STUDENT SOLUTIONS MANUAL TO ACCOMPANY

LOSS MODELS

FROM DATA TO DECISIONS

FIFTH EDITION

STUART A. KLUGMAN · HARRY H. PANJER

GORDON E. WILLMOT

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This edition first published 2019

Edition History
Wiley (1e, 1998; 2e, 2004; 3e, 2008; and 4e, 2012)

Registered Office
John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030, USA

Editorial Office
111 River Street, Hoboken, NJ 07030, USA

For details of our global editorial offices, customer services, and more information about Wiley products visit us at www.wiley.com.

Wiley also publishes its books in a variety of electronic formats and by print-on-demand. Some content that appears in standard print versions of this book may not be available in other formats.

Library of Congress Cataloging-in-Publication Data
Names: Klugman, Stuart A., 1949- author. | Panjer, Harry H., 1946- author. |
Willmot, Gordon E., 1957- author.
Title: Loss models : from data to decisions / Stuart A. Klugman, Society of
Actuaries, Harry H. Panjer, University ofWaterloo, Gordon E.Willmot,
University of Waterloo.
Description: 5th edition. | Hoboken, NJ : John Wiley and Sons, Inc., [2018] |
Series: Wiley series in probability and statistics | Includes
bibliographical references and index. |
Identifiers: LCCN 2018031122 (print) | ISBN 9781119523789
(hardcover) | ISBN 9781119538059
(solutions manual)
Subjects: LCSH: Insurance–Statistical methods. | Insurance–Mathematical
models.
Classification: LCC HG8781 (ebook) | LCC HG8781 .K583 2018 (print) | DDC
368/.01–dc23
LC record available at https://lccn.loc.gov/2018031122

Cover design by Wiley

Set in 10/12 pt TimesLTStd-Roman by Thomson Digital, Noida, India

Printed in the United States of America.

10 9 8 7 6 5 4 3 2 1

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CHAPTER 1

INTRODUCTION

The solutions presented in this manual reflect the authors' best attempt to provide insights and answers. While we have done our best to be complete and accurate, errors may occur and there may be more elegant solutions. Errata will be linked from the syllabus document for any Society of Actuaries examination that uses this text.

Should you find errors, or if you would like to provide improved solutions, please send your comments to Stuart Klugman at sklugman@soa.org.

Student Solutions Manual to Accompany Loss Models: From Data to Decisions, Fifth Edition.
Stuart A. Klugman, Harry H. Panjer, and Gordon E. Willmot.

CHAPTER 2

CHAPTER 2 SOLUTIONS

SECTION 2.2

2.1 $F_5(x) = 1 - S_5(x) = \begin{cases} 0.01x, & 0 \le x < 50, \\ 0.02x - 0.5, & 50 \le x < 75. \end{cases}$

$$f_5(x) = F_5'(x) = \begin{cases} 0.01, & 0 < x < 50, \\ 0.02, & 50 \le x < 75. \end{cases}$$

$$h_5(x) = \frac{f_5(x)}{S_5(x)} = \begin{cases} \dfrac{1}{100 - x}, & 0 < x < 50, \\ \dfrac{1}{75 - x}, & 50 \le x < 75. \end{cases}$$

2.2 The requested plots follow. The triangular spike at zero in the density function for Model 4 indicates the 0.7 of discrete probability at zero.

2.3 $f'(x) = 4(1 + x^2)^{-3} - 24x^2(1 + x^2)^{-4}$. Setting the derivative equal to zero and multiplying by $(1 + x^2)^4$ gives the equation $4(1 + x^2) - 24x^2 = 0$. This is equivalent to $x^2 = 1/5$. The only positive solution is the mode of $1/\sqrt{5}$.

Stuart A. Klugman, Harry H. Panjer, and Gordon E. Willmot.

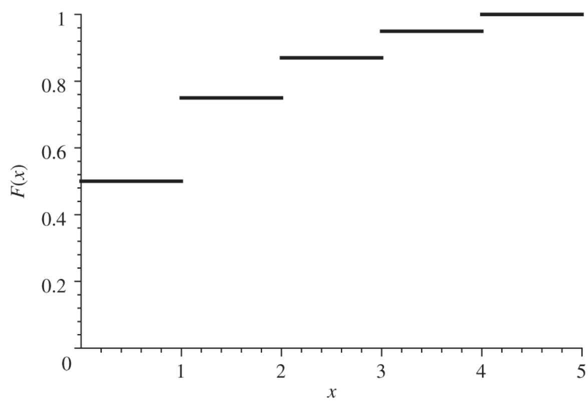

Figure 2.1 The distribution function for Model 3.

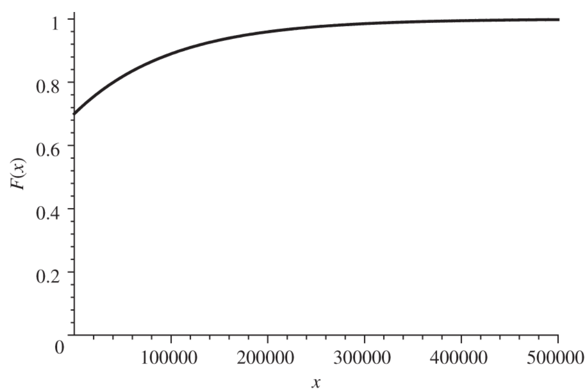

Figure 2.2 The distribution function for Model 4.

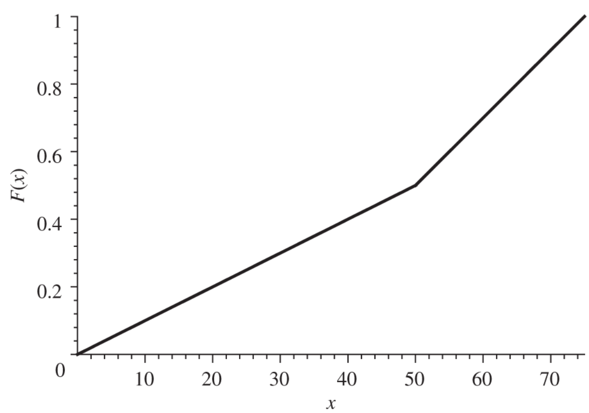

Figure 2.3 The distribution function for Model 5.

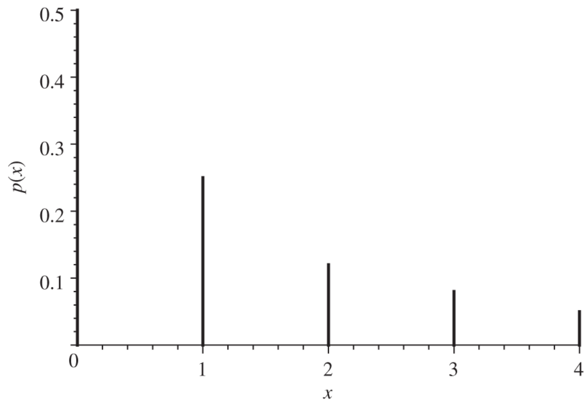

Figure 2.4 The probability function for Model 3.

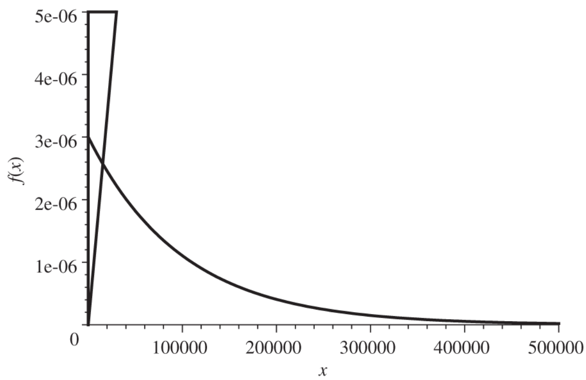

Figure 2.5 The density function for Model 4.

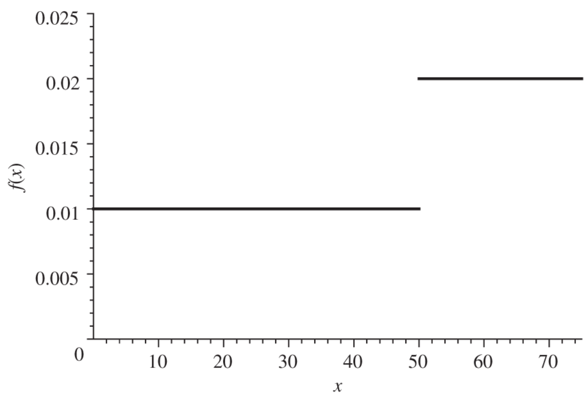

Figure 2.6 The density function for Model 5.

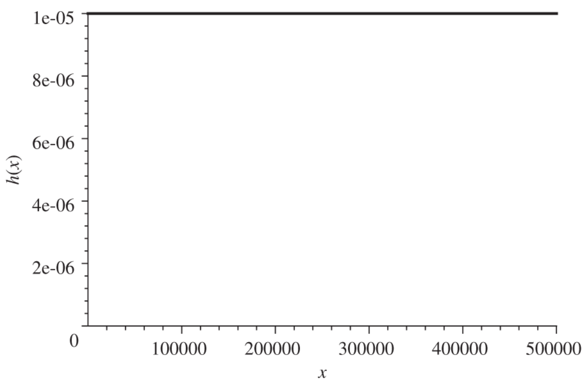

Figure 2.7 The hazard rate for Model 4.

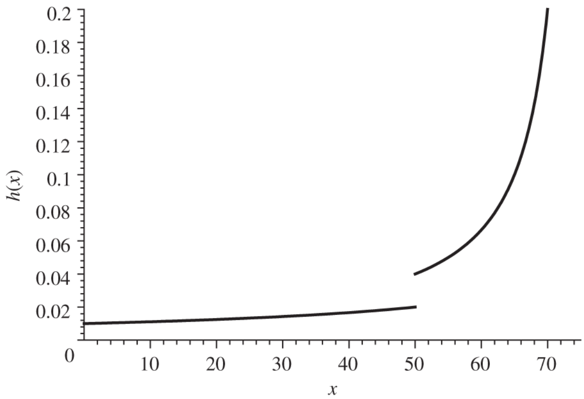

Figure 2.8 The hazard rate for Model 5.

2.4 The survival function can be recovered as

$$\begin{aligned} 0.5 = S(0.4) &= e^{-\int_0^{0.4} A+e^{2x}dx} \\ &= e^{-Ax-0.5e^{2x}\Big|_0^{0.4}} \\ &= e^{-0.4A-0.5e^{0.8}+0.5}. \end{aligned}$$

Taking logarithms gives

$$-0.693147 = -0.4A - 1.112770 + 0.5,$$

and thus $A = 0.2009$.

2.5 The ratio is

$$r = \frac{\left(\dfrac{10{,}000}{10{,}000 + d}\right)^2}{\left(\dfrac{20{,}000}{20{,}000 + d^2}\right)^2} = \left(\frac{20{,}000 + d^2}{20{,}000 + 2d}\right)^2 = \frac{20{,}000^2 + 40{,}000d^2 + d^4}{20{,}000^2 + 80{,}000d + 4d^2}.$$

From observation or two applications of L'Hôpital's rule, we see that the limit is infinity.

CHAPTER 3

CHAPTER 3 SOLUTIONS

SECTION 3.1

3.1

$$\mu_3 = \int_{-\infty}^{\infty}(x-\mu)^3 f(x)dx = \int_{-\infty}^{\infty}(x^3 - 3x^2\mu + 3x\mu^2 - \mu^3)f(x)dx$$
$$= \mu_3' - 3\mu_2'\mu + 2\mu^3,$$

$$\begin{aligned}\mu_4 &= \int_{-\infty}^{\infty}(x-\mu)^4 f(x)dx\\ &= \int_{-\infty}^{\infty}(x^4 - 4x^3\mu + 6x^2\mu^2 - 4x\mu^3 + \mu^4)f(x)dx\\ &= \mu_4' - 4\mu_3'\mu + 6\mu_2'\mu^2 - 3\mu^4.\end{aligned}$$

3.2 For Model 1, $\sigma^2 = 3{,}333.33 - 50^2 = 833.33$, $\sigma = 28.8675$.
$\mu_3' = \int_0^{100} x^3(0.01)dx = 250{,}000$, and $\mu_3 = 0$, $\gamma_1 = 0$.
$\mu_4' = \int_0^{100} x^4(0.01)dx = 20{,}000{,}000$, $\mu_4 = 1{,}250{,}000$, $\gamma_2 = 1.8$.

For Model 2, $\sigma^2 = 4{,}000{,}000 - 1{,}000^2 = 3{,}000{,}000$, and $\sigma = 1{,}732.05$. μ_3' and μ_4' are both infinite, so the skewness and kurtosis are not defined.

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For Model 3, $\sigma^2 = 2.25 - 0.93^2 = 1.3851$ and $\sigma = 1.1769$.
$\mu_3' = 0(0.5) + 1(0.25) + 8(0.12) + 27(0.08) + 64(0.05) = 6.57$, $\mu_3 = 1.9012$,
$\gamma_1 = 1.1663$, $\mu_4' = 0(0.5) + 1(0.25) + 16(0.12) + 81(0.08) + 256(0.05) = 21.45$,
$\mu_4 = 6.4416$, $\gamma_2 = 3.3576$.

For Model 4, $\sigma^2 = 6{,}000{,}000{,}000 - 30{,}000^2 = 5{,}100{,}000{,}000$ and $\sigma = 71{,}414$.
$\mu_3' = 0^3(0.7) + \int_0^\infty x^3(0.000003)e^{-.00001x}dx = 1.8 \times 10^{15}$,
$\mu_3 = 1.314 \times 10^{15}$, $\gamma_1 = 3.6078$.
$\mu_4' = \int_0^\infty x^4(0.000003)e^{-.00001x}dx = 7.2 \times 10^{20}$, $\mu_4 = 5.3397 \times 10^{20}$,
$\gamma_2 = 20.5294$.

For Model 5, $\sigma^2 = 2{,}395.83 - 43.75^2 = 481.77$ and $\sigma = 21.95$.
$\mu_3' = \int_0^{50} x^3(0.01)dx + \int_{50}^{75} x^3(0.02)dx = 142{,}578.125$, $\mu_3 = -4{,}394.53$,
$\gamma_1 = -0.4156$.
$\mu_4' = \int_0^{50} x^4(0.01)dx + \int_{50}^{75} x^4(0.02)dx = 8{,}867{,}187.5$, $\mu_4 = 439{,}758.30$,
$\gamma_2 = 1.8947$.

3.3 The standard deviation is the mean times the coefficient of variation, or 4, and so the variance is 16. From (3.3), the second raw moment is $16 + 2^2 = 20$. The third central moment is (using Exercise 3.1) $136 - 3(20)(2) + 2(2)^3 = 32$. The skewness is the third central moment divided by the cube of the standard deviation, or $32/4^3 = 1/2$.

3.4 For a gamma distribution, the mean is $\alpha\theta$. The second raw moment is $\alpha(\alpha+1)\theta^2$, and so the variance is $\alpha\theta^2$. The coefficient of variation is $\sqrt{\alpha\theta^2}/\alpha\theta = \alpha^{-1/2} = 1$. Therefore $\alpha = 1$. The third raw moment is $\alpha(\alpha+1)(\alpha+2)\theta^3 = 6\theta^3$. From Exercise 3.1, the third central moment is $6\theta^3 - 3(2\theta^2)\theta + 2\theta^3 = 2\theta^3$ and the skewness is $2\theta^3/(\theta^2)^{3/2} = 2$.

3.5 For Model 1,

$$e(d) = \frac{\int_d^{100}(1-0.01x)dx}{1-0.01d} = \frac{100-d}{2}.$$

For Model 2,

$$e(d) = \frac{\displaystyle\int_d^\infty \left(\frac{2text{,}000}{x+2{,}000}\right)^3 dx}{\left(\dfrac{2{,}000}{d+2{,}000}\right)^3} = \frac{2{,}000+d}{2}.$$

For Model 3,

$$e(d) = \begin{cases} \dfrac{0.25(1-d)+0.12(2-d)+0.08(3-d)+0.05(4-d)}{0.5} & 0 \le d < 1, \\ \quad = 1.86 - d, & \\ \dfrac{0.12(2-d)+0.08(3-d)+0.05(4-d)}{0.25} = 2.72 - d, & 1 \le d < 2, \\ \dfrac{0.08(3-d)+0.05(4-d)}{0.13} = 3.3846 - d, & 2 \le d < 3, \\ \dfrac{0.05(4-d)}{0.05} = 4 - d, & 3 \le d < 4. \end{cases}$$

For Model 4,

$$e(d)=\frac{\int_d^\infty 0.3e^{-0.00001x}dx}{0.3e^{-0.00001d}}=100{,}000.$$

The functions are straight lines for Models 1, 2, and 4. Model 1 has negative slope, Model 2 has positive slope, and Model 4 is horizontal.

3.6 For a uniform distribution on the interval from 0 to w, the density function is $f(x) = 1/w$. The mean residual life is

$$\begin{aligned} e(d) &= \frac{\int_d^w (x-d)w^{-1}dx}{\int_d^w w^{-1}dx} \\ &= \frac{\left.\frac{(x-d)^2}{2w}\right|_d^w}{\frac{w-d}{w}} \\ &= \frac{(w-d)^2}{2(w-d)} \\ &= \frac{w-d}{2}. \end{aligned}$$

The equation becomes

$$\frac{w-30}{2}=\frac{100-30}{2}+4,$$

with a solution of $w = 108$.

3.7 From the definition,

$$e(\lambda)=\frac{\int_\lambda^\infty (x-\lambda)\lambda^{-1}e^{-x/\lambda}dx}{\int_\lambda^\infty \lambda^{-1}e^{-x/\lambda}dx}=\lambda.$$

3.8

$$\begin{aligned} \mathrm{E}(X) &= \int_0^\infty xf(x)dx = \int_0^d xf(x)dx + \int_d^\infty df(x)dx + \int_d^\infty (x-d)f(x)dx \\ &= \int_0^d xf(x)dx + d[1-F(d)] + e(d)S(d) = \mathrm{E}[X\wedge d] + e(d)S(d). \end{aligned}$$

3.9 For Model 1, from (3.8),

$$\mathrm{E}[X\wedge u]=\int_0^u x(0.01)dx+u(1-0.01u)=u(1-0.005u)$$

and from (3.10),

$$\mathrm{E}[X\wedge u]=50-\frac{100-u}{2}(1-0.01u)=u(1-0.005u).$$

From (3.9),

$$\mathrm{E}[X\wedge u]=-\int_{-\infty}^0 0\,dx+\int_0^u 1-0.01x\,dx=u-\frac{0.01u^2}{2}=u(1-0.005u).$$

For Model 2, from (3.8),

$$\mathrm{E}[X \wedge u] = \int_0^u x \frac{3(2{,}000)^3}{(x+2{,}000)^4}\,dx + u\frac{2{,}000^3}{(2{,}000+u)^3} = 1000\left[1 - \frac{4{,}000{,}000}{(2{,}000+u)^2}\right],$$

and from (3.10),

$$\mathrm{E}[X \wedge u] = 1{,}000 - \frac{2{,}000+u}{2}\left(\frac{2{,}000}{2{,}000+u}\right)^3 = 1{,}000\left[1 - \frac{4{,}000{,}000}{(2{,}000+u)^2}\right].$$

From (3.9),

$$\begin{aligned}\mathrm{E}[X \wedge u] &= \int_0^u \left(\frac{2{,}000}{2{,}000+x}\right)^3 dx = \left.\frac{-2{,}000^3}{2(2{,}000+x)^2}\right|_0^u \\ &= 1{,}000\left[1 - \frac{4{,}000{,}000}{(2{,}000+u)^2}\right].\end{aligned}$$

For Model 3, from (3.8),

$$\mathrm{E}[X \wedge u] = \begin{cases} 0(0.5) + u(0.5) = 0.5u, & 0 \le u < 1, \\ 0(0.5) + 1(0.25) + u(0.25) = 0.25 + 0.25u, & 1 \le u < 2, \\ 0(0.5) + 1(0.25) + 2(0.12) + u(0.13) \\ = 0.49 + 0.13u, & 2 \le u < 3, \\ 0(0.5) + 1(0.25) + 2(0.12) + 3(0.08) + u(0.05) \\ = 0.73 + 0.05u, & 3 \le u < 4, \end{cases}$$

and from (3.10),

$$\mathrm{E}[X \wedge u] = \begin{cases} 0.93 - (1.86 - u)(0.5) = 0.5u, & 0 \le u < 1, \\ 0.93 - (2.72 - u)(0.25) = 0.25 + 0.25u, & 1 \le u < 2, \\ 0.93 - (3.3846 - u)(0.13) = 0.49 + 0.13u, & 2 \le u < 3, \\ 0.93 - (4 - u)0(0.05) = 0.73 + 0.05u, & 3 \le u < 4. \end{cases}$$

For Model 4, from (3.8),

$$\begin{aligned}\mathrm{E}[X \wedge u] &= \int_0^u x(0.000003)e^{-0.00001x}dx + u(0.3)e^{-0.00001u} \\ &= 30{,}000[1 - e^{-0.00001u}],\end{aligned}$$

and from (3.10),

$$\mathrm{E}[X \wedge u] = 30{,}000 - 100{,}000(0.3e^{-0.00001u}) = 30{,}000[1 - e^{-0.00001u}].$$

3.10 For a discrete distribution (which all empirical distributions are), the mean residual life function is

$$e(d) = \frac{\sum_{x_j > d}(x_j - d)p(x_j)}{\sum_{x_j > d} p(x_j)}.$$

When d is equal to a possible value of X, the function cannot be continuous because there is a jump in the denominator but not in the numerator. For an exponential distribution, argue as in Exercise 3.7 to see that it is constant. For the Pareto distribution,

$$\begin{aligned} e(d) &= \frac{\mathrm{E}(X) - \mathrm{E}(X \wedge d)}{S(d)} \\ &= \frac{\frac{\theta}{\alpha-1} - \frac{\theta}{\alpha-1}\left[1 - \left(\frac{\theta}{\theta+d}\right)^{\alpha-1}\right]}{\left(\frac{\theta}{\theta+d}\right)^{\alpha}} \\ &= \frac{\theta}{\alpha-1}\frac{\theta+d}{\theta} = \frac{\theta+d}{\alpha-1}, \end{aligned}$$

which is increasing in d. Only the second statement is true.

3.11 Application of the formula from the solution to Exercise 3.10 gives

$$\frac{10{,}000 + 10{,}000}{0.5 - 1} = -40{,}000,$$

which cannot be correct. Recall that the numerator of the mean residual life is $\mathrm{E}(X) - \mathrm{E}(X \wedge d)$. However, when $\alpha \leq 1$, the expected value is infinite and so is the mean residual life.

3.12 The right truncated variable is defined as $Y = X$, given that $X \leq u$. When $X > u$, this variable is not defined. The kth moment is

$$\mathrm{E}(Y^k) = \frac{\int_0^u x^k f(x)dx}{F(u)} = \frac{\sum_{x_i \leq u} x_i^k p(x_i)}{F(u)}.$$

3.13 This is a single-parameter Pareto distribution with parameters $\alpha = 2.5$ and $\theta = 1$. The moments are $\mu_1 = 2.5/1.5 = 5/3$ and $\mu_2 = 2.5/.5 - (5/3)^2 = 20/9$. The coefficient of variation is $\sqrt{20/9}/(5/3) = 0.89443$.

3.14 $\mu = 0.05(100) + 0.2(200) + 0.5(300) + 0.2(400) + 0.05(500) = 300.$
$\sigma^2 = 0.05(-200)^2 + 0.2(-100)^2 + 0.5(0)^2 + 0.2(100)^2 + 0.05(200)^2 = 8{,}000.$
$\mu_3 = 0.05(-200)^3 + 0.2(-100)^3 + 0.5(0)^3 + 0.2(100)^3 + 0.05(200)^3 = 0.$
$\mu_4 = 0.05(-200)^4 + 0.2(-100)^4 + 0.5(0)^4 + 0.2(100)^4 + 0.05(200)^4 = 200{,}000{,}000.$
Skewness is $\gamma_1 = \mu_3/\sigma^3 = 0$. Kurtosis is $\gamma_2 = \mu_4/\sigma^4 = 200{,}000{,}000/8{,}000^2 = 3.125$.

3.15 The Pareto mean residual life function is

$$e_X(d) = \frac{\int_d^\infty \theta^\alpha (x+\theta)^{-\alpha} dx}{\theta^\alpha (x+d)^{-\alpha}} = (d+\theta)/(\alpha - 1),$$

and so $e_X(2\theta)/e_X(\theta) = (2\theta + \theta)/(\theta + \theta) = 1.5$.

3.16 Sample mean: $0.2(400) + 0.7(800) + 0.1(1{,}600) = 800$. Sample variance: $0.2(-400)^2 + 0.7(0)^2 + 0.1(800)^2 = 96{,}000$. Sample third central moment: $0.2(-400)^3 + 0.7(0)^3 + 0.1(800)^3 = 38{,}400{,}000$. Skewness coefficient: $38{,}400{,}000/96{,}000^{1.5} = 1.29$.

SECTION 3.2

3.17 The pdf is $f(x) = 2x^{-3},\ x \geq 1$. The mean is $\int_1^\infty 2x^{-2}dx = 2$. The median is the solution to $0.5 = F(x) = 1 - x^{-2}$, which is 1.4142. The mode is the value at which the *pdf* is highest. Because the pdf is strictly decreasing, the mode is at its smallest value, 1.

3.18 For Model 2, solve $p = 1 - \left(\frac{2{,}000}{2{,}000+\pi_p}\right)^3$, and so $\pi_p = 2{,}000[(1-p)^{-1/3} - 1]$, and the requested percentiles are 519.84 and 1419.95.

For Model 4, the distribution function jumps from 0 to 0.7 at zero and so $\pi_{0.5} = 0$. For percentiles above 70, solve $p = 1 - 0.3e^{-0.00001\pi_p}$, and so $\pi_p = -100{,}000 \ln[(1-p)/0.3]$ and $\pi_{0.8} = 40{,}546.51$.

For Model 5, the distribution function has two specifications. From $x = 0$ to $x = 50$, it rises from 0.0 to 0.5, and so for percentiles at 50 or below, the equation to solve is $p = 0.01\pi_p$ for $\pi_p = 100p$. For $50 < x \leq 75$, the distribution function rises from 0.5 to 1.0, and so for percentiles from 50 to 100 the equation to solve is $p = 0.02\pi_p - 0.5$ for $\pi_p = 50p + 25$. The requested percentiles are 50 and 65.

3.19 The two percentiles imply

$$\begin{aligned} 0.1 &= 1 - \left(\frac{\theta}{\theta + \theta - k}\right)^\alpha, \\ 0.9 &= 1 - \left(\frac{\theta}{\theta + 5\theta - 3k}\right)^\alpha. \end{aligned}$$

Rearranging the equations and taking their ratio yields

$$\frac{0.9}{0.1} = \left(\frac{6\theta - 3k}{2\theta - k}\right)^\alpha = 3^\alpha.$$

Taking logarithms of both sides gives $\ln 9 = \alpha \ln 3$ for $\alpha = \ln 9/\ln 3 = 2$.

3.20 The two percentiles imply

$$\begin{aligned} 0.25 &= 1 - e^{-(1{,}000/\theta)^\tau}, \\ 0.75 &= 1 - e^{-(100{,}000/\theta)^\tau}. \end{aligned}$$

Subtracting and then taking logarithms of both sides gives

$$\begin{aligned} \ln 0.75 &= -(1{,}000/\theta)^\tau, \\ \ln 0.25 &= -(100{,}000/\theta)^\tau. \end{aligned}$$

Dividing the second equation by the first gives

$$\frac{\ln 0.25}{\ln 0.75} = 100^\tau.$$

Finally, taking logarithms of both sides gives $\tau \ln 100 = \ln[\ln 0.25/\ln 0.75]$ for $\tau = 0.3415$.

SECTION 3.3

3.21 The sum has a gamma distribution with parameters $\alpha = 16$ and $\theta = 250$. Then $\Pr(S_{16} > 6{,}000) = 1 - \Gamma(16; 6{,}000/250) = 1 - \Gamma(16; 24)$. From the central limit theorem, the sum has an approximate normal distribution with mean $\alpha\theta = 4{,}000$ and variance $\alpha\theta^2 = 1{,}000{,}000$ for a standard deviation of 1,000. The probability of exceeding 6,000 is $1 - \Phi[(6{,}000 - 4{,}000)/1{,}000] = 1 - \Phi(2) = 0.0228$.

3.22 A single claim has mean $8{,}000/(5/3) = 4{,}800$ and variance

$$2(8{,}000)^2/[(5/3)(2/3)] - 4{,}800^2 = 92{,}160{,}000.$$

The sum of 100 claims has mean 480,000 and variance 9,216,000,000, which is a standard deviation of 96,000. The probability of exceeding 600,000 is approximately

$$1 - \Phi[(600{,}000 - 480{,}000)/96{,}000] = 1 - \Phi(1.25) = 0.106.$$

3.23 The mean of the gamma distribution is $5(1{,}000) = 5{,}000$ and the variance is $5(1{,}000)^2 = 5{,}000{,}000$. For 100 independent claims, the mean is 500,000 and the variance is 500,000,000 for a standard deviation of 22,360.68. The probability of total claims exceeding 525,000 is

$$1 - \Phi[(525{,}000 - 500{,}000)/22{,}360.68] = 1 - \Phi(1.118) = 0.13178.$$

3.24 The sum of 2,500 contracts has an approximate normal distribution with mean $2{,}500(1{,}300) = 3{,}250{,}000$ and standard deviation $\sqrt{2{,}500}(400) = 20{,}000$. The answer is $\Pr(X > 3{,}282{,}500) \doteq \Pr[Z > (3{,}282{,}500 - 3{,}250{,}000)/20{,}000] = \Pr(Z > 1.625) = 0.052$.

SECTION 3.4

3.25 While the Weibull distribution has all positive moments, for the inverse Weibull moments exist only for $k < \tau$. Thus, by this criterion, the inverse Weibull distribution has a heavier tail. With regard to the ratio of density functions, it is (with the inverse Weibull in the numerator, and with asterisks (*) used to mark its parameters)

$$\frac{\tau^* \theta^{*\tau^*} x^{-\tau^*-1} e^{-(\theta^*/x)^{\tau^*}}}{\tau \theta^{-\tau} x^{\tau-1} e^{-(x/\theta)^\tau}} \propto x^{-\tau-\tau^*} e^{-(\theta^*/x)^{\tau^*} + (x/\theta)^\tau}.$$

The logarithm is

$$(x/\theta)^\tau - (\theta^*/x)^{\tau^*} - (\tau + \tau^*) \ln x.$$

The middle term goes to zero, so the issue is the limit of $(x/\theta)^\tau - (\tau + \tau^*) \ln x$, which is clearly infinite. With regard to the hazard rate, for the Weibull distribution we have

$$h(x) = \frac{\tau x^{\tau-1} \theta^{-\tau} e^{-(x/\theta)^\tau}}{e^{-(x/\theta)^\tau}} = \tau x^{\tau-1} \theta^{-\tau},$$

which is clearly increasing when $\tau > 1$, constant when $\tau = 1$, and decreasing when $\tau < 1$. For the inverse Weibull,

$$h(x) = \frac{\tau x^{-\tau-1} \theta^\tau e^{-(\theta/x)^\tau}}{1 - e^{-(\theta/x)^\tau}} \propto \frac{1}{x^{\tau+1}[e^{(\theta/x)^\tau} - 1]}.$$