frequency term in Eq. (1.4) is zero in one complete cycle, so there is

$$V_{\rm rms} = \sqrt{\sum_{n=1}^{\infty} \frac{V_n^2}{2}}.$$
 (1.5)

Equation (1.5) could be expressed in terms of the RMS value as follows:

$$V_{\rm rms} = \sqrt{\sum_{n=1}^{\infty} V_{n,\rm rms}^2},\tag{1.6}$$

where $V_{n,\text{rms}}$ is the RMS value of the component at frequency f_n .

The fundamental component in Eq. (1.6) is the desired output and the remaining components can be regarded as "distortion". Considering the fundamental component $V_{1,\text{rms}}$, the above equation turns to be

$$V_{\rm rms} = V_{1,\rm rms} \sqrt{1 + \sum_{n=2}^{\infty} \left(\frac{V_{n,\rm rms}}{V_{1,\rm rms}}\right)^2}$$
 (1.7)

The total harmonic distortion (THD) of the voltage is then defined as

$$\text{THD} = \sqrt{\sum_{n=2}^{\infty} \left(\frac{V_{n,\text{rms}}}{V_{1,\text{rms}}}\right)^2}.$$
 (1.8)

To express the DC voltage utilization of the PWM inverter, it is common to use the ratio of the fundamental amplitude of the output voltage to the DC input voltage V_d of the inverter. For the three-phase SPWM inverter, the maximum output phase voltage is $\frac{V_d}{2}$, so the maximum output line voltage is $\frac{\sqrt{3}}{2}V_d$, that is, the DC voltage utilization is 0.866. For the three-phase inverter with SVPWM control, six voltage space vectors \vec{V}_{s1} , \vec{V}_{s2} , \vec{V}_{s3} , \vec{V}_{s4} , \vec{V}_{s5} , \vec{V}_{s6} form a hexagon, as shown in Fig. 1.1. When the resultant voltage vector rotates with an inscribed circle of radius OM, the maximum output phase voltage and line voltage are $\frac{\sqrt{3}}{3}V_d$ and V_d , respectively, and the corresponding DC voltage utilization rate is 1. When the resultant voltage vector rotates with an inscribed circle of radius ON, which represents for the maximum conventional SPWM, the maximum output phase voltage and line voltage are $\frac{V_d}{2}$ and $\frac{\sqrt{3}}{2}V_d$, respectively, which means that the DC voltage utilization rate is 0.866. When the voltage vector operates in the hexagonal shape condition, the maximum output phase voltage and line voltage are $\frac{2}{3}V_d$ and $\frac{2\sqrt{3}}{3}V_d$, then the DC voltage utilization rate is 1.155.

The switching frequency of the PWM method could affect the efficiency and reliability of inverter directly. The higher the switching frequency f_s , the smaller the distortion rate of the AC output current of the inverter, and the smaller the capacity and volume of the filter inductor and capacitor. However, as the switching frequency increases, the switching losses increase, and the performance requirements for the switching device are improved.

The loss generated by the switching process is called the dynamic switching loss, which is related to the switching-on time t_{on} and switching-off time t_{off} of the switch. Assuming that the current flowing through the switching device is I_C when turned on, the voltage across the switching device is V_C when turned off, and the current and voltage rise or fall linearly during the turn-on and turn-off processes, then the current and voltage of the switching device change according to the following rules during the turn-on process.



1 Introduction

$$\begin{cases} i \approx \frac{I_{\rm C}}{t_{\rm on}} t \\ u \approx V_{\rm C} - \frac{V_{\rm C}}{t_{\rm on}} t \end{cases}$$
(1.9)

The expressions of the current and voltage of the switching device during the turn-off process are

$$\begin{cases} i \approx I_{\rm C} - \frac{I_{\rm C}}{t_{\rm off}} t \\ u \approx \frac{V_{\rm C}}{t_{\rm off}} t \end{cases}$$
(1.10)

Then the dynamic switching loss is

$$P_{\rm S} = f_{\rm s} \left(\int_{0}^{t_{\rm on}} \frac{I_{\rm C}}{t_{\rm on}} t \left(V_{\rm C} - \frac{V_{\rm C}}{t_{\rm on}} t \right) \mathrm{d}t + \int_{0}^{t_{\rm off}} \left(I_{\rm C} - \frac{I_{\rm C}}{t_{\rm off}} t \right) \frac{V_{\rm C}}{t_{\rm off}} t \mathrm{d}t \right)$$
$$= \frac{t_{\rm on} + t_{\rm off}}{6} f_{\rm s} V_{\rm C} I_{\rm C} \tag{1.11}$$

It can be seen that the dynamic switching loss is proportional to the switching frequency f_s . The higher the switching frequency, the larger the loss, the lower the efficiency of the converter, and the switching loss limits the improvement of switching frequency of the converter.

Therefore, various PWM methods for the inverter have been proposed and developed to reduce the harmonic content, increase the DC voltage utilization rate, and reduce the number of switching times. For the AC–AC matrix converter and PWM rectifier, since the former one can be regarded as the AC–DC–AC conversion, the latter one has the same topology as the PWM inverter, their PWM methods can be proposed based on those of inverters.

1.2 SPWM Methods

1.2.1 Natural Sampling SPWM

The natural sampling SPWM method for the inverter is shown in Fig. 1.2 [2], where v_r is a sinusoidal modulating signal and v_c is a triangular carrier signal. The principle of natural sampling SPWM is to compare a sinusoidal modulating voltage of fundamental frequency f_r with a triangular carrier wave of much higher frequency f_c . According to the comparison results, an rectangular pulse sequence whose width

varies with the sinusoidal law is generated, and the pulse sequence is power amplified to drive the inverter to produce a sinusoidal voltage or current output.

Figure 1.3 is a partial enlargement of Fig. 1.2. Accurately calculating the natural intersection time of sine and triangle waveforms is the key to generate a natural SPWM rectangular pulse sequence. Assuming that T_c is the period of the triangular carrier waveform, there are two intersections in one carrier period T_c , which are t_1 and t_2 , respectively. Then, the "ON" and "OFF" durations of each SPWM pulse t_{on1} , t_{on2} , t_{off1} , and t_{off2} are determined by

$$\begin{cases} t_{\text{off1}} = \frac{T_{\text{c}}}{4} (1 - m \sin(\omega t_1)) \\ t_{\text{on1}} = \frac{T_{\text{c}}}{4} (1 + m \sin(\omega t_1)) \\ t_{\text{on2}} = \frac{T_{\text{c}}}{4} (1 + m \sin(\omega t_2)) \\ t_{\text{off2}} = \frac{T_{\text{c}}}{4} (1 - m \sin(\omega t_2)) \end{cases}$$
(1.13)

where $m = V_{\rm rm}/V_{\rm cm}$ is the modulation ratio; $V_{\rm rm}$ is the peak amplitude of the sinusoidal modulation signal; $V_{\rm cm}$ is the peak amplitude of the triangular signal; m ranges from 0 to 1 and the larger the value it is, the higher the fundamental voltage of the output. ω is the angular frequency of the sine wave, which is the desired fundamental frequency of the inverter output voltage.



Fig. 1.2 Principle of the SPWM waveform generation



The pulse width of the generated SPWM wave in one period is

$$t_{\rm on} = t_{\rm on1} + t_{\rm on2} = \frac{T_{\rm c}}{2} \left(1 + \frac{m}{2} (\sin \omega t_1 + \sin \omega t_2) \right)$$
(1.14)

Since Eq. (1.14) is a transcendental equation, it will take a lot of time to solve it by the conventional numerical solution. Therefore, it has been considered that the mathematical model of the natural sampling method is not suitable for real-time control.

1.2.2 Symmetric Regular Sampling SPWM

The symmetric regular sampling SPWM method takes the time corresponding to the symmetry axis of each triangular wave as the sampling time. When a vertex is used as the sampling point, the generated pulse width is significantly smaller and the control error is larger, so the bottom point is usually used as the axis of symmetry [3].

As shown in Fig. 1.4, a line passing through the intersection point of the sine wave and the symmetry axis of the triangle wave is parallel to the time axis. The intersection of the parallel line and the triangular wave is sampled as the "ON" or "OFF" moment of the SPWM wave. Since these two intersections are symmetrical, the sampling method is called the symmetric regular sampling method.

From Fig. 1.4, the relationship can be obtained as follows

1.2 SPWM Methods

$$\frac{T_{\rm c}}{\delta} = \frac{2}{1 + m\sin(\omega t_{\rm D})} \tag{1.15}$$

where $\frac{\delta}{2} = t_{on1} = t_{on2}$. Thus, the pulse width can be defined by

$$\delta = \frac{T_{\rm c}}{2} \left(1 + m \sin\left((2n-1)\frac{\pi}{N}\right) \right) \tag{1.16}$$

where n = 1, 2, ..., N, $N = \frac{f_c}{f_r}$ is the frequency modulation ratio.

During one period of the triangular wave, the gap width on both sides of the pulse is

$$\delta' = \frac{1}{2}(T_{\rm c} - \delta) = \frac{T_{\rm c}}{4} \left(1 - m \sin\left((2n - 1)\frac{2\pi}{N}\right) \right)$$
(1.17)

For the three-phase half-bridge inverter, a three-phase SPWM wave should be formed. Usually, the triangular carriers are common to the three phases, and the three-phase sinusoidal modulated waves are sequentially $\frac{2\pi}{3}$ out of phase. The pulse widths of the three phases in the same triangular wave period are δ_a , δ_b and δ_c , respectively, and the gap widths are δ'_a , δ'_b and δ'_c , respectively. Since the sum of the three-phase sinusoidal modulating voltages is zero at any time, the following equation is established.

