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IUTAM Symposium on Dynamics Modeling and Interaction Control in Virtual and Real Environments

Proceedings of the IUTAM Symposium on Dynamics Modeling and Interaction Control in Virtual and Real Environments, Held in Budapest, Hungary, June 7–11, 2010

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Preface

This volume contains the invited papers presented at the IUTAM Symposium on Multibody Dynamics and Interaction Control in Virtual and Real Environments held in Budapest, Hungary, June 7−11 2010.

The symposium aimed to bring together specialists in the fields of multibody system modeling, contact/collision mechanics and control of mechanical systems. The offered topics included modeling aspects, mechanical and mathematical models, the question of neglections and simplifications, reduction of large systems, interaction with environment like air, water and obstacles, contact of all types, control concepts, control stability and optimization.

Discussions between the experts of these fields made it possible to exchange ideas about the recent advances in multibody system modeling and interaction control, as well as about the possible future trends. The presentations of recent scientific results may facilitate the interaction between scientific areas like system/control engineering and mechanical engineering.

A Scientific Committee was appointed by the Bureau of IUTAM including the following members:

G. Stépán (Hungary, Chairman)
F.L. Chernousko (Russia, IUTAM Representative)
P. Bidaud (France)
W. Harwin (United Kingdom)
J. Kövecses (Canada)
F. Pfeiffer (Germany)
B. Siciliano (Italy)
H. Yabuno (Japan)

The committee selected the participants to be involved and the papers to be presented at the symposium. As a result of this procedure, 45 active scientific participants from 17 countries accepted the invitation and 39 papers and 4 keynote lectures were presented in 8 sessions.

Papers on dynamics modeling and interaction control were naturally selected into the main areas: mathematical modeling, dynamic analysis, friction modeling, solid and thermomechanical aspects, and applications. The presentations were almost equally divided between experimental and computational work and several ones addressed the question of modeling and simulation of complex multibody systems such as gearboxes and vehicles. There were separate sessions on theoretical aspects with a number of lecturers applying fractional order calculus in dynamical modeling of oscillatory systems and investigating chaotic solutions that may arise in case of many contact tasks. In addition, a separate session was offered to friction modeling that was also addressed by several authors during dynamics modeling of biologically inspired robots. A significant outcome of the meeting was the opening towards applications that has a key importance in the future of nonlinear dynamics.

Rather than presenting the papers in the order they were delivered at the Symposium, the contributions are grouped according to topic and application. The ordering is not unique as some papers may fit in several categories.

The scientific presentations were devoted to the following topics, and ordered in this book as follows:

- Dynamics modeling and control of robots (7 talks)
- Applications and control of bio-inspired robots (3 talks)
- Vehicle dynamics and control (3 talks)
- Mathematical modeling of oscillatory systems (7 talks)
- Biomechanics and rehabilitation (6 talks)
- Micro-electromechanical Systems (6 talks)
- Modeling dry friction (4 talks)
- Thermoelasticity aspects (3 talks)

The four keynote lectures indicate the multi-disciplinary character of the Symposium while they span the important topics of dynamics modeling and interaction control in virtual and real environments:

- Kouhei Ohnishi (Keio University, Yokohama, Japan)
  Real World Haptics and Telehaptics
- József Kövecses (McGill University, Montreal, Quebec, Canada)
  Approaches to Lagrangian Dynamics and Their Application to Interactions with Virtual Environments
- Michael Beitelschmidt (Technical University of Dresden, Germany)
  Real Time Simulation and Actuation of Shifting Forces of a Gearbox
- Philippe Bidaud (Institute of Intelligent Systems and Robotics, Paris, France)
  Stability analysis and dynamic control of multi-limb robotic systems

The editors wish to thank both the keynote lectures and participants, and the authors of the papers for their contributions to the important fields of multibody dynamics, interaction control and the closely related branch of disciplines reflected by the titles of topical sessions.

The success of the symposium would not have been possible without the work of the Local Organizing Committee. The members of that were:

Gábor Stépán (Chairman)
László L. Kovács (Secretary)
András Tóth (Project manager)

In addition thanks are due to Springer Science+Business Media for efficient cooperation and to Ms. Nathalie Jacobs for her help and encouragement to the publication of this volume.

Budapest
October 2010

Gábor Stépán

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Dynamics Modeling and Control of Robots

These papers explore dynamics modeling tools and control concepts proposed for some new and special robotic applications including personal, entertainment, micro-scale and space robots. Dynamics modeling aspects and control of an underactuated service robot platform developed within the frame of the ACROBOTER project (FP6-IST-6 45530) are discussed in the papers by L.L. Kovács and A. Zelei. The paper by T. Gorius presents simulation and experimental results obtained with the prototype of an interactive 3D pendulum presented at the Word Exhibition EXPO 2010 in Shanghai. The nonlinear control of a micro-cantilever probe of an atomic force microscope is investigated by H. Yabuno, while the presented work of F. Matsumoto addresses the dynamics and trajectory generation of a space robot model.

The ACROBOTER Platform – Part 1: Conceptual Design and Dynamics Modeling Aspects

László L. Kovács, Ambrus Zelei, László Bencsik, and Gábor Stépán

Abstract. This paper presents the conceptual design and the dynamics modeling aspects of a pendulum-like under–actuated service robot platform ACROBOTER. The robot is designed to operate in indoor environments and perform pick and place tasks as well as carry other service robots with lower mobility. The ACROBOTER platform extends the workspace of these robots to the whole cubic volume of the indoor environment by utilizing the ceiling for planar movements. The cable suspended platform has a complex structure the dynamics of which is difficult to be modeled by using conventional robotic approaches. Instead, in this paper natural (Cartesian) coordinates are proposed to describe the configuration of the robot which leads to a dynamical model in the form of differential algebraic equations. The evolution of the ACROBOTER concepts is described in detail with a particular attention on the under–actuation and redundancy of the system. The influence of these properties and the applied differential algebraic model on the controller design is discussed.

1 Introduction

Obstacle avoidance is an important problem in service and mobile robotics. Robots operating in indoor environments have to overcome various static obstacles on the floor, e.g., chairs, tables, doorsteps and even the edges of carpets in a room. Thus floor based domestic robots need to have strategies to detect and avoid these randomly placed objects.

A new direction in the development of indoor service robots is the use of robotic structures that can move on the walls and/or on the ceiling of a room. An advantage

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of this strategy is that walls and rather the ceiling are almost obstacle free enabling robots to move freely and quickly in any direction. An application which addresses the need for a robot to climb on the walls and crawl on the ceiling inside a building is the MATS robot [1]. Another examples include the mobile robot platform described in [7] and the FLORA walking assisting system developed by FATEC Corporation [5]. Both of them are based on a ceiling absorbed mobile cart utilizing permanent magnets to keep and move the cart on the ceiling. The system [7] has a working unit that is positioned by three parallel telescopic arms. The FLORA robot has a specially designed cable suspended sit harness that can be used to compensate for the body weight of elderly or physically impaired people during walking. These platform concepts solve the problem of avoiding obstacles on the floor, while they are able to roam over almost the whole inner space of a room, and compared to gantry cranes, they enable the use of co-operating multiple units.

Similary, the ACROBOTER service robot is a ceiling based platform. Figures 1 and 4 present that the Climbing Unit (CU) can move in the plane of the suspended ceiling equipped with anchor points. In this first concept the CU is a planar robotic arm, which swaps between the anchor points and provides the crawling motion of the suspension of the robot. The windable suspending cable holds the Swinging Unit (SU) to which the carried objects can be connected. The system has a pendulum like structure, but compared to the above described systems [5, 7], here, the positioning of the payload is controlled by the actuators (fluid mass flow generators) of the SU. The proposed concept combine the planar stepping motion of the arm and the thrusted-hoisted pendulum-like motion of the working unit in 3D relative to the arm.

A design of a tethered aerial robot with a swinging actuator is presented in [6]. In this concept the weight of the robot is carried by a tether and the thrusting forces are generated by two fans with parallel axes. The main task of the robot is to carry a camera (and/or tiny robotic agents) used in rescue operations above unstructured

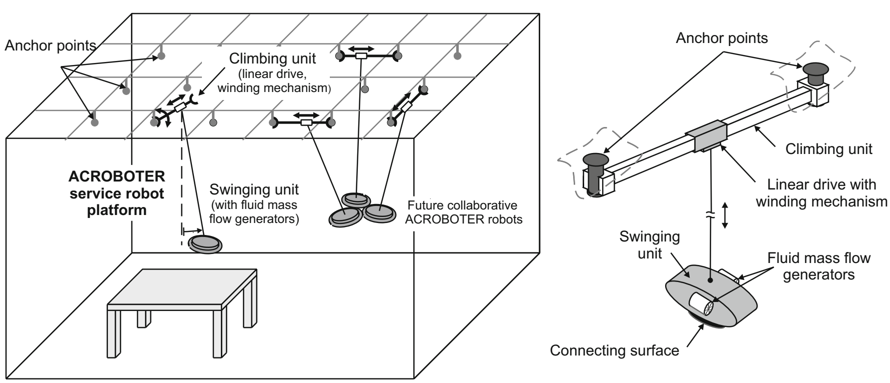

Fig. 1 The ACROBOTER concept: pendulum-like indoor service robot platform (left), main functional components of the system (right)

environments (e.g. at earthquake sites). Despite of the conceptual similarities to tethered aerial systems the different application scenarios of ACROBOTER requires a completely different design. The ACROBOTER platform ensures higher payload capability than aerial robots which may results in high inertial forces and nonlinear oscillations of the payload. In addition the pendulum like behavior of the system may also be utilized to provide better maneuverability and larger workspace in case of proper motion planning of the climbing unit.

2 Conceptual Design

The main task of ACROBOTER is to carry objects in the 3D space of inner environments. This can be accomplished in a set-point manner or the task may require the tracking of some desired trajectories. Hence, the swinging unit (see Fig. 1) needs to be fully actuated and controlled in the 3D environment. The CU has to move the suspension point of the swinging unit smoothly in the plane of the ceiling, while the actuators of the SU have to provide the desired orientation of the unit and its position relative to the CU.

In case of ACROBOTER the ceiling based traction unit, i.e. the climbing unit, is a planar RRT robot. The redundancy of this robot enables smooth planar movements when the CU has to swap between different anchor points (see left in Fig. 4). Different solutions like [5, 7] that provide the smooth movement of the suspension can also be considered.

The present chapter focuses on the design aspects of the swinging unit. The first prototype of the SU is presented in Fig. 2. In this concept two large diameter ducted fans are used to provide the desired nutation of the unit, while three windable orienting cables are used to control the roll and pitch of the platform. Ducted fans are

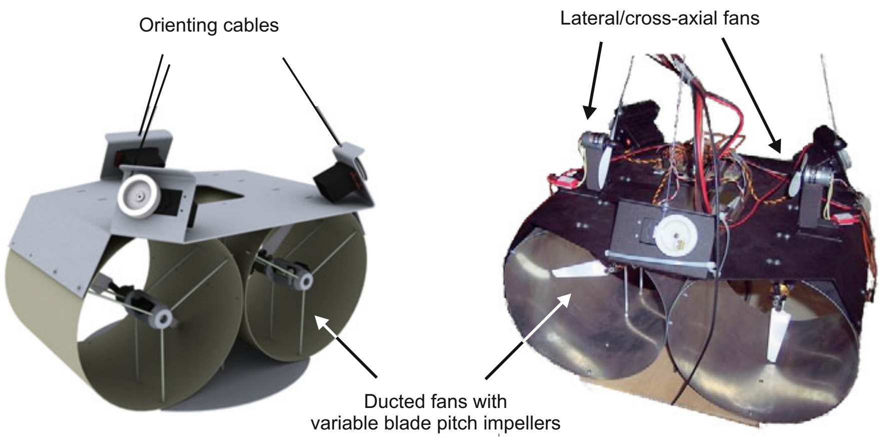

Fig. 2 First prototype of ACROBOTER: CAD design (left), built prototype with complementary lateral/cross-axial fans (right)

lightweight solution for on board thrust generation. The magnitude of the generated thrust is, however, proportional to the diameter of the fans which increases the size of the device when heavy payloads need to be nutated. The moment generated by the two parallel axis fans controls the yaw angle. In addition, the applied ducted fans have variable blade pitch impellers that can adjust the magnitude of the thrust forces or even they can quickly be reversed. This solution can provide large thrust forces, but the maneuverability of the concept is limited. The two ducted fans with parallel axes can only provide a thrust force paralell to the axes of the fans and a resultant moment acting upon the suspended payload as independent control inputs. Thus, the number of actuators is lower than the degrees of freedom of the suspended payload. To resolve the underactuation of the concept a pair of lateral/cross-axial fans were attached to the top of the first prototype of the SU (see right in Fig. 2).

The recognition of the under actuated character of the first design result in a new concept, which was also motivated by the need for decreasing the vertical dimension of the unit. The CAD models of the new concept are shown in Fig. 3 and the corresponding second prototype is depicted at right in Fig. 4. In this concept six identical ducted fans are used as thrusters that are placed around the circumference of a disk. The advantage of this solution is that the swinging unit can be fully actuated by using same ducted fan modules each providing a one-directional thrust force. The price that has to be paid is the lower resultant thrust that the smaller fans can provide. Therefore, developing the second prototype it was assumed that the SU will not provide large nutation for the carried objects. Instead, the fans will more effectively stabilize the motion of the unit around a desired trajectory. The gross motion is generated by the CU while the SU continuously compensates for the tracking errors.

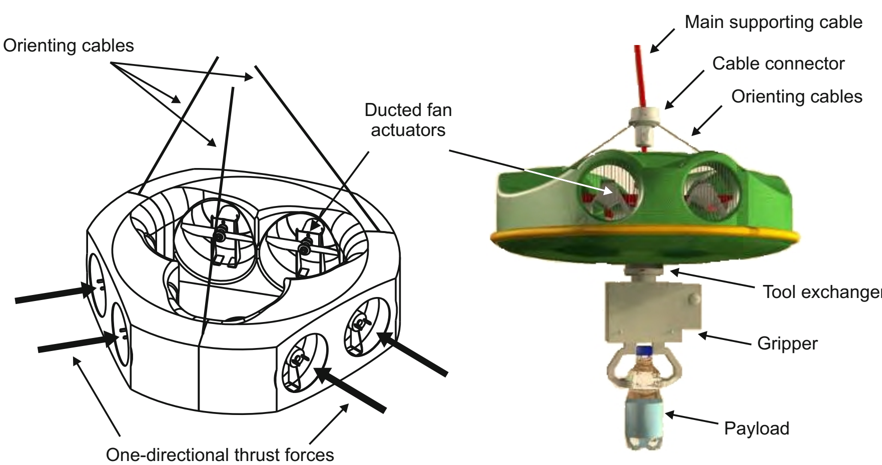

Fig. 3 Second prototype of ACROBOTER with six equally sized one-directional ducted fan actuators

3 Dynamics Modeling

The whole system prototype of ACROBOTER is presented in Fig. 4. This figure shows that the climbing unit is an RRT robot which has 3 degrees-of-freedom, when its upper (anchor) arm is attached to an anchor point. In this respect the CU can be described by conventional robotic approaches using the minimum set of descriptor coordinated, i.e. the generalized coordinates associated with the serial arm structure. The lower (rotation) arm of the CU is a linear axis that carries the winding mechanism hoisting the SU. Thus, including the winding mechanism, the climbing unit has 4DoFs. The main suspending cable and the orienting cables of the SU are connected by the cable connector (CC), which is a relatively small sized component and therefore can be modeled as a point mass with additional 3 DoFs. Then, considering the spatial 6DoFs of the SU, the system has 12DoFs in total, which requires the same number of generalized coordinates to describe its configuration.

Considering the completely different joint structure and actuation of the CU and the SU, the two systems were modeled independently from each other. The CU and the SU have separate motion controllers that are synchronized by the global motion controller of the system. The kinematic description of the CU follows the conventional robotic description and therefore not described here. The SU and the CC forms a cable suspended structure with a closed kinematic chain, where the ducted fan actuators cannot be associated with the real and/or virtual "joints" of the robotic structure. Thus, in case of the kinematic description of the SU the choice of coordinates has a key importance in obtaining a still complex but computationally affordable dynamical model.

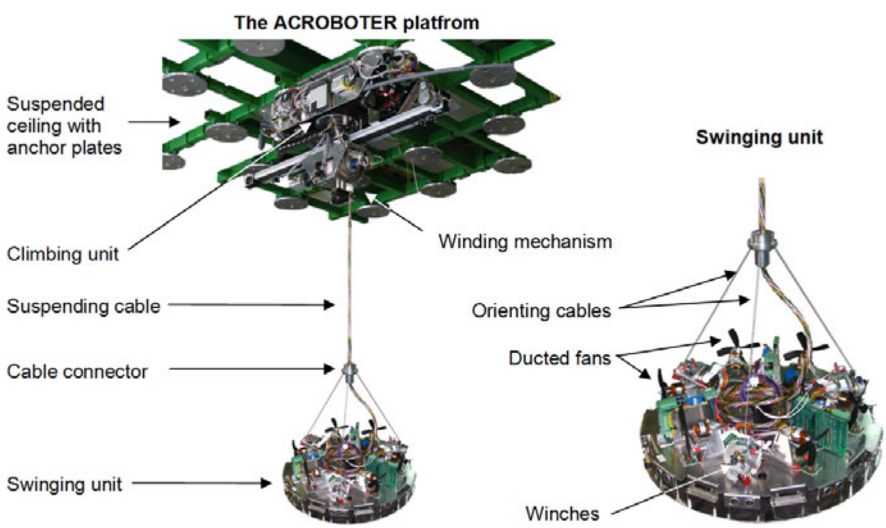

Fig. 4 The ACROBOTER service robot platform

An efficient parameterization of the affine transformation between the global (world) and the local (body) coordinate system of the SU is the use of natural coordinates originally introduced by [2]. This formalism uses a non-minimum set of specially chosen descriptor coordinates for the kinematic description of multibody systems including robotic structures, and the corresponding dynamics modeling is based on the Lagrangian equations of the first kind. This leads to the equations of motion in the well-known set of differential-algebraic equations of index 3

$$\mathbf{M}\ddot{\mathbf{q}} + \boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}}(\mathbf{q})\boldsymbol{\lambda} = \mathbf{Q}_g + \mathbf{H}(\mathbf{q})\mathbf{u}\,, \tag{1}$$

$$\boldsymbol{\phi}(\mathbf{q}) = \mathbf{0}\,, \tag{2}$$

where $\mathbf{q}$ denotes a redundant set of coordinates associated with the CC and the SU. The position of the CC is given by its Cartesian coordinates, while the pose of the SU is described by the coordinates of the co-planar points identified by the outlets of the winches and a unit vector which is perpendicular to the base of the SU. According to [2], the selection of these descriptor coordinates results in a constant mass matrix. The mass matrix $\mathbf{M}$, here, is a block diagonal matrix containing the mass matrices of the CC and SU, respectively. In eq. (1) the matrix $\boldsymbol{\Phi}_{\mathbf{q}} = \partial\boldsymbol{\phi}/\partial\mathbf{q}$ is the constraint Jacobian, $\boldsymbol{\lambda}$ is the vector of Lagrangian multipliers , $\mathbf{Q}_g$ is the constant generalized gravity force and $\mathbf{H}$ is the transmission matrix corresponding to the input vector $\mathbf{u}$. Note that in case of ACROBOTER the constraint equations (2) stand for the squared distances of the basic points (selected as cable outlets of the winches), and the perpendicularity and length of the unit vector the coordinates of which are also used as descriptor coordinates. Consequently, the constraint Jacobian is a linear function of the descriptor coordinates. Although, the number of coordinates are relatively high (15 in case of the model of the SU and the CC), the properties and the special structure of the resulting equations of motion make it possible to derive a real-time dynamic model of ACROBOTER.

Various methods exist for the solution of the equations of motion (1, 2). Simulation techniques involve the classical method of Lagrangian multipliers with Baumgarte stabilization [2, 8], and the projection method [3] which transforms the original set of differential algebraic equations (DAE) of motion to ordinary differential equations (ODE). Another possibility is to substitute the algebraic equations with singularly perturbed differential equations and use available stiff ODE solvers. The direct solution of equations (1) and (2) is possible by the index-reduction of the DAE problem which leads to the full descriptor form of index 1

$$\dot{\mathbf{q}} = \mathbf{p} \tag{3}$$

$$\dot{\mathbf{p}} = \mathbf{a} \tag{4}$$

$$\begin{bmatrix} \mathbf{M} & \boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}} \\ \boldsymbol{\Phi}_{\mathbf{q}} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{a} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_g + \mathbf{H}(\mathbf{q})\mathbf{u} \\ -\dot{\boldsymbol{\Phi}}_{\mathbf{q}}(\mathbf{q},\mathbf{p})\mathbf{p} \end{bmatrix} \tag{5}$$

$$\boldsymbol{\phi}(\mathbf{q}) = \mathbf{0}\,. \tag{6}$$

The system of equations (4–6) can numerically be solved by backward difference methods (like the Backward-Euler method) via applying the Newton-Raphson iteration scheme to the resulting system of implicit algebraic equations.

4 Control Aspects

Based on the kinematic structure of ACROBOTER described in Section 3 it can be seen that the CU is a redundant manipulator, and moreover the base coordinate system of this arm is changing during swapping between the anchor points. This is an important issue in the control design of the climbing unit. Since the main suspending cable and the orienting cables can equally move the SU in the vertical direction the cable suspension system has a redundant character too. By assuming that the CU regulates the motion of the upper end of the main cable perfectly, the motion of this suspension point can be seen as a constraint on the independent dynamic model of the SU. This way the system formed by the SU and the CC has 9DoFs controlled by 7 actuators only. These include the 4 cable winches and a fictitious compound actuator that provides the two components of the resultant force and the resultant moment generated by the ducted fans. Thus the system is under–actuated, which means that two coordinates out of nine cannot be prescribed arbitrarily because they depend on the internal dynamics of the system. For example, consider that the SU have to move horizontally and the the elevation of the CC above the SU is prescribed. Then the motion of the CC parallel to the base of the SU cannot be actuated. Existing under–actuated robot control techniques (like [9]) are available for the class of systems where the equations of motions without control inputs can easily be identified, which is often the case for serial manipulators. When the control inputs are coupled by a transmission matrix the equation that describes the internal dynamics of the system can be achieved by projecting the equation of motion into the null-space of this matrix. Then, separating the coordinates into controlled and uncontrolled ones, the projected equation can be solved for the uncontrolled coordinates. These calculated coordinates can be seen as prescribed (uncontrolled) coordinates, which enables the generalization of the computed torque control method to under actuated robotic systems [4] modeled by minimum set of generalized coordinates. The computed torque control of ACROBOTER is based on the direct discretization of the differential algebraic equations of motion, which method is described in detail in Part 2 of the present work.

5 Conclusions

The main idea of a novel locomotion technology was presented in this paper and the design concepts of the cable suspended ACROBOTER platform were discussed. The main differences between the presented concepts are their vertical dimension and their maneuverability. Independently of the selected second prototype of the swinging unit it was concluded that the ACROBOTER has a complex spatial structure and it is advantageous to derive its equation of motion in terms of a redundant set of descriptor coordinates. The applied DAE model enables the efficient simulation of the model, while the use of the selected natural coordinates make it straightforward to calculate the forward/inverse kinematics of the system. The actuation scheme of the ACROBOTER robot were also discussed with a view on the possible realization of a computed torque controller.

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The ACROBOTER Platform - Part 2: Servo-Constraints in Computed Torque Control

Ambrus Zelei and Gábor Stépán

Abstract. The paper presents the motion control of the ceiling based service robot platform ACROBOTER that contains two main subsystems. The climbing unit is a serial robot, which realizes planar motion in the plane of the ceiling. The swinging unit is hoisted by the climbing unit and it is actuated by windable cables and ducted fans. The two subsystems form a serial and subsequent closed-loop kinematic chain segments. Because of the complexity of the system we use natural (Cartesian) coordinates to describe the configuration of the robot, while a set of algebraic equations represents the geometric constraints. Thus the dynamical model of the system is given in the form of differential-algebraic equations (DAE). The system is under-actuated and the the inverse kinematics and dynamics cannot be solved in closed form. The control task is defined by the servo-constraints which are algebraic equations that have to be considered during the calculation of control forces. In this paper the desired control inputs are determined via the numerical solution of the resulting DAE problem using the Backward Euler discretization method.

1 Introduction

Indoor service robots can effectively use the ceiling of the indoor environment to provide obstacle free motion of the base of these robots, while the carried working units can practically move in the whole inner space of the environment [6]. The present paper describes the motion control of a new service robot platform developed within the ACROBOTER (IST-2006-045530) project (see details in Part 1 of

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the present paper). The developed robot utilizing the ceiling for the planar motion, and its cable suspended pendulum-like subsystem is the working unit.

A major challenge in ceiling based locomotion is that the ceiling based unit has to hold the total weight of the robot and the payload safely, and has to provide fast motion of the carried objects at the same time. To satisfy these requirements, permanent magnets are applied to develop a ceiling absorbed mobile base in [6]. In case of ACROBOTER, a serial robot based climbing unit was developed that can crawl on an anchor point system installed on the ceiling (see left in Fig. 1).

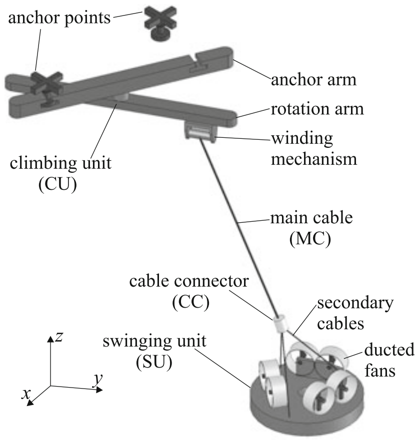
The other main subsystem of ACROBOTER is the swinging unit. It is connected to the climbing unit via a windable cable that is called the main cable hereafter. The swinging unit has a mechanical interface to connect different tools. This unit can be positioned and oriented with three orineting secondary cables and ducted fan actuators. For more detailed description of the ACROBOTER design the reader is referred to Part 1.The kinematic structure of its planar mechanical model is described in detail in Section 2.

In this paper, first, the dynamical model of the investigated complex robotic structure is described by natural coordinates [2]. The resulting equations of motion are formulated as Lagrangian equations of motion of the first kind. In addition to the introduced geometric constraints, the task of the robot is defined by the so-called servo-constraints which introduce further algebraic equations associated with the original DAE problem of calculating the desired control inputs of the computed torque control (CTC) of ACROBOTER. In the second part of this work the numerical solution of the DAE system of equations is presented by using the Backward Euler discretization method. At the end a real parameter case simulation study is provided to demonstrate the applicability of the proposed controller.

2 Structure of the ACROBOTER Platform

The mechanical structure of the ACROBOTER can be seen left in Fig. 1. The climbing unit is an RRT robot that provides the ceiling based locomotion of the system. Its task is to position the suspension point (cable outlet) in the plane of the ceiling. The climbing unit consists of the anchor arm, the rotation arm and a linear axis moving the winding mechanism. The anchor arm swaps between neighboring anchor points, while the rotation arm and the linear axis provide additional two degrees-of-freedom (DoFs). Thus the climbing unit is a kinematically redundant planar manipulator, except in the case when both ends of the anchor arm is fixed to the ceiling. The winding mechanism hoists the swinging unit via the main cable, which unit contains three additional cable actuators. The main role of these cables to control the orientation of the unit, but they also can regulate its elevation yielding a further redundancy. In addition to these cables, three pairs of ducted fans are employed to orient and position the swinging unit. The orienting cables are assumed to be ideal in the model.

In the planar model shown right in Fig. 1, the climbing unit is considered as a single linear axis. The cable connector modeled as a point mass with 2 DoFs and



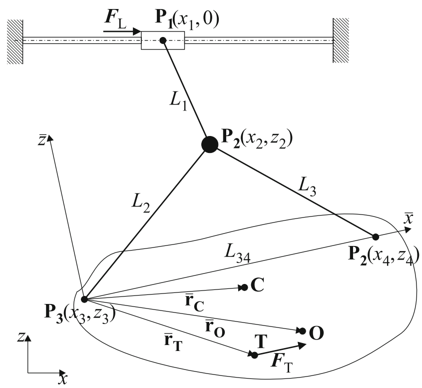


Fig. 1 The ACROBOTER structure (left), planar mechanical model (right)

the swinging unit is a rigid body with 3 DoFs. The kinematics of the swinging unit is described by the redundant set of coordinates associated with points $\mathbf{P}_3$ and $\mathbf{P}_4$.

The total number of DoFs is 6 and we use 7 descriptor coordinates plus one geometric constraint which represents the constant distance L_{34}.

The position of the climbing unit is controlled by the force F_L, while the swinging unit is actuated via the cable forces F_1, F_2 and F_3 and the thrust force F_T. In the model **C** denotes the center of gravity of the swinging unit. Point **T** determines the line of action of the thrust force F_T being parallel to the local axis $\bar{x}$. And **O** is an arbitrarily selected point that has to be controller to move on the desired trajectory.

3 CTC with Backward Euler Discretication

The equations of motion (1) and (2) of the system is derived in the form of the Lagrangian equation of motion of the first kind:

$$\mathbf{M}\ddot{\mathbf{q}} + \boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}}(\mathbf{q})\boldsymbol{\lambda} = \mathbf{Q}_g + \mathbf{H}(\mathbf{q})\mathbf{u}\,, \tag{1}$$

$$\boldsymbol{\phi}(\mathbf{q}) = \mathbf{0}\,, \tag{2}$$

where $\mathbf{q} \in \mathbb{R}^n$ denotes the descriptor coordinates and $\mathbf{M} \in \mathbb{R}^{n\times n}$ is the constant mass matrix. In eq. (2) the vector $\boldsymbol{\phi}(\mathbf{q}) \in \mathbb{R}^m$ represents geometric constraints. Matrix $\boldsymbol{\Phi}_{\mathbf{q}}(\mathbf{q}) = \partial\boldsymbol{\phi}(\mathbf{q})/\partial\mathbf{q} \in \mathbb{R}^{m\times n}$ is the constraint Jacobian. Vector $\boldsymbol{\lambda} \in \mathbb{R}^m$ contains the Lagrange multipliers and $\mathbf{Q}_g \in \mathbb{R}^n$ is the constant generalized force vector of the gravitational terms. The control input vector is $\mathbf{u} \in \mathbb{R}^l$ is mapped by the input matrix $\mathbf{H}(\mathbf{q}) \in \mathbb{R}^{n\times l}$. The above formalism can directly be applied to the spatial case yielding the same form of the equations of motion as (1) and (2).

The inverse kinematical and dynamical calculations have unique solution if the number of control inputs and the dimension of the task is equal [1]. Thus the task have to be defined by l number of algebraic equations. This set of additional constraint equations are the so-called servo-constraints (control-constraints) $\boldsymbol{\phi}_s(\mathbf{q},\mathbf{p}(t)) = \mathbf{0}$. We assume that these servo-constraint equations can be written in the form $\boldsymbol{\phi}_s(\mathbf{q},\mathbf{p}(t)) = \mathbf{g}(\mathbf{q}) - \mathbf{p}(t)$ where $\mathbf{g}(\mathbf{q})$ represents, for example, the end-effector position of the robot and $\mathbf{p}(t)$ is the performance goal to be realized [1].

For under-actuated robotic systems modeled by Lagrangian equation of motion of the second kind, the computed torque control method was generalized in [5]. The generalized method is called Computed Desired Computed Torque Control (CDCTC) method. Here the expression "computed desired" refers to the fact that a set of uncontrolled coordinates can be separated from the controlled ones, and the desired values of these uncontrolled coordinates have to be calculated by considering the internal dynamics of the system.

The CDCTC method proposed in [5] can be applied to dynamical systems that are described by ordinary differential equations only. This problem can be resolved by projecting the equations of motion 1 and 2 to the subspace of admissible motions associated with the geometric constraints [4]. The simultaneous application of this projection (including the configuration corrections during the numerical solution) and the CDCTC algorithm is complex and computationally expensive. In addition, it has to be noted that the selection of the controlled and uncontrolled coordinates might be highly intuitive in case of complex (non-convetional) robotic structure like ACROBOTER. The introduction of this kind of distinct coordinates is possible only if the servo-constraint equations can be solved in closed form for the set of controlled coordinates.

Instead of the application of the CDCTC method, we apply the Backward Euler discretization for the DAE system the resulting set of implicit equations are solved by the Newton-Raphson method for the desired control inputs $\mathbf{u}$. Considering a PD controller with gain matrices $\mathbf{K}_P$ and $\mathbf{K}_D$ the control law can be formulated as

$$\mathbf{M}\ddot{\mathbf{q}}^d + \boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}}(\mathbf{q}^d)\boldsymbol{\lambda}^d = \mathbf{Q}_g + \mathbf{H}(\mathbf{q}^d)\mathbf{u} - \mathbf{K}_P(\mathbf{q}^d - \mathbf{q}) - \mathbf{K}_D(\dot{\mathbf{q}}^d - \dot{\mathbf{q}}) \,, \tag{3}$$

$$\boldsymbol{\phi}_s(\mathbf{q}^d, \mathbf{p}(t)) = \mathbf{0} \,, \tag{4}$$

$$\boldsymbol{\phi}(\mathbf{q}^d) = \mathbf{0} \,, \tag{5}$$

where superscript d refers to desired quantities. Then, the first order form of equation (3) reads

$$\dot{\mathbf{q}}^d = \mathbf{y}^d \tag{6}$$

$$\dot{\mathbf{y}}^d = \mathbf{M}^{-1}\left[-\boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}}(\mathbf{q}^d)\boldsymbol{\lambda}^d + \mathbf{Q}_g + \mathbf{H}(\mathbf{q}^d)\mathbf{u} - \mathbf{K}_P(\mathbf{q}^d - \mathbf{q}) - \mathbf{K}_D(\dot{\mathbf{q}}^d - \dot{\mathbf{q}})\right] . \tag{7}$$

Equations (6) and (7) are first order ordinary differential equations, while equations (4) and (5) are algebraic ones. We use the Backward Euler formula with timestep h to discretize the DAE system, that result in the set of nonlinear algebraic equations for the unknowns desired values $\mathbf{z}_{i+1} = \left[\, \mathbf{q}^d_{i+1} \; \mathbf{y}^d_{i+1} \; \mathbf{u}_{i+1} \; \boldsymbol{\lambda}^d_{i+1} \,\right]^{\mathrm{T}}$ in the form:

$$\mathbf{F}(\mathbf{z}_{i+1}) = \begin{bmatrix} \mathbf{q}^d_{i+1} - \mathbf{q}^d_i - h\mathbf{y}^d \\ \mathbf{y}^d_{i+1} - \mathbf{y}^d_i - h\mathbf{M}^{-1}\left[-\boldsymbol{\Phi}^{\mathrm{T}}_{\mathbf{q}}(\mathbf{q}^d_{i+1})\boldsymbol{\lambda}^d_{i+1} + \mathbf{Q}_g + \mathbf{H}(\mathbf{q}^d_{i+1})\mathbf{u}_{i+1} - \mathbf{K}\mathbf{e}\right] \\ \boldsymbol{\phi}_s(\mathbf{q}^d_{i+1}, \mathbf{p}(t_{i+1})) \\ \boldsymbol{\phi}(\mathbf{q}^d_{i+1}) \end{bmatrix} \tag{8}$$

$$\text{with} \quad \mathbf{K} = \left[\, \mathbf{K}_P \,\vdots\, \mathbf{K}_D \,\right] \quad \text{and} \quad \mathbf{e} = \begin{bmatrix} \mathbf{q}^d_{i+1} - \mathbf{q}_{i+1} \\ \hdashline \dot{\mathbf{q}}^d_{i+1} - \dot{\mathbf{q}}_{i+1} \end{bmatrix}. \tag{9}$$

For the solution the initial values are $\mathbf{q}^d_0$ and $\mathbf{y}^d_0$ at $i = 0$. They should satisfy the servo-constraints and the geometric constraints. During simulation the initial values of the states $\mathbf{q}_0$ and $\mathbf{y}_0$ only have to satisfy the geometric constraints. The numerical solution of 8 and 9 is based on the well-known Newton-Raphson method. The corresponding Jacobian matrix $\mathbf{J}(\mathbf{z}_{i+1}) = \partial\mathbf{F}(\mathbf{z}_{i+1})/\partial\mathbf{z}_{i+1}$ is calculated numerically, however it could also be constructed semi-analytically [3]. Then the iteration

$$\mathbf{z}^{n+1}_{i+1} = \mathbf{z}^n_{i+1} - \mathbf{J}(\mathbf{z}^n_{i+1})\mathbf{F}(\mathbf{z}^n_{i+1}) \tag{10}$$

provides the solution at each time instants, where $\mathbf{z}^n_{i+1}$ is the n^{th} approximation of $\mathbf{z}_{i+1}$. The initial guess $\mathbf{z}^0_{i+1}$ in each time step comes from the best approximation $\mathbf{z}^N_i$ of the previous time step. Usually the Newtor-Raphson iteration converges in $N = 2 \div 6$ steps depending also on the required tolerance.

4 Real Parameter Case Simulation

This section presents the simulation results obtained for the planar model of ACROBOTER shown right in Fig. 1. The selected descriptor coordinates are the Cartesian coordinates of the points $\mathbf{P}_i$, $i = 1 \ldots 4$ yielding $\mathbf{q} = [\, x_1 \; x_2 \; z_2 \; x_3 \; z_3 \; x_4 \; z_4 \,]^{\mathrm{T}}$. The control inputs are colleted in vector $\mathbf{u} = [\, F_L \; F_1 \; F_2 \; F_3 \; F_T \,]^{\mathrm{T}}$. The single geometric constraint represents the constant distance L_{34} between the points $\mathbf{P}_3$ and $\mathbf{P}_4$ and can be written as:

$$\boldsymbol{\phi}(\mathbf{q}) = \left[(x_3 - x_4)^2 + (z_3 - z_4)^2 - L^2_{34}\right] \tag{11}$$

The mass of the swinging unit is $m_{SU} = 9.3$kg and its moment of inertia with respect to the axis at $\mathbf{P}_3$ is $I_{SU} = 0.4$kgm^2. The mass of the cable connector is $m_{CC} = 0.5$kg and the mass of the linear drive that represents the climber unit is $m_{CU} = 20$kg. The distance L_{34} is set to be 0.4m. The position of the center of gravity is given by $\bar{\mathbf{r}}_C = [\, 0.2 \; 0.05 \,]^{\mathrm{T}}$, while the point of application of the thrust force and the position of point $\mathbf{O}$ are defined by the vectors $\bar{\mathbf{r}}_T = [\, 0.2 \; -0.05 \,]^{\mathrm{T}}$ and $\bar{\mathbf{r}}_O = [\, 0.2 \; 0 \,]^{\mathrm{T}}$ respectively in the local frame $(\bar{x}, \bar{z})$ measured in meters.

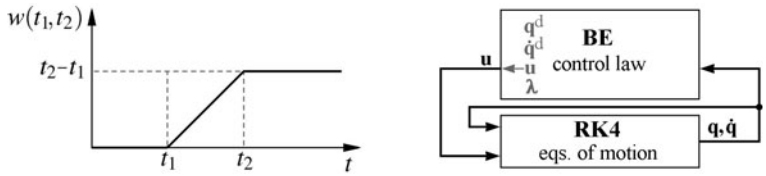


Fig. 2 Modified ramp function (left), block diagram of the simulation (right)

The task of the robot is to track a given trajectory of the point **O**. At the same time the elevation of the cable connector and the horizontality of the swinging unit are also prescribed. The servo-constraint $\boldsymbol{\phi}_s(\mathbf{q},\mathbf{p}(t)) = \mathbf{g}(\mathbf{q}) - \mathbf{p}(t)$ is defined by

$$\mathbf{g}(\mathbf{q}) = \left[x_1 \;\; z_2 - \frac{z_3+z_4}{2} \;\; \frac{x_3+x_4}{2} \;\; \frac{z_3+z_4}{2} \;\; z_3 - z_4 \right]^{\mathrm{T}}, \tag{12}$$

$$\mathbf{p}(t) = \left[x^d_{CU} \;\; h^d_{CC} \;\; x^d_{SU} \;\; z^d_{SU} \;\; 0 \right]^{\mathrm{T}}, \tag{13}$$

where the desired climbing unit position $x^d_{CU} = 0.4w(3,4)$, the desired cable connector elevation $h^d_{CC} = 0.8 - 0.2w(2,4)$, the desired swinging unit horizontal position $x^d_{SU} = 0.2w(2,4)$ and the vertical position $z^d_{SU} = -1.5 + 0.4w(2,4)$ are given in meters and they are used to calculate the reference values of the controller. The corresponding weighting functions $w(2,4)$ and $w(3,4)$ are defined by $w(t_1,t_2)$ as shown in Fig. 2. In the investigated simple case, it is possible to solve the servo-constraint equations and the geometric constraint equation for the intuitively chosen set of controlled coordinates $\mathbf{q}_c = [x_1 \; z_2 \; x_3 \; z_3 \; z_4]^{\mathrm{T}}$. with $\mathbf{q}_u = [x_2 \; x_4]^{\mathrm{T}}$. The corresponding solution for the controlled coordinates reads:

$$x_1 = x^d_{CU}, \;\; z_2 = z^d_{SU} + h^d_{CC}, \;\; x_3 = x^d_{SU} - \frac{L_{34}}{2}, \;\; z_3 = z^d_{SU} \text{ and } z_4 = z^d_{SU} \tag{14}$$

The uncontrolled coordinate x_4 comes directly from the geometric constraint equation. Despite of the available closed form solution, here, we do not separate the controlled and the uncontrolled coordinates. Instead the servo-constrains are directly attached to the control law (3-5) proposed in Section 3.

The equations of motion was solved using the fourth order Runge-Kutta method, and the control input was calculated by the simultaneously applied Backward-Euler algorithm as shown in Fig. 2. The simulation of the DAE system was accomplished by using Baumgarte's method [2] under the assumption that the geometric constraints do not depend on time explicitly:

$$\begin{bmatrix} \mathbf{M} & \boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}} \\ \boldsymbol{\Phi}_{\mathbf{q}} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_{\mathbf{g}} + \mathbf{H}\mathbf{u} \\ -\dot{\boldsymbol{\Phi}}_{\mathbf{q}}\dot{\mathbf{q}} - 2\alpha\boldsymbol{\Phi}_{\mathbf{q}}\dot{\mathbf{q}} - \beta^2\boldsymbol{\phi} \end{bmatrix} \tag{15}$$

In equation (15) $\alpha = 40$ and $\beta = 60$ are constant numbers that effects the suppression of the geometric constraint errors. The time step of the simulation was set to $h = 0.01\,\mathrm{s}$.

Using the experimentally tuned gain matrices $\mathbf{K}_P = \mathrm{diag}(150, 0, 10, 1, 3, 1, 3)$ and $\mathbf{K}_D = \mathrm{diag}(1000, 0.5, 10, 15, 20, 15, 20)$, the simulated motion of the system is presented in Fig. 3, where panel (a) shows the stroboscopic motion of the planar ACROBOTER. The realized path of point **O** is denoted by the thick curve that slightly oscillates around, but converges to the desired path depicted as a thin straight line. The desired configurations are shown by dashed lines, while the continuous lines presents the realized configurations of the robot. According to the task equations (12) and (13) the robot is commanded to stand still till $t = 2$s, then the reference point **O** is commanded to move along a straight line with constant velocity. During the same period of time the desired elevation of the cable connector is decreasing.

The climbing unit is commanded to start moving with constant velocity at $t = 3$s. Then, at $t = 4$s the task is to keep the swinging unit in a certain fixed position. Panel (b) in Fig. 3 shows the constraint violation with the maximum of 4mm. Note that the constraint violation depend on the α and β parameters of eq. (15). Panels (c) and (d) show the servo-constraint errors. The error in the climbing unit's position is $\phi_{s,1}$, the elevation error of the cable connector is $\phi_{s,2}$, the horizontal and vertical position errors corresponding to the coordinates of point **O** are $\phi_{s,3}$ and $\phi_{s,4}$ and the orientation error of the swinging unit is $\phi_{s,5}$ (see eqs. (12) and (13)).

The servo-constraint errors show that the large initial errors decreasing in the first 2 second. Then the sudden change of the desired velocities causes further, but settling oscillations. When the system has to stop at $t = 4$s the oscillations are suppressed, too. However, the relatively high frequency oscillation of $\phi_{s,4}$ dies out slowly. This corresponds to the oscillations of the cable connector having relatively small mass compared to the swinging unit. It is hard to suppress the horizontal

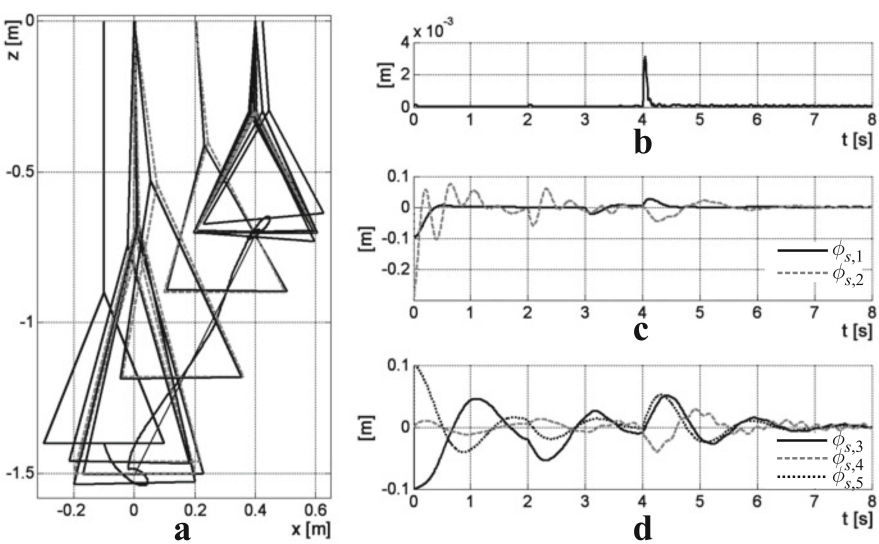


Fig. 3 Simulation results: (a) stroboscopic movement of the system, (b) violation of the geometric constraint, (c) and (d) servo constraint violations