Social Indicators Research Series 77

Adele Bianco
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Italian Studies on Quality of Life

Springer

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Volume 77

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Italian Studies on Quality of Life

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ISSN 1387-6570 ISSN 2215-0099 (electronic)
Social Indicators Research Series
ISBN 978-3-030-06021-3 ISBN 978-3-030-06022-0 (eBook)
https://doi.org/10.1007/978-3-030-06022-0

This Springer imprint is published by the registered company Springer Nature Switzerland AG.
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Acknowledgments

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Contents

Italian Studies on Quality of Life: An Interdisciplinary Overview

Adele Bianco, Paola Conigliaro, and Michela Gnaldi

The volume collects a selection of the papers presented at the conferences of the Italian Association for Quality of Life Studies (AIQUAV), held in Florence in 2015 and 2016. The aim is to offer an overview of the ways the Italian school of quality of life studies addresses various topics both in terms of content and quality.

Italy is known around the world for its artistic, natural and landscape beauties, for its good culinary art, and more generally for its aesthetic sense and good taste. "Made in Italy" is synonymous with high-quality manufacturing products, characterized by care and elegance. In the opinion of most people, Italy is the country of the "dolce vita" and because of these characteristics it can therefore be considered a natural and social laboratory of well-being and quality of life.

Despite this, Italy is affected by difficulties and suffers from a restriction of its potential development. After the global financial crisis of 2008–2013, Italy is recovering more slowly in comparison to other advanced countries (OECD 2011).

According to the OECD (2017), Italy seems to be a very contradictory country. Italy faces difficulties in the labor market—e.g. the employment rate of women and young people is one of the lowest among the OECD countries (https://data.oecd.org/emp/employment-rate.htm)—but in spite of it the level

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A. Bianco et al. (eds.), *Italian Studies on Quality of Life*, Social Indicators Research Series 77, https://doi.org/10.1007/978-3-030-06022-0_1

of wealth among Italian families is relatively high. The Italian population is one of the most long-lived. Recently the National Institute of Statistics (ISTAT, https://www.istat.it/it/archivio/204917) published evidence that healthy life expectancy is rising among people over sixty. More generally, Italians experience a good state of health. Nevertheless, they are dissatisfied with regards to work and school, environmental and living conditions, and safety. On the other hand, familial relationships and friendship networks are very satisfying and seem to be a "social capital" (Granovetter 1973; Bourdieu 1980; Godechot and Mariot 2004, pp. 243–282), a resource that can be used in case of need. The social sciences suggest possible welfare measures to improve these situations, as already suggested by Maggino and Nuvolati (2012).

Although they are very different from each other, the papers presented here offer an overview of the richness, breadth of the topics and of the multidisciplinary approach of Italian scholars engaged in these issues. These researchers and experts in different research fields come from Italian universities, public research centers such as ISTAT or CNR (National Research Council) and other associations.

The volume is structured as follows. The first part is devoted to *methods and indicators for research on the quality of life*, the second one to *social sustainability, lifestyles, cultural aspects and local applications* and the third section concerns *economy, welfare and quality of life.*

The topics covered in the first section range from the social inclusion of disadvantaged people, to economic aspects, to the quality of Italian provincial life, to issues related to transparency, corruption and its prevention. Despite the thematic heterogeneity the papers of the section share the same methodological rigor. This makes it possible to place the Italian contribution on the quality of life in the broader international scene.

In the first article of the first section, the contribution by Fattore and Zenga deals with new methods for the evaluation of complex traits, with data on financial literacy. The paper develops an innovative procedure for reducing the dimensionality of multidimensional ordinal data, by using the concept of "embedded scale": that is, deriving an evaluation scale against which to assess the individual level of literacy in the financial field. The procedure tries to pick-up the scale that minimizes the information loss in the dimensionality reduction process, in the logic of Principal Component Analysis, while carefully preserving the ordinal nature of the data. In this respect, the paper advances the development of a statistical toolbox for the treatment of multidimensional systems of ordinal attributes.

Mazziotta, Pallara and Solipaca focus on the theme of social inclusion of people with a disability, a key objective of the EU and the ONU Development Sustainable Goals. In this paper, the multidimensional and complex nature of social inclusion of people with a disability is handled through a synthetic measure of disparity, obtained through the Adjusted Mazziotta Pareto Index, between disabled and non-disabled people in EU15 countries. The aims are yielding a measurement of active inclusion of persons with disabilities and describing the interactions between health conditions and contextual factors affecting inclusion. The results show that there is a disparity between the two subpopulations throughout Europe, even though the situation across member states varies significantly.

The next paper by Costa, Declich, Marchesich and Osti proposes the use of the Mazziotta-Pareto Index to compare the well-being of Italian provinces. The Authors report results with province-based rankings and maps that allow identification of distinctive characteristics of well-being in various territories for comparative purposes, both within single domains and globally. The snapshot of Italy they show is not fully consistent with the stereotype of the rich North vs. poor South.

The subsequent three contributions of the first section deal with the measurement of corruption and transparency. In his paper, Carloni frames the issue of measuring corruption by discussing corruption indicators in the context of Italian prevention policies. The discussion starts by stressing the inadequacy of commonly adopted corruption indicators—such as "perception-based", objective and judicial measures—and concentrates on new indicators of administrative corruption, indicators of abnormality and risk of corruption, which signal a shift from a repressive enforcement approach, typical of traditional measures of corruption, to a broader and preventative approach. The development of these last measures arises from the acknowledgement that it is not as important to measure corruption, as to understand where it emerges from, and to issue an alert when such conditions arise. In the Italian context, the annual report filled in by the prevention-of-corruption supervisor within the "Piano Triennale per la Prevenzione della Corruzione" (PTPC) has to be considered one of such latest measures of corruption prevention, and is the object of analysis of the subsequent article by Gnaldi and Del Sarto. The Authors provide a synthetic measure of corruption prevention—the Composite Indicator of Corruption Prevention (CICP)—for a sample of 232 Italian municipalities. Afterwards, they classify and qualify them into homogeneous groups as regards to the adopted anti-corruption measures, relying on a Latent Class approach. They show that despite a general tendency to comply with the corruption prevention law, it is possible to identify a group of least virtuous municipalities showing very low values for the CICP, deserving special attention by the Italian Anticorruption Authority (ANAC).

Closely related to the issue of corruption prevention is that of transparency, which is the focus of the paper by Galli, Rizzo and Scaglioni who propose a first integrated study of transparency and Equitable and Sustainable Well-being (BES, https://www.istat.it/en/well-being-and-sustainability/the-measurement-of-well-being) indicators at the Italian regional level. Based on four different dimensions—transparency, digital, social and political—the Authors specifically assess whether transparency of public administrations is correlated with the quality of the socio-political environment. They provide initial evidence that there is a relationship between transparency and some BES indicators: that is, the intensity of Internet use, trust in institutions, social participation and, to a lower extent, civic and informal political participation and women's participation in municipal councils.

BES indicators are also at the core of the contribution proposed by Cataldo et al., who investigate the relative weight of some BES dimensions in shaping the perceived quality of life of citizens of the historic centre of Naples, an Italian southern city whose town centre has been recognized by UNESCO as a world heritage site. Within a Structural Equation Model approach, the Authors show that while quality of services, safety and social relations have a low impact on quality

of life perception, tourism—a dimension investigating citizens' views as regard to the effects of tourist flows in the area at issue—has the strongest impact among all dimensions.

In the article entitled "The incidence of sin taxes: evidence from Italy", di Bella, Gandullia, Leporatti and Montefiori discuss the fairness of the Italian taxation system as regards to sin goods (products such as tobacco, alcohol, and gambling), whose consumption is considered self-destructive for individuals and source of negative outcomes for the entire society. After investigating the Italian taxation on such goods, the Authors analyse and compare their degree of regressiveness using data from the Italian Household Budget Survey (HBS). Results show that, among such goods, those on gambling taxes can be considered the most regressive form of taxation, followed by cigarettes and beer, and that lower income households spend a larger proportion of their income on risky behaviours than higher income ones.

The article by Ravazzini and Piekałkiewicz closes the section and addresses the general question of whether gender should be included as a characteristic for the definition of reference groups, by focusing on an application on the impact of income comparisons on subjective well-being and using as data three pretests of the German Socio-Economic Panel. The Authors show that in any specifications, the effect of income comparisons on subjective well-being is better explained with the regression constructed without gender in the reference group than including gender and conclude that reference groups for income comparisons should not include gender as a relevant characteristic.

The second section is focused on the topic of social sustainability, lifestyles, and culture in Italy. A first group of contributions is devoted to the BES and a second one to food, an important element of Italian culture. This section is closed by contributions on the quality of life of the elderly population and on the Italian natural and landscape beauties.

The paper of Bachelet, Riccardini, Maggino focuses on how the concepts of vulnerability and resilience may be employed in the analysis of the sustainability of human well-being. The Authors propose an approach focusing on an individual dimension of well-being, considering single persons' exposure to risk and their ability to recover. Using data provided by the National Institute of Statistics (ISTAT), they offer an example of how to analyze the subjective aspects of wellbeing in terms of sustainability.

The contribution of Salustri and Viganò gives a multi-stakeholders' analysis of BES data. It concerns some subjective indicators on health regarding citizens based in Bozen (South Tyrol). The aim is to show convergences and divergences among four selected classes of agents (citizens, local stakeholders, central administrations and for-profit institutions), interdependencies among data on well-being and economic development, and the gap between the institutional indicators and a subjective approach, based on satisfaction surveys. It could help policy makers in promoting quality of life.

At the time of EXPO 2015, food has been the core of many studies. The subsequent three contributions of the second section deal with this basic aspect

of our life. Corazziari, Facioni, Maggino show the close relationship between the quality of life and the food habits of Italians, using data provided by the National Institute of Statistics.

Di Francesco focuses on the dietary habits of European university students in a comparative way. He points out that inappropriate dietary habits are associated with a set of behaviors (almost a syndrome) such as substance and / or medicines abuse, excessive use of social networks, frequent breaking of the sleep / waking cycle, psycho-physical disorders, irregular and excessively fatiguing forms of sports activities.

Corvo focuses on the link between food waste and quality of life. He points out that food waste is a feature of advanced countries and that this is a function of the productive system. After providing some official data on food waste, the Author shows that the dramatic consequences of the economic crisis are leading to a waste reduction.

A third group of contributions in the second section of the volume focuses on the ageing Italian population. In fact, Italy, as along with Germany and Japan, has the highest proportion of old people in the world. The papers by Cristini, Cesa-Bianchi and Dryjanska, Giua are respectively devoted to the standard of living of the elderly population and to the relationship between generations. The first two Authors show the close connection between culture, creativity and age. The history of art confirms this link. Michelangelo, for example, made his last masterpiece, the *Rondanini Pietà* at 89. Neuroscientific researches confirm that culture plays a role in mitigating neuropsychiatric diseases. The Authors point out that culture and creativity might perform various functions such as: (a) facilitate aging; (b) constitute a model for cross-generational relationships; (c) represent a therapeutic tool for the elderly patient, including those with dementia.

Dryjanska and Giua focus on intergenerational solidarity as the way to prevent cognitive decline and improve the quality of life. Considering the positive effects on the physical and cognitive level, the Authors report a case study based in Rome. The intergenerational relationship makes it possible to improve the quality of life of all the participants.

Intergenerational relationships are also addressed by Albanese and Bocci in their contribution dedicated to sustainable tourism. This paper is part of a series of research and experimental projects in intergenerational tourism started in 2000. The quality of tourism is related to natural and socio-cultural resources and could solve specific local problems and promote social-cultural learning experiences between participants of different generations. In particular, intergenerational holiday activities are focused on the local area, its history and environment with the aim of promoting it.

Ferrari, Bocci, Lepisto, Cavallero and Rombai draw attention to the perception and evaluation of the features of Italian natural and landscape beauties. In so doing they contribute to the study of environmental psychology and show that the topic is closely linked to the quality of life also in developing personal identity and the sense of belonging to a social group.

The third section of the volume deals with the quality of life related to economic and welfare aspects. Ciani Scarnicci, Laino, Belcaro and Vita examine in an original way the correlation between economic growth and indicators of quality, lifestyles and sustainability. Their starting point is that GDP is not an indicator of well-being and they propose to consider alternative indicators such as the Equitable and Sustainable Well-being. The environmental factors improving equity and sustainability should be considered in evaluating the evolution of a community.

The paper by Coli and Pacini deals with the new organization of Welfare systems experienced in the last 20 years. They discuss whether the administrative decentralization process has increased poverty in Europe.

Concerning the link between the quality of work and psycho-physical well-being, Conigliaro underlines the increasing relevance of working conditions in term of social sustainability, both for workers' and for clients' quality of life. She considers three clusters of features: universal rights, job quality, including general working conditions, and well-being in relation to work, also in terms of organizational well-being.

Giammanco and Gitto and Mussino and Cosmai address in two different papers the topic of health and lifestyles. Giammanco and Gitto focus on the quality of life of cancer patients. This is a relevant and sensitive topic due to the spread of the disease and its impact on quality of life. There are many different styles of application of therapeutic protocols in healthcare. The Authors present a study carried out at four operating units of the Oncology center of Catania Hospital. The 300 cancer patients were involved in a customer satisfaction survey. The results suggest that the uncertainty in illness impacts on patient satisfaction as well as patient participation and other service-related aspects.

Mussino and Cosmai make a comparison of active lifestyles within Europe. The importance of practicing sports for well-being and health is recognized. Regular physical activity can also be considered an investment in health and therefore a saving of welfare costs. The Authors compare the quantity and quality of sports practice with the different welfare models in Europe.

Borrelli's paper deals with the issue of sustainable energy sources. The Author focuses attention on the social acceptance of key technologies and its governance. Attention is devoted by him to the link between policies at macro (international and national) and micro (local) level. The sustainability of energy policies depends at the local level on the dialogue between citizens and decision-makers and between businesses and decision-makers.

Bianco's paper closes the volume. The question is whether per capita income and level of consumption could be considered the key indicators to describe the rise of the middle class and the improvement of the quality of life in developing countries. Evidences show that the condition for the rise of the middle class are a modern economic system and a social context where gender gaps are reduced and where women are respected and empowered.

The contributions collected here show that Italian studies on quality of life are interdisciplinary, fruitful and open to the complexity of the globalized world.

The curators—who shared the commitment and responsibility in implementing this volume and therefore are strictly alphabetically ordered—would like to thank the Authors and the reviewers who have collaborated on this volume.

Last but not least, a special thanks to our AIQUAV President Prof. Filomena Maggino for having supported us in the making of this volume.

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Part I
Methods and Indicators for Quality of Life Research

Quantification of Partially Ordered Data and the Evaluation of Financial Literacy

Marco Fattore and Mariangela Zenga

Abstract In this paper, we address the problem of the quantification of partially ordered data arising from multi-indicator systems of binary attributes and develop a procedure for the evaluation of Financial Literacy, on a real dataset pertaining to Italy. The procedure is based on the concept of "quantification" of chains (i.e. on the assignment of non-negative scores to sequences of comparable elements), in the poset arising from the attribute system, and aims at providing a two-dimensional representation of the data, in the spirit of a dimensionality reduction procedure. Due to the existence of incomparabilities in the poset, statistical units can be assessed only to some degree of indeterminacy and the goal of the procedure is to find out the "coordinate axes" that maximally reduce such indeterminacy.

Introduction

The use of Partially Ordered Set (poset) theory (Neggers and Kim 1998; Schröder 2002), in evaluation studies, has been recently investigated by several authors (Bachtrögler et al. 2014; Badinger and Reuter 2015; Bruggemann et al. 2014; Bruggemann and Patil 2011; Carlsen and Bruggemann 2016; Di Bella et al. 2016; Fattore et al. 2011, 2016; Fattore 2016; Fattore and Arcagni 2016; Iglesias et al. 2016; Patil and Taillie 2004), as a way to overcome the inconsistencies of composite indicators, particularly when ordinal attributes are involved. Following this stream, here we propose a procedure to quantify partially ordered data derived from multidimensional systems of attributes, in view of providing numerical evaluations of complex socio-economic traits. The idea is to map the elements of the poset associated to the attribute system into a bidimensional real space, by identifying a pair of "coordinate axes", as in classical data reduction algorithms. Here, however, axes coincide with suitable sequences of ordered profiles (chains), which get

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A. Bianco et al. (eds.), *Italian Studies on Quality of Life*, Social Indicators Research Series 77, https://doi.org/10.1007/978-3-030-06022-0_2

exogenously quantified (see (Knuth and Bahreyni 2014)), to serve as embedded measurement scales for the quantification of all poset elements (notice that our aim is not to reduce the complexity of the poset itself, like in the POSAC algorithm (Di Bella et al. 2016), but to quantify, in a parsimonious way, poset elements). As a first step in this direction, we focus on a particular class of posets, namely on product orders of binary attributes, since binary data are often encountered in socio-economics and since their mathematical features lead to powerful results that deserve a specific mention. To make the treatment as plain as possible, we develop the procedure on a real dataset pertaining to Financial Literacy, in Italy. Data are presented in section "Financial literacy data and the knowledge poset"; section "Chain quantification in product orders of binary attributes" introduces the concept of chain quantification; section "Data reduction through chain quantifications" develops the data reduction procedure; section "Bidimensional representation of knowledge data" introduces the bidimensional representation of financial data; section "Some technical comments on chain quantification" provides a brief discussion on some technical aspects related to quantification and section "Conclusion: open questions and further research" concludes. A final Appendix provides a pictorial example of the kind of posets employed in the paper.

Financial Literacy Data and the Knowledge Poset

The data used in this paper are extracted out of a survey on Financial Literacy, held in Italy in 2013 (see (VV.AA 2014)). The survey involved a sample of 1247 individuals and collected data on three main domains, namely *Financial Attitude*, *Financial Knowledge* and *Financial Behavior*. Financial Attitude refers to attention to medium/long term financial planning; Financial Knowledge pertains to basic technical competencies and to the comprehension of essential financial concepts; Financial Behaviour regards the way subjects choose financial instruments, make expenditure choices, manage their savings and the like. Socio-demographic data are also collected, for subpopulation analysis. Here, as a leading example to develop the posetic procedure, we focus on the Knowledge domain, which comprises six binary attributes related to the comprehension of the following items: (i) inflation, (ii) simple interest, (iii) compound interest, (iv) relation between risk and return, (v) investment diversification and (vi) investments comparison, based on risk-return profiles. Based on their knowledge (yes/no) of these concepts, statistical units are assigned binary sequences of six digits (0 = "no"; 1 = "yes"), in the following called *Knowledge profiles* (or, simply, profiles). These $2^6 = 64$ profiles are then structured into a partially ordered set, the *Knowledge poset*, according to the following criterion: *profile "a" has equal or less financial knowledge than profile "b", written* $a \preceq b$*, if and only if all of the scores of "a" are not higher than the*

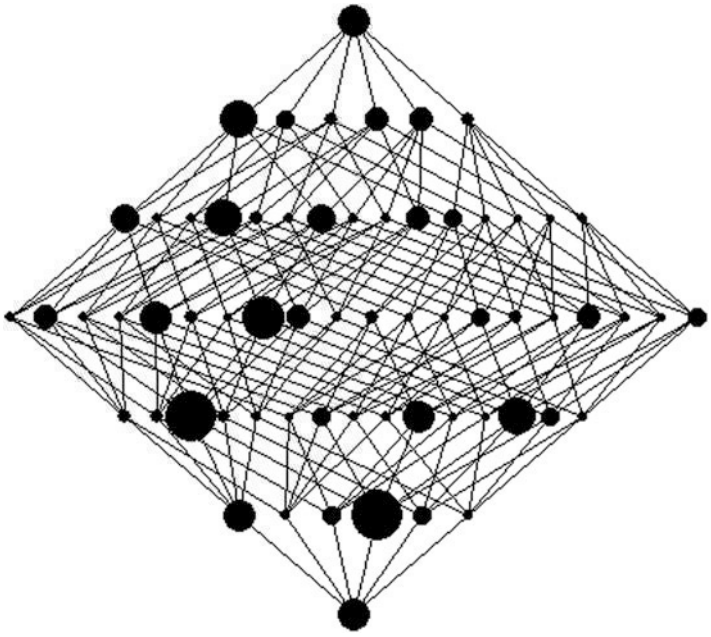

Fig. 1 Hasse diagram of the Knowledge poset (the greater the diameter of the node, the higher the frequency of the corresponding profile in the dataset)

corresponding scores of "b". This partial ordering criterion is very natural and is called the *product order* of (linear orders associated to) the input binary attributes (notice that this product order is also a lattice). Figure 1 depicts the Hasse diagram (see (Bruggemann et al. 1995)) of the Knowledge poset, where "better" profiles (represented as nodes) are linked to "worse" profiles by edges, to be read as downward oriented. Profiles not linked by downward sequences of edges are, instead, incomparable. The bottom element is profile 000000 (the respondent answered incorrectly to all of the questions), the top is profile 111111 (the respondent was able to correctly answer to all of the questions). Profiles on the same level of the Hasse diagram share the same number of 0s and 1s (the latter increases from bottom to top). For sake of readability, profiles have not been explicitly shown in the diagram (see Appendix for an example with profile labels); anyway, the picture is intended just to give an overall impression of the data structure. Each profile is realized by a different number of respondents; to visually account for that, the diameter of a node increases with the relative frequency associated to it. Only profile 011001 is not observed in the dataset (we notice, that this profile would correspond to a somehow incoherent set of answers, where one declares knowledge of investment comparison based on risk-return analysis, but not on what the relation between risk and investment is).

Chain Quantification in Product Orders of Binary Attributes

In this section, we introduce the concepts of *chain quantification* and of *induced interval quantification* of poset elements, which are at the heart of the data reduction procedure. To be concrete, we work directly on the Knowledge poset.

Maximal Chains and "Interval" Poset Quantification

A *chain* in a poset is a set of elements which are mutually comparable; in the corresponding Hasse diagram, it is thus a downward (or upward) sequence of nodes, linked by edges. A chain connecting the top node and the bottom node is here called a *maximal chain*.

We first define quantification on maximal chains, making them *measurement scales* embedded into the partially ordered set; then, we extend the quantification to the whole poset. To this goal, let γ be a maximal chain in the Knowledge poset Π (e.g., in Fig. 2, 000000 − 000100 − 100100 − 100110 − 110110 − 111110 − 111111, which is the maximal chain passing through the nodes with greatest diameters in Fig. 1). Let us *quantify* γ, defining a *monotone positive valuation* (from now on, a *valuation*), i.e. an order preserving map $v(\bullet)$ from Π to $[0, 1]$, so that $a \trianglelefteq b$ implies $v(a) \leq v(b)$. To make things simple, for $p \in \gamma$, we define $v(p)$ as the fraction of 1s in the profile (e.g. $v(010010) = 1/3$) and call it the *canonical quantification*; once defined $v(\bullet)$, any element a in the poset gets quantified, by the interval $\Delta_a(\gamma) = \left[v\left(\gamma_a^-\right), v\left(\gamma_a^+\right)\right]$, where γ_a^- is the greatest element of γ less than a and γ_a^+ is the lowest element of γ greater than a (see Fig. 2). Elements γ_a^+ and γ_a^- are also called *forward* and *backward projections* (see (Knuth and Bahreyni 2014)) of a on γ. Notice that if $a \in \gamma$, then $\gamma_a^- = \gamma_a^+ = a$ and the quantification interval is $[v(a), v(a)]$. In summary, given the quantification of γ, different profiles are assessed to different indeterminacy degrees, i.e. with different interval widths. This is a consequence of data multidimensionality that, in posets, reflects into the existence of incomparabilities among profiles. In particular, there are elements of the Knowledge poset which are quantified as $[0, 1]$. Consider, for example, profile $a = 010000$, whose forward and backward projections on γ are $\gamma_a^+ = 111111$ and $\gamma_a^- = 000000$, respectively. It should be clear that any profile p having 0 where the second element (from bottom) of γ has 1 and having 1 where the sixth element of γ has 0 is assigned the interval [0, 1] by γ[1]. So, for example, the entire chain $\lambda = 000000 - 000001 - 000011 - 001011 - 011011 - 111011 - 111111$ is composed of elements all of which are quantified as $[0, 1]$ (with the exception of 000000 and 111111, which are valuated as 0 and 1, respectively). In this case, we say that chain γ is uninformative on chain λ. On the whole, in the Knowledge

[1]More formally, this is true since the join between such p and any element of γ is 111111 and the meet is 000000.

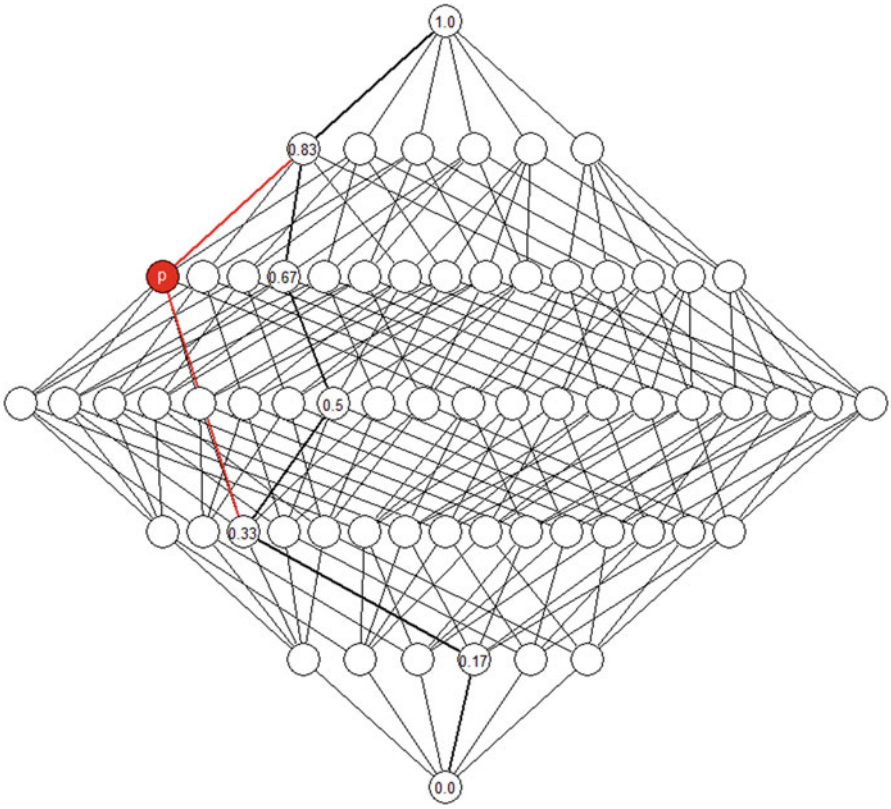

Fig. 2 Example of chain and profile quantification in the Knowledge poset. Given the chain γ_{max}, highlighted in black with valuations in white, profile p is assigned the valuation interval $[0.33, 0.83]$

poset there are 24 chains on which γ is uninformative (in a general product order of k binary attributes, for each maximal chain there are $(k-2)!$ chains on which the former is uninformative). Finally, it is easy to see that uninformativeness is a symmetric relation, i.e. if γ is uninformative on λ, then λ is uninformative on γ (to see this, let be γ uninformative on λ: if λ would be informative on γ, there would exist $q \in \lambda$ and $p \in \gamma$, both different from 000000 and 111111, such that $\gamma_p^+ = q$, or $\gamma_p^- = q$; but then certainly either $p \trianglelefteq \gamma_q^-$, or $\gamma_q^+ \trianglelefteq p$, i.e. γ would be informative on λ).

To summarize, in product orders of binary attributes, quantifying a maximal chain allows to assign quantification intervals to each poset element and, also, to identify a subset of maximal chains which are mutually uninformative with it.

Data Reduction Through Chain Quantifications

The Knowledge poset Π is highly symmetric, in that its structure and the shape of its Hasse diagram do not depend upon the way attributes are listed. As a result, no maximal chain has any preferred role and there is no *a priori* criterion to pick one up, in order to quantify the poset. However, the frequency distribution of the statistical units is not homogeneous on Π and this does provide a way to break the symmetry and to identify one, or more, distinguished chains. For simplicity's sake, we develop the discussion in four distinct steps.

Canonical Quantification

As a first step, suppose to assign the following valuation on the bottom and the top profiles of Π only: $v(000000) = 0$; $v(111111) = 1$. From this "baseline" quantification, one can only deduce that $0 \leq v(a) \leq 1$, for every knowledge profile a (since any of them satisfies $000000 \leq a \leq 111111$). Each profile is (possibly) realized by one or more statistical units, which inherit the quantification interval $[0, 1]$ as well. Suppose now to introduce a quantified chain. A generic profile a and the statistical units sharing it get associated the quantification interval $\Delta_a(\gamma) = \left[v\left(\gamma_a^-\right), v\left(\gamma_a^+\right)\right]$, where in general $0 \leq v\left(\gamma_a^-\right) \leq v\left(\gamma_a^+\right) \leq 1$ (we can think of $\Delta(\gamma)$ as a map from Π to intervals comprised in $[0, 1]$). Thus, introducing a quantified chain reduces the width of quantification intervals (indeterminacy) associated to profiles and statistical units.

In view of data reduction, it is then natural to search for the quantified chain that maximally reduces the population indeterminacy, given the frequency distribution on Π. Such a chain will be called a *maximum information chain*. Here, however, an issue arises. The indeterminacy reduction induced by a chain depends upon (i) the chain itself and (ii) how it is quantified. In principle, one could quantify the chain so as to reflect exogenous evaluation criteria or, in Bayesian terms, some prior knowledge. When developing an evaluation procedure to be used by "domain experts", the possibility to fix the embedded scale should be highly appreciable. Here, however, we simply look for a "neutral" rule; we thus quantify profiles canonically, from 0 to 1, assigning to each profile the frequency of 1s in it, as we did in the previous example.

Indeterminacy Reduction Power of a Quantified Chain

With the quantification choice discussed in the previous paragraph, we search for the maximum information chain on the Knowledge poset, by listing all of the maximal chains (their number being $6! = 720$) and comparing their Indeterminacy Reduction Power *IRP*, defined as:

$$IRP\,(\gamma) = 1 - \frac{\sum_{a\in\Pi}\left[v\left(\gamma_a^+\right) - v\left(\gamma_a^-\right)\right]\cdot n_a}{\sum_{a\neq\perp, a\neq\top} n_a} \tag{1}$$

In the above expression, the numerator is the sum of profiles' interval widths along γ, weighted by the number of statistical units in the profiles, and the denominator is the same for the baseline quantification (since, in this case, the score range associated to profiles is 1, but for the bottom ($\perp$) and top ($\top$) profiles, which are scored with no indeterminacy). Figure 3 shows the *IRP*s of all of the maximal chains, sorted in a decreasing way. The chain γ_{max} with maximum *IRP* (0.656) turns out to be $000000 - 000100 - 100100 - 100110 - 101110 - 111110 - 111111$. By projecting poset profiles on γ_{max}, we get different valuation intervals for different profiles. In particular, 7 profiles (those on the identified chain) are quantified with no indeterminacy, 5 profiles have quantification ranges equal to $0.\overline{3}$, 8 profiles have ranges equal to 0.5, 12 equal to $0.\overline{6}$, 16 equal to $0.8\overline{3}$ and 16 equal to 1 (Fig. 4). The latter ones are profiles over which chain γ_{max} is uninformative and whose existence was anticipated in the previous paragraph.

Indeterminacy Reduction Power of a Pair of Quantified Chains

After extracting chain γ_{max}, it would be natural to search for a second chain, which quantifies with a small indeterminacy degree profiles and statistical units "badly" (i.e. with wide intervals) quantified by the maximum information chain.

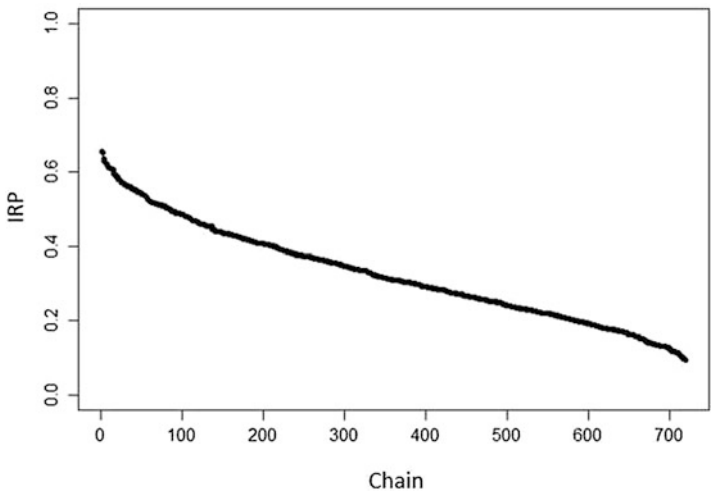

Fig. 3 Indeterminacy reduction power of maximal chains of the Knowledge poset, sorted in a decreasing way

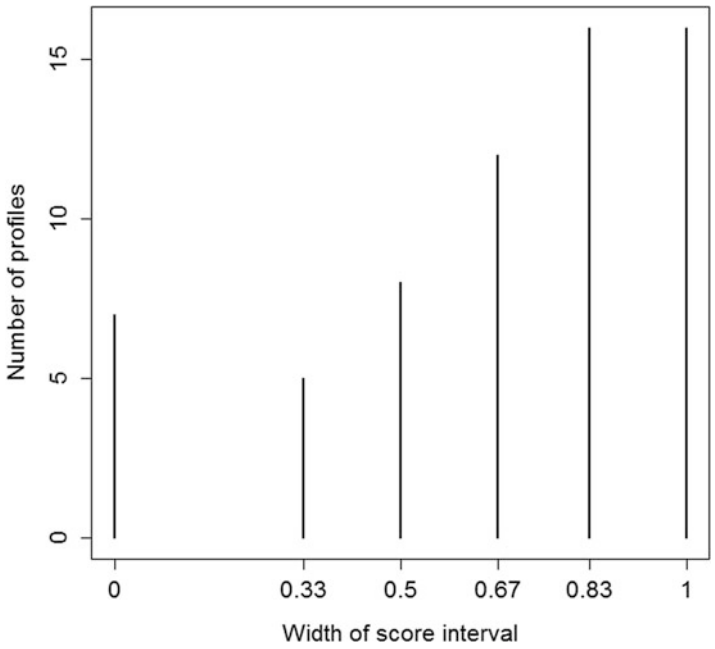

Fig. 4 Frequency distribution of the width of score intervals against γ_{max}

Profiles with wide intervals on the first chain (i.e. on which the first chain is uninformative or almost uninformative), could get narrower intervals on the second chain, so reducing the overall indeterminacy of the poset quantification. To make this idea formal, however, we first have to define a way to measure quantification indeterminacy, when two chains are involved. Suppose $\Delta_a(\gamma_1)$ and $\Delta_a(\gamma_2)$ are the interval quantifications of element a on two chains γ_1 and γ_2. The most natural way to define the global quantification indeterminacy of a is through the geometric mean $\Delta_a(\gamma_1\gamma_2) = \sqrt{\Delta_a(\gamma_1)\,\Delta_a(\gamma_2)}$ (see (Knuth and Bahreyni 2014)). Analogously to what done before, we define the *IRP* of the pair of chains γ_1 and γ_2 as:

$$IRP(\gamma_1\gamma_2) = 1 - \frac{\sum_{a\in\Pi} \Delta_a(\gamma_1\gamma_2)\cdot n_a}{\sum_{a\neq\perp, a\neq\top} n_a}.$$

Mutual Uninformativeness: Differences and Analogies with Classical Data Analysis

Among all pairs of maximal chains, it is natural to search for that maximizing the *IRP* and such that the two chains are mutually uninformative; otherwise, the

second chain could be "redundant", quantifying with narrow intervals profiles and statistical units already "well" quantified by the first chain. In a sense, this is analogous to what is typically done in euclidean data analysis, where orthogonal (or uncorrelated) components are often looked for. In euclidean data analysis the measure of total variation (e.g. total variance) can be reconstructed optimally, by choosing suitable orthogonal components, extracted in sequence. In the poset case, however, there is no concept of "variance decomposition" and we cannot minimize the global indeterminacy of the resulting quantification $\Delta_a(\gamma_1\gamma_2)$, by searching γ_2 among the maximal chains uninformative with respect to the maximum information chain γ_{max}. In order to minimize $\Delta_a(\gamma_1\gamma_2)$, we have to search directly among all of the pairs of maximal, and mutually uninformative, chains. To this goal, for each maximal chain of the Knowledge poset, we first computed the set of 24 corresponding uninformative chains; then, we searched for the best pair. The pair of chains γ_1 and γ_2 which minimizes the global indeterminacy is composed of $\gamma_1 = 000000 - 100000 - 100100 - 100110 - 110110 - 110111 - 111111$ and $\gamma_2 = 000000 - 001000 - 001100 - 001110 - 011110 - 011111 - 111111$ (see Fig. 5). The indeterminacy reduction power $IRP(\gamma_1\gamma_2)$ of this pair is equal[2] to 0.678. We see that, in this case, the improvement over $IRP(\gamma_{max})$ is negligible. However, the choice of two chains has the positive effect to make profile uncertainties much more homogeneous, than in the single chain case. In fact, no profile has indeterminacy equal to 1 and, in general, the indeterminacy distribution is much less spread, as shown in Fig. 6. Moreover, as discussed later, the "financial" interpretation of the result becomes much easier.

Bidimensional Representation of Knowledge Data

Chains γ_1 and γ_2 can be used as two axes, on which to represent financial literacy data. To each statistical unit, an interval on γ_1 and an interval on γ_2 correspond, providing an insight on the levels and the indeterminacy of its quantification. Since the Knowledge poset comprises 64 profiles, statistical units are grouped and there are 64 different possible pairs of intervals on the two axes. Figure 7 shows the width of the intervals along γ_1 and γ_2 for each profile (some jittering has been added

[2]The minimum *IRP* equals 0.311, thus showing that the range of possible *IRP* values across mutually uninformative chain pairs is quite wide.

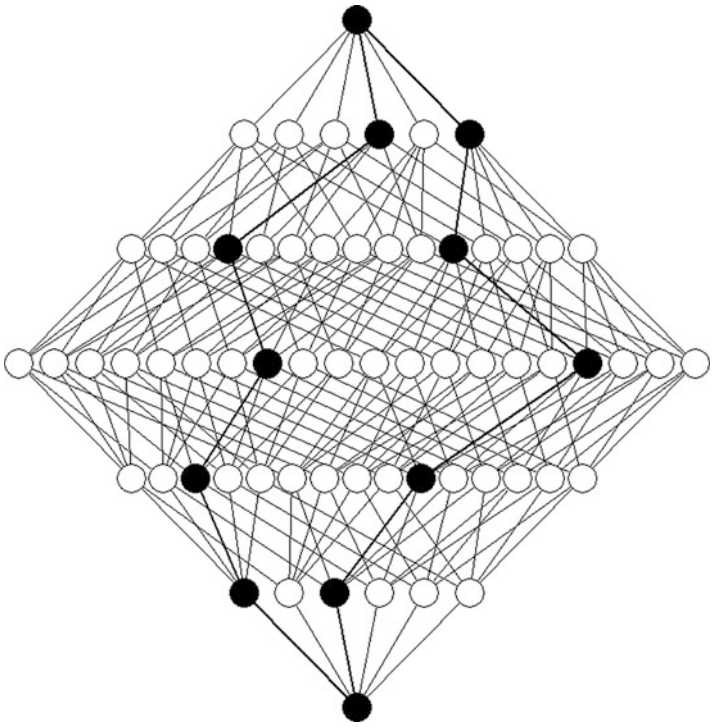

Fig. 5 Maximum information chain pair; the chain on the left, in the poset, is γ_1; that on the right is γ_2

to show them in a distinct way), weighted by the corresponding frequencies[3]. The only profiles with no indeterminacy on both chains are 000000 and 111111; the other elements of γ_1 and γ_2 have indeterminacy 0 on one axis and indeterminacy 1 on the other. All of the other knowledge profiles have different indeterminacy levels and one can identify "badly", i.e. with great indeterminacy, quantified elements (right upper corner) and better, i.e. with low indeterminacy, quantified ones (towards the center of the plot).

A different representation of the Knowledge profiles is provided in Figure 8. There, profiles are represented as rectangles, whose sides are the score intervals on chains γ_1 and γ_2, respectively. Thus, the plot conveys information on both the level and the indeterminacy of profiles' quantification (clearly, when indeterminacy is high, profile level is not informative). In view of communication to decision-makers,

[3]To ease visualization, weights are actually proportional to frequencies at power 0.6; the plot is meant just to provide a general idea of the frequency distribution.

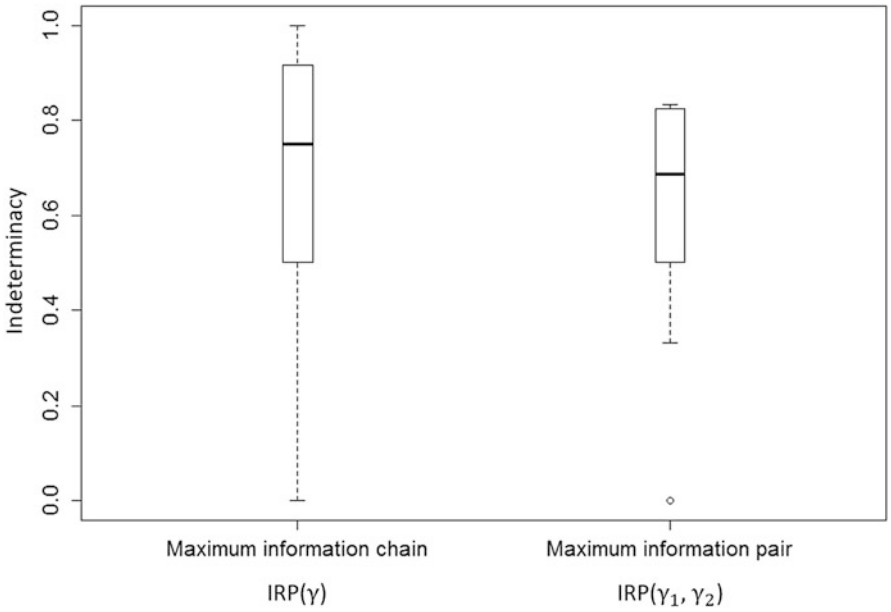

Fig. 6 Boxplot of indeterminacy distributions for the maximum information chain ($\gamma = \gamma_{max}$) and the maximum information chain pair γ_1 and γ_2

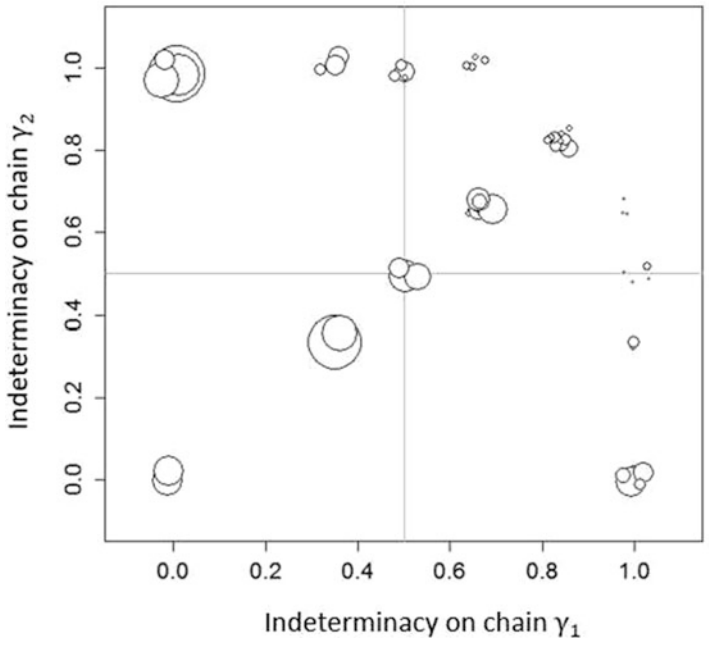

Fig. 7 Width of intervals associated to Knowledge profiles, by quantification on the best pair of chains γ_1 and γ_2 (the bigger the circle, the higher the observed frequency of that profile)

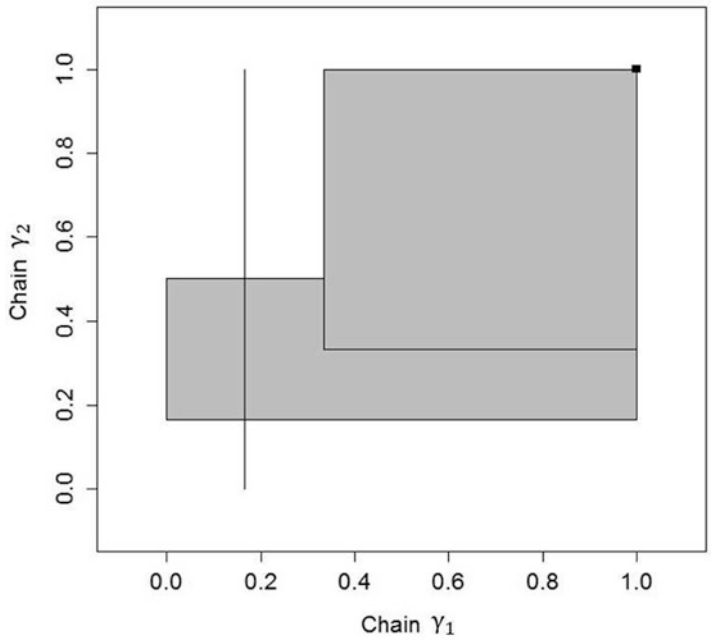

Fig. 8 Plot of profiles against chains γ_1 and γ_2. Each profile is represented as a rectangle (possibly degenerate) whose sides are the score intervals on the axes. To ease readability, just a few profiles have been plotted, namely: 111111 (the small black square top right); 001010 (the large horizontal rectangle, partly hidden), 100000 (the vertical line, i.e. a degenerate rectangle); 111100 (the large square)

such an "indeterminacy-preserving" representation is likely to be more useful than providing classical composite scores.

It is interesting to comment on the two chains γ_1 and γ_2, seen as reference scales for the evaluation of Financial Literacy. To give an interpretation of chain γ_1, we look at the sequence of competence additions, from the "no competencies at all" profile (the bottom of the Knowledge poset) to the "full competence" profile (the top of the Knowledge poset). The competence sequence is the following: "no competencies" (000000) → "inflation" (100000) → "relation between risk and return" (100100) → "investment diversification" (100110) → "simple interest" (110110) → "investments comparison, based on risk-return profiles" (110111) → "compound interest" (111111). Read the other way around, the sequence can be seen as a chain of implications; for example, if a respondent declares knowledge of what simple interest is, then he/she also declares adequate knowledge of inflation, relation between risk and return and investment diversification. The interpretation of the chain is quite straightforward. The first three items refer to qualitative aspects of finance, ordered by sophistication; the last three items are of a more quantitative