MATRIX Book Series 2

David R. Wood *Editor-in-chief*
Jan de Gier · Cheryl E. Praeger
Terence Tao *Editors*

2017 MATRIX Annals

MATRIX
Springer

MATRIX Book Series 2

MATRIX is Australia's international and residential mathematical research institute. It facilitates new collaborations and mathematical advances through intensive residential research programs, each lasting 1–4 weeks.

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Preface

MATRIX is Australia's international and residential mathematical research institute. It was established in 2015 and launched in 2016 as a joint partnership between Monash University and The University of Melbourne, with seed funding from the ARC Centre of Excellence for Mathematical and Statistical Frontiers. The purpose of MATRIX is to facilitate new collaborations and mathematical advances through intensive residential research programs, which are currently held in Creswick, a small town nestled in the beautiful forests of the Macedon Ranges, 130 km west of Melbourne.

This book is a scientific record of the eight programs held at MATRIX in 2017:

- *Hypergeometric Motives and Calabi–Yau Differential Equations*
- *Computational Inverse Problems*
- *Integrability in Low-Dimensional Quantum Systems*
- *Elliptic Partial Differential Equations of Second Order: Celebrating 40 Years of Gilbarg and Trudinger's Book*
- *Combinatorics, Statistical Mechanics, and Conformal Field Theory*
- *Mathematics of Risk*
- *Tutte Centenary Retreat*
- *Geometric R-Matrices: from Geometry to Probability*

The MATRIX Scientific Committee selected these programs based on scientific excellence and the participation rate of high-profile international participants. This committee consists of: Jan de Gier (Melbourne University, Chair), Ben Andrews (Australian National University), Darren Crowdy (Imperial College London), Hans De Sterck (Monash University), Alison Etheridge (University of Oxford), Gary Froyland (University of New South Wales), Liza Levina (University of Michigan), Kerrie Mengersen (Queensland University of Technology), Arun Ram (University of Melbourne), Joshua Ross (University of Adelaide), Terence Tao (University of California, Los Angeles), Ole Warnaar (University of Queensland), and David Wood (Monash University).

These programs involved organisers from a variety of Australian universities, including Australian National University, Monash University, Queensland University of Technology, University of Newcastle, University of Melbourne, University of Queensland, University of Sydney, University of Technology Sydney, and University of Western Australia, along with international organisers and participants.

Each program lasted 1–4 weeks, and included ample unstructured time to encourage collaborative research. Some of the longer programs had an embedded conference or lecture series. All participants were encouraged to submit articles to the MATRIX Annals.

The articles were grouped into refereed contributions and other contributions. Refereed articles contain original results or reviews on a topic related to the MATRIX program. The other contributions are typically lecture notes or short articles based on talks or activities at MATRIX. A guest editor organised appropriate refereeing and ensured the scientific quality of submitted articles arising from each program. The Editors (Jan de Gier, Cheryl E. Praeger, Terence Tao and myself) finally evaluated and approved the papers.

Many thanks to the authors and to the guest editors for their wonderful work.

MATRIX is hosting eight programs in 2018, with more to come in 2019; see www.matrix-inst.org.au. Our goal is to facilitate collaboration between researchers in universities and industry, and increase the international impact of Australian research in the mathematical sciences.

David R. Wood
MATRIX Book Series Editor-in-Chief

Hypergeometric Motives and Calabi–Yau Differential Equations

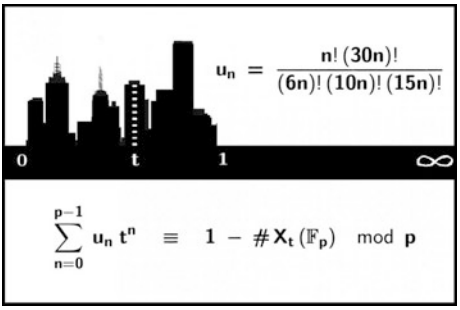
8–27 January 2017

Organisers

Ling Long
Louisiana State Uni

Masha Vlasenko
Institute of Mathematics of the Polish Academy of Sciences

Wadim Zudilin
Uni Newcastle



The majority of the articles presented below are extended abstracts of the talks given by program participants at the workshop that took place from January 16 to 20, 2017. Some of them present a new perspective or results that appeared due to collaboration following the activity in Creswick.

The two main topics of the program, Calabi–Yau differential equations and hypergeometric motives, provide an explicit approach and experimental ground to such important themes in contemporary arithmetic geometry as the Langlands program, motives and mirror symmetry. Hypergeometric motives are families of motives whose periods are given by generalised hypergeometric functions. Their L-functions are expected to cover a wide range of known L-functions. Due to the recent work of researchers (many of whom were present in Creswick) it is now possible to compute L-functions of hypergeometric motives efficiently. Thus one can test the standard conjectures, e.g. on special values and modularity, for motives of any degree and weight. Many algorithms for computing with the hypergeometric motives are now implemented in the computer algebra system `Magma`.

Local factors of hypergeometric L-functions can be investigated by the means of finite hypergeometric functions, another topic to which a few articles in this volume are devoted. The techniques developed by the authors allow to transport classical formulas to the finite field setting, count points on algebraic varieties over finite fields, study their congruence properties and Galois representations. Importantly, finite hypergeometric functions can be viewed as periods of motives over finite fields.

Periods over finite fields form a new angle of understanding the integrality phenomenon arising in mirror symmetry. Originally discovered by physicists in the mid 1980s, mirror symmetry remains one of the central research themes binding string theory and algebraic geometry. Numerous examples show that the expression of the mirror map in so-called canonical coordinates possesses rich arithmetic properties. This expression involves particular solutions to a Picard–Fuchs differential equation of a family of Calabi–Yau manifolds near a singular point. Application of p-adic methods to the study of Calabi–Yau differential equations gives a very promising prospective, as it is announced in the final article by Duco van Straten.

The three weeks at the MATRIX institute were intense and fruitful. To illustrate these words, there was a special lecture by Fernando Rodriguez Villegas scheduled at the very last moment on Thursday afternoon of the workshop week, in which he presented, jointly with David Roberts and Mark Watkins, a new conjecture on motivic supercongruences that was invented in Creswick. This talk influenced what happened in the last week of the workshop. David Broadhurst gave his two lectures on the very first and very last days of the program, reporting in the second talk on the tremendous progress achieved by him in collaboration with David Roberts over the three weeks.

We are confident that ideas and projects that emerged during the program will drive our field of research in the coming years.

Masha Vlasenko
Guest Editor

Participants

James Wan (SUTD, Singapore), Fang-Ting Tu (Louisiana State), Yifan Yang (National Chiao Tung University), Éric Delaygue (Institut Camille Jordan, Lyon), John Voight (Dartmouth), Adriana Salerno (Bates College), Alex Ghitza (Melbourne), Mark Watkins (Sydney), Piotr Achinger (IHES) with Helena, Jan de Gier (Melbourne), David Broadhurst (Open University), Ole Warnaar (Queensland), Ravi Ramakrishna (Cornell), Fernando Rodriguez Villegas (ICTP, Trieste), Sharon Frechette (College of the Holy Cross), Robert Osburn (University College Dublin), Frits Beukers (Utrecht), Paul Norbury (Melbourne), David Roberts (Minnesota Morris), Duco van Straten (Johannes Gutenberg), Holly Swisher (Oregon State), Abdellah Sebbar (Ottawa)

Computational Inverse Problems

11–23 June 2017

Organisers

Tiangang Cui
Monash Uni

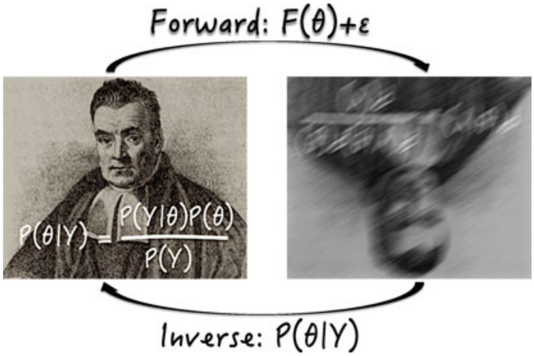
Hans De Sterck
Monash Uni

Markus Hegland
Australian National Uni

Youssef Marzouk
Massachusetts Inst Tech

Ian Turner
Queensland Uni Tech

Karen Willcox
Massachusetts Inst Tech



The integration of complex data sets into large-scale computational models is one of the central challenges of modern applied mathematics. This challenge is present in almost every application area within science and engineering, e.g., geosciences, biological systems, astrophysics, meteorology, aerospace, and subsurface flow. At the heart of this challenge often lies an inverse problem: we seek to convert indirect data into useful characterisations of the unknown model parameters including source terms, initial or boundary conditions, model structure, physical coefficients, etc. Solution of the inverse problem, along with model prediction and uncertainty assessment, can be cast in a Bayesian setting and thus naturally characterised by the posterior distribution over unknown parameters conditioned on the data. Unfortunately, solution of such statistical inverse problems for systems governed by large-scale, complex computational models has traditionally been intractable: models are complicated and computationally expensive to evaluate; available indirect data are often limited, noisy, and subject to natural variation; inversion algorithms often scale poorly to high-dimensional, or in principle infinite- dimensional, model parameters.

Our program contributed to the active international research effort in computational mathematics to connect theoretical developments with algorithmic advancements, buoyed by a range of cutting-edge applications. The program attracted a total of 47 attendees from a diverse range of highly relevant fields. Our attendees include renowned researchers in numerical analysis, scientific computing, optimisation, and stochastic computation, as well as high profile domain experts working in meteorology, super-resolution imaging, aerospace, and subsurface. The program began with a week of mini-conference. Seven 45-min plenary presentations and twenty 30-min invited presentations were scheduled during the mini-conference. In the second week, we organised thirteen 45-min presentations in the mornings and reserved afternoons for collaboration.

During the program, our attendees presented and extensively collaborated the following key topics in computational inverse problems:

- Deterministic and statistical methods for inverse problems.
- Advanced Markov chain Monte Carlo and quasi Monte Carlo methods.
- Optimal transport theory and its current and potential applications in inverse problems.
- Model reduction methods and multi-scale methods.
- Scalable experimental design methods.
- High performance numerical solvers, including multilevel methods.
- Applications in geothermal engineering, additive manufacturing, aeronautics, remote sensing, and super-resolution imaging.

The articles in this proceedings represent different aspects of the program. For example, Bardsley and Cui describe an optimisation-based methods for nonlinear hierarchical Bayesian inverse problem, Fox et al. presents a novel methods for sequential inverse problems using the Frobenius-Perron operator, MacNamara, McLean and Burrage present an adaptive contour integration methods for solving master equations, Guo, Loeper, and Wang present initial investigations of using optimal transport to solve inverse problems in finance, Harrach and Rieger present a set optimisation technique for reconstructing electrical impedance tomography image using single-measurement, Haario et al. investigates new ideas on characterising chaotic stochastic differential equations, Lamminpää et al. presents a case study on the atmospheric remote sensing, Ye, Roosta-Khorasani, and Cui present an extensive survey on optimisation methods used in inverse problems.

We would like to thank all of the authors who took the time to contribute to this volume. We would also like to thank the MATRIX staff and officials for hosting and facilitating this wonderful event and giving us the opportunity to share our work with this volume.

Tiangang Cui and Hans De Sterck
Guest Editors

Participants

Bart van Bloemen Waanders, Benjamin Peherstorfer, Colin Fox, Gregoire Loeper, Habib N. Najm, Harriet LI, Heikki Haario, Janosch Rieger, Jinglai LI, John Bardsley, Josef Dick, Kate Lee, Kody Law, Lutz Gross, Marko Laine, Nan Ye, Oliver Maclaren, Olivier Zahm, Omar Ghattas, Qinian Jin, Tianhai Tan, Tim Garoni, Zheng Wang, Elizabeth Qian, Gianluca Detommaso, Ruanui Nicholson, Elvar Bjarkason, Fred Roosta, Shev MacNamara, Alessio Spantini, Amani Alahmadi, Hoang Viet Ha, Mia (Xin) Shu, Carl (Ao) Shu, Thien Binh Nguyen, Oliver Krzysik, Brad Marvin, Ellen B Le, Jesse Adams, Hongbo Xie, Hans Elmlund, Cyril Reboul

Integrability in Low-Dimensional Quantum Systems

26 June–21 July 2017

Organisers

Murray Batchelor
Australian National Uni

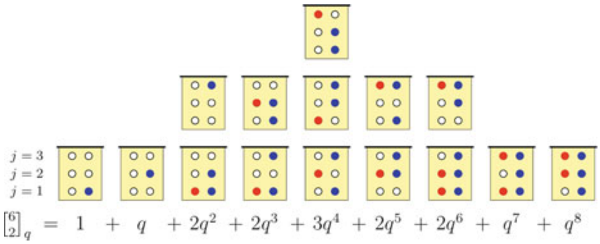
Patrick Dorey
Uni Durham

Giuseppe Mussardo
SISSA Trieste

Paul Pearce
Uni Melbourne

Chaiho Rim
Sogang, Seoul

Clare Dunning
Uni Kent



This MATRIX program focused on aspects of integrability in low-dimensional quantum systems and areas of application. It was organized around currently active hot topics and open problems. The emphasis was on focused research and interaction in small groups to achieve real collaboration. The research topics included:

- AdS/CFT
- Bethe ansatz and quantum spin chains
- Bulk and boundary conformal and quantum field theory
- Cold atoms, strongly correlated systems
- Integrability in models of matter-light interaction
- Logarithmic CFT
- ODE/IM and its massive variants
- Quantum quenches and quantum entanglement
- Random matrix approach to CFT and integrability

Among the integrability community, this workshop was a major event on the international scene and enabled us to bring together scientists at the leading edge of research in integrable quantum systems in low-dimensions. Indeed, with 59 participants over 3 weeks, a significant proportion of the active world-wide community working on quantum integrability was in attendance.

Classical integrabilty of two-dimensional systems and the related quantum integrability of one-dimensional systems are finding areas of application in statistical physics, condensed matter physics and particle physics in addition to contributing to highly mathematical topics such as Yang-Baxter algebras, quantum groups, cluster algebras, affine Lie algebras and combinatorial representation theory. With a series of Introductory Lectures on Hot Topics and advanced seminars, this workshop offered extensive training to graduate students and Early Career Researchers working in integrability and related topics.

Highlights, among many of the meeting, include the announcement of (1) the analytic calculation of the conformal partition functions of two-dimensional critical percolation, (2) the demonstration of the quantum toroidal integrability behind the AGT correspondence as well as (3) some striking progress on the mathematical description of fusion within the affine Temperly-Lieb algebra. Contributed articles included in these MATRIX Annals cover the topics of (1) form factors, (2) the combinatorics and generating functions of RNA structures, (3) supersymmetric quantum chains and (4) proofs of factorization and sum-to-1 properties of the $A_n^{(1)}$ face models. During the program there were also several groups of collaborators informally reporting rapid progress including (1) a collaboration explaining the mysteries of Baxter's Q-matrix for $sl(2)$ models at roots of unity and (2) a collaboration deriving analytically the correlation functions and conformal weights of critical dense polymers. Many physical applications to quantum quenches, ultracold atoms and matter-light interaction were also showcased during the meeting. All of these represent significant advancement in our discipline.

We gratefully acknowledge the generous support of our sponsors—MATRIX, the Australian Mathematical Sciences Institute (AMSI), the Australian Mathematical Society (AustMS) and the Asia Pacific Center for Theoretical Physics (APCTP). We particularly thank Jan de Gier for his encouragement in bringing this program together. We also thank the very helpful MATRIX staff at Melbourne and Creswick campuses, as well as our outstanding chef Adam, for their many significant contributions to the success of this meeting. Lastly, we thank the authors who kindly took the time and made the effort to contribute to this volume.

Chaiho Rim and Paul Pearce
Guest Editors

Participants

Changrim Ahn (Ewha, Seoul, Korea), Zoltan Bajnok (Wigner, Hungary), Jean-Emile Bourgine (KIAS, Seoul, Korea), Daniel Braak (Augsburg, Germany), Jun-peng Cao (CAS, Beijing, China), Sang Kwan Choi (Sichuan University, China), Ed Corrigan (York, UK), György Fehér (Budapest, Hungary), Angela Foerster (Rio Grande do Sul, Brazil), Holger Frahm (Hannover, Germany), Azat Gainutdinov (Tours, France), Frank Göhmann (Wuppertal, Germany), Xiwen Guan (CAS, Wuhan, China), Jesper Jacobsen (ENS, Paris, France), Shashank Kanade (Alberta, Canada), Andreas Klümpe (Wuppertal, Germany), Karol Kozlowski (Lyon, France), Atsuo Kuniba (Tokyo, Japan), Masahide Manabe (Warsaw, Poland), Chihiro Matsui (Tokyo, Japan), Yutaka Matsuo (Tokyo, Japan), Jianin Mei (Dalian, China), Alexi Morin-Duchesne (Louvain, Belgium), Rafael Nepomechie (Miami, USA), Ovidiu Patu (Bucharest, Romania), Francesco Ravanini (Bologna, Italy), Yvan Saint-Aubin (Montréal, Canada), Kareljan Schoutens (Amsterdam, Netherlands), Junji Suzuki (Shizuoka, Japan), Gabor Takacs (Budapest, Hungary), Masato Wakayama (Kyushu, Japan), Yupeng Wang (CAS, Beijing, China), Waltraut Wustmann (Maryland, USA), Wen-Li Yang (Xian, China), Hong Zhang (ITP, Beijing, China), Huan-Xiang Zhou (Chongqing, China), Rui-Dong Zhu (Tokyo, Japan), Zeying Chen (Uni Melbourne), Jan de Gier (Uni Melbourne), Omar Foda (Uni Melbourne), Alexandr Garbali (Uni Melbourne), Phil Isaac (Uni Queensland), Kazuya Kawasetsu (Uni Melbourne), Sergii Koval (Australian National Uni), Jon Links (Uni Queensland), Tianshu Liu (Uni Melbourne), Vladimir Mangazeev (Australian National Uni), Thomas Quella (Uni Melbourne), Jorgen Rasmussen (Uni Queensland), David Ridout (Uni Melbourne), Boris Runov (Australian National Uni), William Stewart (Uni Melbourne), Michael Wheeler (Uni Melbourne), Paul Zinn-Justin (Uni Melbourne)

Elliptic Partial Differential Equations of Second Order: Celebrating 40 Years of Gilbarg and Trudinger's Book

16–28 October 2017

Organisers

Lucio Boccardo
Sapienza Uni Roma

Florica-Corina Cirstea
Uni Sydney

Julie Clutterbuck
Monash Uni

L. Craig Evans
Uni California Berkeley

Enrico Valdinoci
Uni Melbourne

Paul Bryan
Macquarie Uni

Our program celebrated the 40th anniversary of the publication of Gilbarg and Trudinger's highly influential "Elliptic Partial Differential Equations of Second Order", one of the most highly cited texts in mathematics (over 10,000 citations). We sought to link past research with future perspectives, by discussing what the important developments in the area during these 40 years have been and what are the new trends of contemporary research. Particular attention was given to some of the topics in which the book served as a great source of inspiration, such as fully nonlinear PDEs, viscosity solutions, Hessian equations, optimal transport, stochastic point of view, geometric flows, and so on.

The first week of the program consisted of a series of introductory lectures aimed at Ph.D. students and postdocs, featuring in particular lectures given by Neil Trudinger himself. Special thanks go to Connor Mooney who gave a beautiful series of lectures with only 24 h notice after a late cancellation by the original lecturer due to illness. The lectures were:

- **Estimates for fully nonlinear equations**, Neil Trudinger
- **Mean Curvature Flow with free boundary**, Valentina Wheeler
- **Optimal regularity in the Calculus of Variations**, Connor Mooney

The second week was devoted to research. During this week, three to four research lectures were held per day with the remainder of the time devoted to research collaboration and the general enjoyment of the beautiful Australian bushland surrounding the institute. Arising from the research workshop were several submissions included in these proceedings. The papers deal with topics such as variational problems, particularly non-linear geometric problems, optimal transport, regularity properties and spectral properties of elliptic operators. Neil Trudinger, one of the two authors for whom are program honoured is famous for his work on such problems. As such, each submission represents a continuation of the legacy of the book "Elliptic partial differential equations of second order" and its continuing influence on mathematics.

- "Boundary regularity of mass-minimizing integral currents and a question of Almgren" by Camillo De Lellis, Guido De Philippis, Jonas Hirsch and Annalisa Massaccesi: This paper is an announcement of results to be published in detail in a forthcoming article. The results describe boundary regularity of area minimising currents in high codimension ensuring that regular points are dense on the boundary and leading to a structure theorem answering in particular a question of Almgren, implying singular points on the boundary have low dimension, and yielding a monotonicity formula. The announced results, representing a continuation of Allard's regularity theorem and the monumental "Big Regularity Paper" of Almgren were completed during the second week of our program.
- "Optimal transport with discrete mean field interaction" by Jiakun Liu and Grégoire Loeper: This paper is also an announcement of ongoing work motivated by the motion of self-gravitating matter governed by the Euler-Poisson system. Here the authors build on the first author's formulation of the problem as a variational problem which is then solved using optimal transport techniques exploiting Monge-Kantorovich duality. The result considers a time-discretisation of more general variational problems obtaining regularity results.
- "A sixth order curvature flow of plane curves with boundary conditions" by James McCoy, Glen Wheeler and Yuhan Wu: This submission announces results for a high order curvature flow. Curvature flows, particularly those arising via variational principles have been extensively studied over the past 30 years. A prototypical example is the Mean Curvature Flow, a second order gradient flow, whilst the Wilmore flow is perhaps the most well known example of a higher gradient flow. The author's describe a sixth order gradient flow with free boundary arising in elasticity theory. The maximum principle arguments that feature heavily in second order flows are of no utility in higher order flows and must be replaced by other techniques. The authors employ a Poincaré inequality

and interpolation inequalities to develop suitable integral estimates leading to the conclusion that the flow smoothly converges to the optimal configuration.
- "Quasilinear parabolic and elliptic equations with singular potentials" by Maria Michaela Porzio: This paper considers quasilinear equations with singular, Hardy potentials. A well known result is that solutions to the heat equation with positive driving term and positive initial data do not exist for Hardy parameters larger than the optimal constant in Hardy's inequality. This particular paper considers divergence form operators and in particular the asymptotic behaviour of solutions to such equations provided the Hardy parameter is sufficiently small. Unique global existence of solutions is obtained with decay estimates implying the asymptotic behaviour as $t \to \infty$ is independent of the initial data and hence uniqueness of the associated elliptic problem is assured.
- "How to hear the corners of a drum" by Medet Nursultanov, Julie Rowlett, and David Sher: This paper is a detailed announcement of ongoing work. A well known question asks if one can "hear the shape of a drum?". The answer in general is no, there exists non-congruent domains for which the Laplacian has the same spectrum. The result of this paper says that smooth domains may be distinguished from domains with corners by the spectrum of the Laplacian. More precisely, the spectrum of the Laplacian on a smooth domain with either Neumann or Robin boundary conditions is never equal to that of the Laplacian on a domain with corners and either Neumann or Robin boundary conditions. The result hinges on a generalisation of Kac's locality principle.

Special thanks go to Julie Clutterbuck for her wonderful depiction of a well worn copy of Gilbarg's and Trudinger's book featured on our program materials.

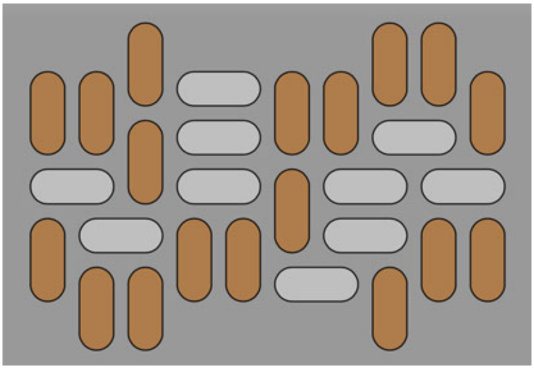
Paul Bryan
Guest Editor

Participants

Yann Bernard (Monash Uni), Norman Dancer (Uni Sydney), Camillo De Lellis (ETH Zurich), Guido de Philippis (SISSA Trieste), Serena Dipierro (Uni Melbourne), Yihong Du (Uni New England), Nicola Fusco (Uni Napoli), Jesse Gell-Redman (Uni Melbourne), Joseph Grotowski (Uni Queensland), Feida Jiang (Tsinghua Uni), Gary Lieberman (Iowa State Uni), Jiakun Liu (Uni Wollongong), Gregoire Loeper (Monash Uni), Connor Mooney (Uni Texas Austin), Aldo Pratelli (Erlangen-Nürnberg), Maria Michaela Porzio (Roma), Frédéric Robert (Uni Lorraine), Julie Rowlett (Chalmers Uni), Mariel Sáez (Pontificia Uni, Chile), Neil Trudinger (Australian National Uni), John Urbas (Australian National Uni), Jerome Vetois (McGill Uni), Xu-Jia Wang (Australian National Uni), Valentina Wheeler (Uni Wollongong), Bin Zhou (Australian National Uni), Graham Williams (Uni Wollongong), James McCoy (Uni Wollongong)

Combinatorics, Statistical Mechanics, and Conformal Field Theory

29 October–18 November 2017



Organisers

Vladimir Korepin
Stony Brook Uni

Vladimir Mangazeev
Australian National Uni

Bernard Nienhuis
Uni Amsterdam

Jorgen Rasmussen
Uni Queensland

This program brought together leading experts in and around the area where statistical mechanics, integrability, conformal field theory, and combinatorics meet and in some sense overlap. A primary goal was to encourage research collaborations across the traditional divides between the research communities interested in the separate disciplines. Significant recent developments stem from this kind of cross fertilisation, and the aim was to cultivate further such collaborations and widen the scope of their successes.

The scientific presentations and discussions were largely centred around Yang-Baxter integrable models; the Razumov-Stroganov conjecture and generalisations thereof; combinatorial points and the role of supersymmetry in integrable lattice models and quantum chains; the combinatorics of spanning trees and pattern-avoiding permutations; and logarithmic conformal field theory.

With the strong emphasis on collaborations and discussions, there were only a couple of seminars per day in the first and third week. The embedded AMSI Workshop took place in the second week and included talks by Bazhanov, Guttmann, Hagendorf, Mangazeev, Nienhuis, Pearce, Ridout, Ruelle, Tartaglia, Weston and Wheeler. Sessions with informal and brief presentations were held throughout the program, with the aim to expand collaborative work on existing research projects and to foster new ideas and collaborations.

The contribution to this volume by Bernard Nienhuis and Kayed Al Qasimi was directly stimulated by discussions with Christian Hagendorf at the MATRIX Workshop. It provides a proof of a conjecture on certain one-point functions related to the Razumov-Stroganov conjecture.

Jorgen Rasmussen
Guest Editor

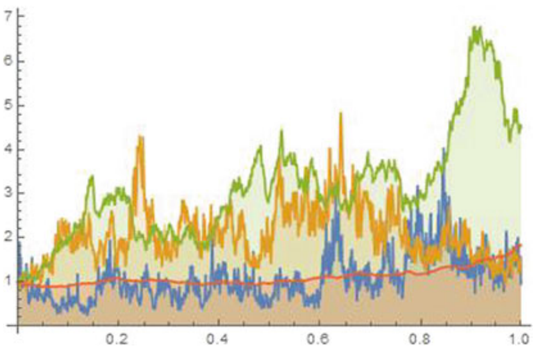
Participants

Georgy Feher (Budapest Uni Technology and Economics), Christian Hagendorf (Catholic Uni Louvain), Philippe Ruelle (Catholic Uni Louvain), Elena Tartaglia (SISSA Trieste), Murray Batchelor (Australian National Uni), Vladimir Bazhanov (Australian National Uni), Zeying Chen (Uni Melbourne), Omar Foda (Uni Melbourne), Jan de Gier (Uni Melbourne), Alexandr Garbali (Uni Melbourne), Tony Guttmann (Uni Melbourne), Jon Links (Uni Queensland), Paul Pearce (Uni Melbourne), Thomas Quella (Uni Melbourne), David Ridout (Uni Melbourne), Alessandra Vittorini-Orgeas (Uni Melbourne), Michael Wheeler (Uni Melbourne), Paul Zinn-Justin (Uni Melbourne), Robert Weston (Heriot-Watt Uni), Atsuo Kuniba (Tokyo Uni)

Mathematics of Risk

20 November–8 December 2017

Organisers



Kostya Borovkov
Uni Melbourne

Kais Hamza
Monash Uni

Masaaki Kijima
Tokyo Metropolitan Uni

Alexander Novikov
Uni Technology Sydney

Peter Taylor
Uni Melbourne

The mathematical modelling of the various types of risk modern society encounters at different levels of its operation has become an important part of applied mathematics as well as a source of challenging theoretical problems. The main need in modelling risk is where the latter refers to a serious danger to society and nature. As illustrated by the recent Global Financial Crisis of 2007–2008, the finance industry is one of the most serious sources of risk. Since the finance industry tried to (at least, partly) blame mathematical models for what had happened, it is all the more important for mathematicians to address the issue of financial risk and use mathematics to find ways to mitigate it.

The need for quantitative risk modelling has, in recent years, attracted enormous worldwide attention. The risk related to both extreme and non-extreme events is generating a vast research activity, which is international by its very nature. Moreover, there is an international regulatory aspect concerning mathematical modelling of financial risks. One of the key elements of the current versions of the Basel accord (a global regulatory framework for bank capital adequacy, stress testing, and market liquidity risk) is the emphasis on responsible use of mathematical models.

Our program mostly addressed various aspects of mathematical modelling and subsequent analysis of risks related to activities in the finance industry and, more generally, economics. Major attention was also paid to studying the mathematical theory that can be used to model more general types of risk, related to chronic and long-term hazards and extremes, as well as the interplay between them.

The key themes of the program included:

- the modelling of uncertainty and risk events using the theory of stochastic processes, in particular, the evaluation of the distributions of boundary functionals of random processes, including the computation of boundary crossing probabilities;
- new methods for and approaches to computing the prices of financial derivatives;
- the systemic risk, including the stability of national and world financial systems, consequences for the markets from the wide use of algorithmic trading, network modelling of relevant real-life systems;
- risk modelling and quantification, including risk measures;
- the analysis of model risk, i.e. the type of risk that arises due to using inappropriate mathematical models for asset price dynamics etc.;
- mathematical modelling of extreme events due to factors such as natural disasters, human errors, infrastructure and computer control systems' failures.

Our program included two 'embedded events'. In the first week of the program, we ran four 5-h workshops for PhD students, research workers and industry specialists on the following topics:

- Extreme Value Theory—Applications to risk analysis (M. Kratz);
- Financial measures of risk and performance (M. Zhitlukin);
- Ruin probabilities: exact and asymptotic results (Z. Palmowski);
- Clearing in financial networks (Yu. Kabanov).

In the second week of the program, we hosted a research conference where about 20 talks were given. The slides used by both the workshop presenters and conference speakers are available at the program web-site, https://www.matrix-inst.org.au/events/mathematics-of-risk. For the present volume, two of the workshop presenters (M. Kratz and M. Zhitlukhin) prepared more detailed expositions of the material from their workshops. We are most grateful to them for their time and effort required to write these very interesting and instructive papers. Our thanks also go to the other program participants who took time to contribute to this volume. Finally, we would like to thank the MATRIX and Creswick campus staff for facilitating and hosting this event. The participants enjoyed it tremendously. We had an excellent opportunity to engage in joint research work and exchange our ideas, both during the conference week and outside it.

Alexander Novikov and Kostya Borovkov
Guest Editors

Participants

Martin Larsson, Michael Zhitlukhin, Yuri Kabanov, Zbigniew Palmowski, Marie Kratz, Ljudmila Vostrikova, Budhi Surya, Jie Xiong, Kazutoshi Yamazaki, Eugene Feinberg, Jun Sekine, Takashi Shibata, Katsumasa Nishide, Daniel Dufresne, Aihua Xia, Yan Dolinsky, Thomas Taimre, Boris Buchmann, Gregoire Loeper, Alexander Buryak, Koji Anamizu, Hitomi Ito, Nakamura Eri, Yusuke Komatsu, Yasuhiro Shimizu, Kyoko Yagi, Tadao Oryu, Evgeny Prokopenko, Peter Spreij, Jeremy Baker, Aaron Chong, Tianshu Cong, Gurtek Ricky Gill, Qingwei Liu, Jin Sun, Priyanga Dilini Talagala, Hui Yao (Alice), Junyu (Adam) Nie, Eduard Biche, Kaustav Das, Meng Shi, Yunxuan Liu, Yuqing Pan, Wei Ning, Jason Leung, Igor Vladimirov, Libo Li, Peter Straka, Michael Callan, Jaden Li, Nino Kordzakhia, Juri Hinz, Fima Klebaner, Nicholas Read, Kevin Vo, Zhehao Zhang

Tutte Centenary Retreat

26 November–2 December 2017

Organisers

Graham Farr (Chair)
Monash Uni

Marston Conder
Uni Auckland

Dillon Mayhew
Victoria Uni Wellington

Kerri Morgan
Monash Uni

James Oxley
Louisiana State Uni

Gordon Royle
Uni Western Australia

The year 2017 marked the centenary of the birth of W.T. (Bill) Tutte (1917–2002), the great Bletchley Park cryptologist and pioneering graph theorist. This Retreat was part of a worldwide programme of Tutte Centenary events, see https://billtuttememorial.org.uk/centenary/, including events at Bletchley Park, Waterloo, Cambridge, and Monash. It was scheduled for the week preceding the 5th International Combinatorics Conference (5ICC) (http://www.monash.edu/5icc/) at Monash.

The Retreat programme focused on three topics that have grown out of seminal contributions made by Tutte at the very start of his career:

Tutte-Whitney Polynomials. These count a wide variety of structures associated with a graph, and are related to network reliability, coding theory, knot theory and statistical physics. They were introduced by Whitney (1932) and Tutte (1947, 1954), and now play a central role in enumerative graph theory. They extend readily to matroids, which were the focus of the second topic.

Matroid Structure Theory. Many aspects of graph theory are especially natural and elegant when viewed in the broader setting of *matroids*, which are combinatorial abstractions of sets of vectors under linear independence. Tutte developed the theory of matroid connectivity, and characterised several important matroid classes in terms of forbidden substructures (*excluded minors*). His work continues to inspire developments in the field.

Symmetric Graphs. A lot of recent research on symmetric graphs builds on ground-breaking theory by Tutte on the trivalent case. Tutte developed the theory of arc-transitive graphs, and the techniques he used formed the foundations of a large and growing branch of discrete mathematics.

The Retreat emphasised collaborative research supported by problem sessions. There were three introductory talks: an overview of Tutte's contributions to mathematics, by James Oxley; Tutte-Whitney polynomials, by Gordon Royle; and symmetric graphs, by Marston Conder and Michael Giudici. Oxley's talk led to the paper 'The contributions of W.T. Tutte to matroid theory' by Graham Farr and James Oxley, included in this volume.

We had a total of 32 participants (including organisers). Participants found the workshop to be an exceptionally stimulating event precisely because, instead of hearing a long sequence of talks about the work of others, they got to work for extended periods on interesting problems with a variety of collaborators. They were able to develop new ideas and learn a lot about other recent work. A number of questions were answered relatively quickly, simply through sharing knowledge among participants. The more substantial research collaborations in each of the three themes dealt with the following.

Tutte-Whitney Polynomials

- An old question of Hassler Whitney (1932) about an elegant extension of the four-colour theorem using duality and Tutte-Whitney polynomials.
- Chromatic polynomial of hypergraphs (with the research having been done and largely written up during the workshop).
- A notion of "rank function" for certain algebraic structures which exhibit a pleasing duality, with the potential to generalise matroid representability and shed light on Tutte polynomials of other combinatorial objects.
- One of the Merino-Welsh conjectures (correlation inequalities relating numbers of acyclic orientations and totally cyclic orientations of a graph).
- The complexity of counting bases in binary matroids.

Matroid Structure Theory

- A problem of connectivity in frame matroids, a generalisation of the matroids that arise from graphs.

- A problem about the relationship between size and rank in matroids, inspired by an old theorem of Edmonds.
- Analysis and generalisation of a distinctive property of wheels and whirls, matroids Tutte identified as playing a fundamental role in his theory of 3-connectivity for matroids.
- Characterisation of the members of a natural class of matroids where the interaction of elements, circuits and cocircuits is particularly elegant and symmetric.

Symmetric Graphs

- Normal quotient analysis for finite edge-transitive oriented graphs of valency four. This led to the paper 'Biquasiprimitive oriented graphs of valency four' by Nemanja Poznanovic and Cheryl Praeger in this volume.
- A question of Caprace (at Groups St Andrews, Birmingham, August 2017) on whether there exists a 2-transitive permutation group P such that only finitely many simple groups act arc-transitively on a connected graph X with local action P. Marston Conder gave a partial answer in his paper 'Simple group actions on arc-transitive graphs with prescribed transitive local action' in this volume.
- Answer to a question by Folkman (1967) about (bipartite) semi-symmetric graphs of order $2n$ and valency $d \geqslant n/2$, to be published by Marston Conder and Gabriel Verret.
- Development of the theory of LR-structures, which in some sense extend Tutte's work on symmetric 3-valent graphs, and the answer to an open question on them.
- A question related to determining the "graph-type" of a larger number of transitive groups. This proved quite difficult, but was solved soon after the Retreat in joint work with someone who could not attend.

It is expected that about fifteen papers will result from research that began during the workshop week, including the three in this volume of *MATRIX Annals*.

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Graham Farr, Marston Conder, Dillon Mayhew and Gordon Royle
Guest Editors

Participants

Joanna Ellis-Monaghan (Saint Michael's College, Vermont), Johann A. Makowsky (Technion, Israel), Steven Noble (Birkbeck, Uni London), Deborah Chun (West Virginia Uni Technology), Charles Semple (Uni Canterbury), Geoff Whittle (Victoria Uni Wellington), Nick Brettell (Victoria Uni Wellington), Joanna Fawcett (Uni Cambridge), Michael Giudici (Uni Western Australia), Cheryl Praeger (Uni Western Australia), Cai Heng Li (South Uni Science and Technology, Shenzhen), Gabriel Verret (Uni Auckland), Luke Morgan (Uni Western Australia), Primož Potočnik (Uni Ljubljana), Sanming Zhou (Uni Melbourne), Kevin Grace (Louisiana State Uni), Iain Moffatt (Royal Holloway, Uni London), Xian'an Jin (Xiamen Uni), Thomas Britz (UNSW Sydney), Tara Fife (Louisiana State Uni), Nemanja Poznanovic (Uni Melbourne), William Whistler (Durham Uni), Georgina Liversidge (Uni Newcastle), Rutger Campbell (Uni Waterloo), Keisuke Shiromoto (Kumamoto Uni), Ruixue Zhang (Nanyang Technological Uni), Ben Jones (Monash Uni)

Geometric R-Matrices: From Geometry to Probability

17–22 December 2017

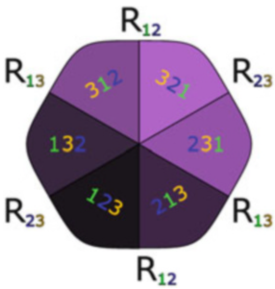
Organisers

Rick Kenyon
Brown Uni

Andrei Okounkov
Columbia Uni

Anita Ponsaing
Uni Melbourne

Paul Zinn-Justin
Uni Melbourne



The focus of this workshop was originally, as the title “Geometric R-matrices” suggests, to discuss the interaction of quantum integrable systems, a topic in mathematical physics, with algebraic geometry and representation theory. As the subtitle “from geometry to probability” indicates, it was quickly expanded to include interactions with other branches of mathematics, in particular combinatorics and probability. Here is a brief sketch of these interactions:

Algebraic Geometry and Representation Theory In the 2000s, the idea emerged that the theory of quantum integrable systems could be used to study the (quantum, equivariant) cohomology of certain varieties that appear naturally in algebraic geometry and representation theory, such as Grassmannians, flag varieties, and related Schubert varieties, orbital varieties, etc. This was reformulated as a beautiful, coherent program by Maulik and Okounkov in the early 2010s, combining ideas from geometric representation theory and in particular Nakajima’s quiver varieties, other ideas from geometry (Gromov–Witten invariants, etc.), with concepts coming from the study of supersymmetric gauge theories (cf. the work of Nekrasov and Shatashvili). The quantum integrable system is defined out of the geometry by starting with its building block which is the R-matrix of our title.