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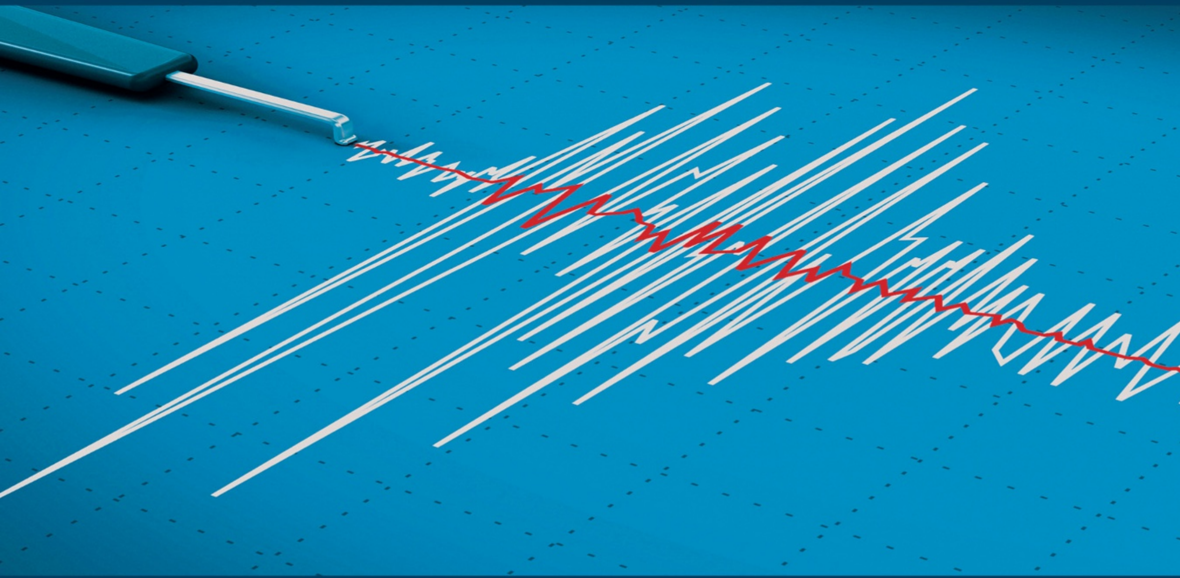
STATISTICAL METHODS FOR EARTHQUAKES SET



Volume 2

Earthquake Statistical Analysis through Multi-state Modeling

Irene Votsi, Nikolaos Limnios
Eleftheria Papadimitriou
and George Tsaklidis

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Earthquake Statistical Analysis through Multi-state Modeling

“We Athenians in our persons take our decisions on policy and submit them to proper discussion. The worst thing is to rush into action before the consequences have been properly debated. And this is another point where we differ from other people. We are capable at the same time of taking risks and estimating them beforehand. Others are brave out of ignorance; and when they stop to think, they begin to fear. But the man who can most truly be accounted brave is he who best knows the meaning of what is sweet in life, and what is terrible, and he then goes out undeterred to meet what is to come.”

– Abstract from Pericle’s Funeral Oration in Thucydides’ “History of the Peloponnesian War” (started in 431 B.C.)

Statistical Methods for Earthquakes Set

coordinated by
Nikolaos Limnios, Eleftheria Papadimitriou, George Tsaklidis

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List of Abbreviations

ΔCFF	Coulomb failure function
AIC	Akaike's information criterion
a.s.	almost surely
BIC	Bayesian information criterion
EM	expectation-maximization
EMC	embedded Markov chain
HMC	hidden Markov chain
HMRC	hidden Markov renewal chain
HMM	hidden Markov model
HMRM	hidden Markov renewal model
HSMM	hidden semi-Markov model
HSMC	hidden semi-Markov chain
MC	Markov chain
MLE	maximum likelihood estimator
MRC	Markov renewal chain
PHMM	Poisson hidden Markov model
SMC	semi-Markov chain
SMM	semi-Markov model
SMK	semi-Markov kernel

List of Symbols

$\mathbb{N}$	set of non-negative integers, $\mathbb{N}^* = \mathbb{N} \backslash \{0\}$
E	state space of the (semi-)Markov chain
E^*	state space of the hidden Markov renewal chain
L	state space of the Markov renewal chain
A	observation space
$\mathcal{M}_E$	real matrices defined on $E \times E$
$\mathcal{M}_{E \times A}$	real matrices defined on $E \times A$
$\mathcal{M}_E(\mathbb{N})$	matrix-valued functions with values in $\mathcal{M}_E$ and defined on $\mathbb{N}$
$A_1 * A_2$	(discrete-time) matrix convolution product on $A_1, A_2 \in \mathcal{M}_E(\mathbb{N})$
$\mathbf{J} = (J_n)_{n \in \mathbb{N}}$	embedded Markov chain
$\mathbf{X} = (X_n)_{n \in \mathbb{N}}$	sojourn times between successive jumps
$\mathbf{S} = (S_n)_{n \in \mathbb{N}}$	jump times
$\mathbf{Y} = (Y_n)_{n \in \mathbb{N}}$	sequence of observations
$\mathbf{Z} = (Z_k)_{k \in \mathbb{N}}$	(semi-)Markov chain
$\mathbf{U} = (U_k)_{n \in \mathbb{N}}$	backward recurrence times

$(\mathbf{Z}, \mathbf{U}) = (Z_k, U_k)_{n\in\mathbb{N}}$	double Markov chain
$(\mathbf{J}, \mathbf{S}) = (J_n, S_n)_{n\in\mathbb{N}}$	Markov renewal chain
$(\mathbf{J}, \mathbf{S}, \mathbf{Y}) = (J_n, S_n, Y_n)_{n\in\mathbb{N}}$	hidden Markov renewal chain
$(\mathbf{Z}, \mathbf{Y}) = (Z_k, Y_k)_{k\in\mathbb{N}}$	hidden (semi-)Markov chain
M	fixed censoring time
$\mathcal{H}_M$	observation of a trajectory in $[0, M]$
$N(M)$	jumps number of the EMC up to time M
$N_i(M)$	visits number of the EMC to state i up to time M
$N_{ij}(M)$	jumps number of the EMC from state i to state j up to time M
$N_{(i,s)}(M)$	visits number of the MRC to state (i, s) up to time M
$N_{(i,s_1)(j,s_2)}(M)$	transitions number of the MRC from state (i, s_1) to state (j, s_2) up to time M
$\mathbf{q} = (q_{ij}(k))_{i,j\in E,k\in\mathbb{N}}$	semi-Markov kernel
$\mathbf{Q} = (Q_{ij}(k))_{i,j\in E,k\in\mathbb{N}}$	cumulative semi-Markov kernel
$\mathbf{P} = (p_{ij})_{i,j\in E}$	transition matrix of the EMC
$\mathbf{f} = (f_{ij}(k))_{i,j\in E,k\in\mathbb{N}}$	conditional sojourn time distribution
$\mathbf{h} = (h_i(k))_{i\in E,k\in\mathbb{N}}$	sojourn time distribution
$\mathbf{F} = (F_{ij}(k))_{i,j\in E,k\in\mathbb{N}}$	cumulative conditional sojourn time distribution
$\mathbf{H} = (H_i(k))_{i\in E,k\in\mathbb{N}}$	cumulative sojourn time distribution

$\overline{\mathbf{H}} = (\overline{H}_i(k))_{i\in E, k\in\mathbb{N}}$	survival function of sojourn times
$\boldsymbol{\alpha} = (\alpha(i))_{i\in E}$	initial distribution of the EMC
$\mathbf{m} = (m_i)_{i\in E}$	mean sojourn time
$\boldsymbol{\mu} = (\mu_i)_{i\in E}$	mean recurrence time
$\boldsymbol{\pi} = (\pi_i)_{i\in E}$	stationary distribution of the SMC
$\boldsymbol{\nu} = (\nu_i)_{i\in E}$	stationary distribution of the EMC
$\widetilde{\mathbf{a}}$	initial distribution of the double Markov chain
$\widetilde{P}$	transition matrix of the double Markov chain
$\widetilde{\boldsymbol{\pi}}$	stationary distribution of the double Markov chain
$\mathbf{a}^\sharp$	initial distribution of the MRC
$\mathbf{P}^\sharp$	transition matrix of the MRC
$\boldsymbol{\pi}^\sharp$	stationary distribution of the MRC
$T_{(i,j)}$	first passage time of the Markov chain **(Z,Y)** in state (i,j)
$\mathbf{a}$	initial distribution of the Markov chain **(Z,Y)**
L	likelihood function
$\mathbf{R}$	emission probability matrix
$\widehat{p}_{ij}(M), \widehat{q}_{ij}(k,M), \dots$	estimators of $p_{ij}, q_{ij}(k), \dots$
$\mathcal{N}(0,1)$	standard normal distribution ($\mu = 0, \sigma = 1$)
$\xrightarrow{\mathcal{L}}$	convergence in distribution
$\mathbf{1}_A$	indicator function of a set A
ΔCFF	change in Coulomb failure function
ΔCS	Coulomb stress change
Δp	pore pressure change
$\Delta\tau$	shear stress change
$\Delta\sigma$	fault-normal stress change

Δt	time shift
Σ	planar fault surface
μ	coefficient of friction
μ'	apparent coefficient of friction
B	Skempton's coefficient
τ	long-term stressing rate
U	uniform dislocation
δ_{ij}	Kronecker delta

Preface

Statistical seismology attracts the attention of seismologists, statisticians, geologists, engineers, government officials and insurers among others, since it serves as a powerful tool for seismic hazard assessment and, consequently, for risk mitigation. This field aims to connect physical and statistical models and to provide a conceptual basis for the earthquake generation process. To date, purely deterministic models have inadequately described earthquake dynamics. This is mainly due to the restricted knowledge concerning fundamental state parameters related to the causative process, such as stress state and properties of the medium. Comparing the deterministic approaches with the stochastic ones, we should note that, today, the latter are the most favorable. Stochastic processes allow for the efficient modeling of real-life random phenomena and the quantification of associated indicators.

This book is intended as a first, but at the same time, a systematic approach for earthquake multi-state modeling by means of hidden (semi-)Markov models. It provides a presentation of bibliography sources, methodological studies and the development of stochastic models in order to reveal the mechanism and assessment of future seismogenesis. This book aims to ease the reader in getting and exploiting conceivable tools for the application of multi-state models to concrete physical problems encountered in seismology. It also aims to encourage discussions and future modeling efforts in the domain of statistical seismology, by tackling from, theoretical advances to very practical applications.

This book is concerned with several central themes in a rapidly developing field: earthquake occurrence modeling. It contains seven chapters and three appendices and begins with two lists containing abbreviations and symbols used throughout the book.

Next is an introduction that describes the state-of-the-art earthquake modeling approaches that focus on multi-state models.

Chapter 1 introduces the reader to the crustal stress state, stress changes and evolution and the association with earthquake generation. The complexity of this process is then investigated by using advanced stochastic models in the chapters that follow.

Chapter 2 presents a multi-state modeling approach that enables the description of strong seismicity in the broader Aegean area from 1865 to 2008. This chapter aims to help the reader to acquaint with the application of hidden Markov models and it presents a detailed example of multi-state modeling in seismology. In particular, hidden Markov models are used to shed some new light on the “hidden” component that controls the generation of earthquakes: the *stress field*. Our purpose is to assess the evolution of the stress field and its inherently causative role in both the number and size of earthquakes.

Chapter 3 presents (hidden) semi-Markov models and their associated stochastic processes. It contains all statistical estimation tools that enable the reader to estimate indicators of interest associated with the occurrence of strong earthquakes.

Chapter 4 presents theoretical results for the statistics of stochastic processes that can have direct applications in seismology. It aims to teach how the study of a real-life phenomenon could lead to the development of the statistics of stochastic processes and thereafter how these theoretical results could be further used to answer open questions regarding the phenomenon under study.

Chapter 5 gives some guidelines for the comparison of multi-state models and provides specific numerical examples. This last chapter is a collection of concluding remarks, open questions and perspectives in the field.

At the end of the book, three appendices are provided. Appendix 1 presents some main definitions of Markov models. Appendix 2 presents how the three problems regarding hidden Markov models could be solved. Appendix 3 presents the dataset used throughout the book.

The authors express their gratitude to M. Hamdaoui for his technical help and assistance. The changes and evolution of the stress field were calculated using the program written by J. Deng [DEN 97a] based on the DIS3D code by S. Dunbar and Erikson (1986) and the expressions of G. Converse.

Some of the figures were plotted using the Generic Mapping Tools algorithm [WES 98]. This book will be useful to applied statisticians and geophysicists interested in the theory of multi-state modeling. It can also be useful to students, teachers and professional researchers who are interested in statistical modeling for earthquakes.

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