Benjamin Rott · Günter Törner
Joyce Peters-Dasdemir · Anne Möller
Safrudiannur *Editors*

Views and Beliefs in Mathematics Education

The Role of Beliefs in the Classroom

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Preface

It is always a pleasure for editors to finalize a new book by writing a preface. In particular, we are happy to have the 23rd international conference series on Mathematical Views (MAVI). In 1995, the first MAVI conference was held at the University of Duisburg in Germany, organized by Erkki Pehkonen (Helsinki) and Günter Törner (Duisburg). In the proceedings, the editors of this first MAVI conference stated: "The aim of this research group […] is to study and examine the mathematical-didactic questions that arise through research on mathematical beliefs and mathematics-education."

In all these years, MAVI conferences have remained manageable conferences with 40–50 attendants from several (mostly European) countries; this time, there were participants even from Thailand, Japan, Indonesia, and Canada. The atmosphere and the discussions are always very cooperative and friendly, which makes MAVI conferences particularly successful in attracting younger scientists.

From October 4 to 6, 2017, the conference returned to the University of Duisburg-Essen. The theme of the 23rd MAVI was "Views and Beliefs in Mathematics Education." Compared to the 1990s, the landscape of views and beliefs has changed significantly. Today, beliefs are not a neglected and largely unexplored field of research anymore. Instead, they are non-neglecting variables which are omnipresent in contemporary research in mathematics education. However, there is still a lot of work to be done, as this volume shows.

The papers presented in this volume provide a good entry into contemporary research on beliefs, values, affect, and other related constructs.
Meanwhile, a new homepage http://www.mathematical-views.org/ has been started where MAVI documents and information regarding upcoming conferences will be compiled. With young researchers joining this group, we wish that there will be further MAVI conferences and volumes following up in the research tradition of the previous ones.

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Chapter 1
Are Researchers in Educational Theory Free of Beliefs: In Contrast to Students and Teachers?—Is There an Overseen Research Problem or Are There "Blank Spots"?

Günter Törner

Abstract In this article, the author gives an overview of current research on the topic of beliefs and raises the question whether beliefs of researchers themselves have been overlooked.

1.1 Belief Research in Mathematics Didactics—Anno 2018

By now, the amount of research articles dealing with the role of beliefs in mathematical teaching and learning processes has become almost unmanageable. It is questionable what exactly the respective researchers refer to when using the term "belief," only very few of them explicitly explain the terminology underlying their works. More so, a unification of terms, as recommended by the author, has only reached a couple of inclined readers (Törner, 2002). Eventually, every author uses his personal definition and these subjective definitions of beliefs have become excellent examples for actual beliefs.

However, there has been a significant change since 2002, as in those days beliefs had still been described as "hidden variables" by Leder, Pehkonen, and Törner (2002). By now, beliefs—however defined—have proven to be a multifaceted and important factor of explanation and already in Goldin, Rösken and Törner (2009) we have been able to announce: Beliefs are no longer a hidden variable!

Given that, within the frame of scientific publications, beliefs are only seldom further defined as being constructs, a functional understanding of beliefs seems to offer a complementary frame of research. The doctoral dissertation by Rolka (2006) has made a major contribution in this respect. Already in Abelson (1986) we can find corresponding approaches. Very often, beliefs disclose learning impediments and barriers within learning processes. The failure of the curricular

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Problem-Solving-Initiative is an excellent example since its implementation failed due to inadequate beliefs (cf. Frank, 1988, 1990; Schoenfeld, 1985). Schoenfeld (2010) follows a similar approach when using the term "orientation" instead of "beliefs" in order to refer to the often unreflected "personal subjective theories" of the active players in question. This is especially true for decision-making processes, as emphasized by Schoenfeld.

Furthermore, it has become apparent by now that we should not diametrically oppose beliefs to what we consider as "knowledge" (Abelson, 1986). The author modifies a metaphor deriving from the field of history and being attributed to the renowned German historian Nipperdey (1927–1992); we formulate analogously:

> The colors symbolizing knowledge and beliefs are not those of a chessboard, namely black and white, instead they are constituted by infinite nuances and shades of gray.

Such a view helps us get rid of what is occasionally suggested when knowledge is grated as being good and beliefs as being bad. At this point we also need to recall the title of a book by Lakoff and Johnson (1980) dealing with the role of metaphors: Metaphors We Live By. We have come to realize: Yes indeed, we all live with beliefs. In the end this is both inevitable and very normal. Alan Schoenfeld has personally stated to the author: We are all victims of our beliefs structures which are shaped by both our experience and our communities. Very often we do not reflect on this circumstance.

Very often it seems—and the author has been able to pin this insight in his surveys—as if beliefs simply prevent us from having a cognitive vacuum. Elements of unknowingness in our knowledge networks are compensated by beliefs, whereby the respective networks undergo stabilization. In those subject-specific mathematical contexts in which we are not able to store reliable elements of knowledge, the resulting gaps are filled by beliefs. It happened once that in an interview the author tried to explain the aspect of exponential growth in further detail, when the interviewee answered by pointing out that during World War I, the North Sea could not be fished heavily due to military ships which resulted in an exponential growth of populations.

Even though we often speak of a so-called "body of knowledge," it appears beneficial to also include the numerous beliefs in these considerations instead of separating them. Apparently, it seems likely that beliefs and elements of knowledge can coexist "peacefully," and that even very contradictory and dissenting beliefs do not necessarily need to cause conflicts.

1.2 Bearers of Beliefs: The Case of Researchers

Lately, the author has often been dealing with a lack of discussion with regard to beliefs in specific areas of research literature. As already pointed out in an article included in the book by Leder et al. (2002), beliefs can initially be differentiated by the objects they refer to (their beliefs' objects), meaning the context of the specific belief. According to the author, a further coordinate axis is constituted by the specific bearers of beliefs.

In literature (and also during congresses) the differentiation of beliefs very often only goes as far as "beliefs of teachers" on the one hand, and "beliefs of students" on the other hand. Occasionally also outsiders experience discussion: parents, political stakeholders, or any people of a given society. If we take a closer look into our investigations, we will find that in the literature of mathematics education, there are hardly any articles dealing with the beliefs of (mathematics education) researchers. They seem to have been neglected. Why so?

Is this due to the fact that beliefs are not considered being as noble as knowledge and that we consequently should not assume researchers to have such inferior beliefs? Are beliefs parts of a fake-news-reality? Is not the sole presence of knowledge considered the manifestation of researchers' rationality?

So far, the author's database includes exactly three articles discussing the beliefs of mathematics teacher educators (Aydin, Baki, Kögce, & Yildiz, 2009; Aydin, Baki, Yildiz, & Kögce, 2010; Nathan & Koedinger, 2000). These works are definitely interesting; however, they do not primarily focus the differentiation of teachers' and researchers' beliefs. Instead, they focus the confirmation of slightly differing perceptions. The works cited do not answer the posed question. This much being said as an introduction. A first answer will be dealt with in the following section.

1.3 Beliefs as Myths

In the following we will deal with the question whether in relevant literature there is proof for researchers having beliefs after all, eventually just referred to by using different terms.

Given our reference to the terminology, orientation, preferred by Schoenfeld, it becomes apparent that the term belief may be worn and unclear. In German research literature the term "belief" has experienced untranslated establishment in order to underline its status of being a specialist term. All possible Germanizations of the term are unclear and in parts contextually fraught.

The author repeats himself when emphasizing that beliefs are multifaceted fuzzy constructs appearing in different coverings. There is no denying about Pajares' (1992, p. 308) comment that: "... the most fruitful concepts are those to which it is impossible to attach a well-defined meaning. The respective terms may vary, but the functional patterns and modes of action only differ slightly."

This being said, in some educational scientific contexts, beliefs are often referred to as myths. Oser (2014) explains this by the fact that our understanding of the variables and their optimal combination in teaching and learning processes within the classroom is not yet satisfying (see also Rauin (2004)). Oser continues (p. 764):

> The search for the optimal combination of those variables, enabling subjectively and objectively successful teaching and learning processes, resembles the search for the Holy Grail: There is something we keep looking for and even though it is selectively apparent in single elements, we cannot really get hold of it.

This search for the Holy Grail encourages subjective theories—beliefs in the end—to grow and to get out of control. At this point we need to mention the example of *empirical myths*.

The author does not want to deal with these empirical myths in further detail; however, please note: Empirical myths arise from educational sciences being divided into an empirical and a non-empirical branch, as well as from an often detectable incorrect mutual interpretation of the different theoretical principles. It happens that explorative models are reinterpreted as loadable theory statements, so that we need to assume specific and mostly unreflected beliefs on the parts of some researchers. These are the beliefs we keep looking for.

In a 2006 talk, the well-known (German) educationalist Helmke touched upon the so-called *method-myths*. He listed a couple of examples and spoke of the following method-myths:

- Confusion of quantity and quality: Researchers equate the so-called "innovative methods" (such as open classroom instruction, activity-oriented lessons, project teaching, and learning cycles) with good teaching.
- The same group is convinced that teacher-centered instruction necessarily results in receptive and superficial learning.
- Often, we can come across representatives of a faction of educationalists who propagate that especially weaker students could benefit from open classroom instructions (or the so-called extended forms of learning).
- During classroom observations, the author has come to notice that currently active teachers and maybe even the mentor himself live by the thesis: The more various the methods, the better…

Surely the reader can confirm having come across such statements (beliefs). The examples given should have highlighted that there are convictions in the different factions among researchers which are, upon closer examination, nothing but beliefs. In literature, however, they are only seldom discussed under this specific headline even though they do have about the same effects.

At this point we could surely mention numerous beliefs—on mathematics and on the teaching and learning of it—being stated by mathematical researchers with full conviction of their propriety. However, we are eager to deliberately restrict our considerations to researchers in the field of mathematics education.

In the following we will mention three further areas of beliefs' objects by mathematics educationalists which the author refers to as "blank spots" since they show stereotypical standard statements. In fact, these are nothing but beliefs.

1.3.1 Blank Spots in Beliefs Research?

Numerous papers by researchers deal with teachers, the institution school and the belonging students.

1.4 Teachers, School, Students

Surely, numerous didactical research papers address school reality. They give the impression that the newly gained insights are of relevance for school practice and that they should consequently be implemented. However, which idea of teachers is implicitly rooted in the statements of the researchers involved?

Teachers are the immediate addressees of researchers. They are always open-minded, interested and thankful for being able to gain new insights based on current research. Why should experience from different cultural environments not be rewarded and thus exploited for our own practical application?

Eventually, at this point researchers inadmissibly project their own self-perception onto other people. We imply that researchers are constant learners, that they have time for this process at their disposal, that they are diversely interested and curious about others' actions in the process of teaching and learning at schools. These features constitute the ideal of any scientific profession. However, these features only seldom apply to teachers working at school.

Initially we have to remark that teachers do not merely concentrate on teaching, instead they have to cover numerous duties accompanying the teaching processes at school: consultations with parents, correction of class tests, preparation of lessons, cooperation among colleagues, and many more. Other features include administrative tasks like curriculum or teacher conferences. The time of actual teaching may consume about 26 h per week. Roughly estimated this covers about 60% of the total working hours at best. With other words: There is only little time for autonomous and freely organized learning.

It is quickly neglected that only very few teachers are able to take note of the articles in research journals. Given the number of journals this is already tough for researchers who are usually confined to one specific area of research. We cherish an illusion in believing that teachers go sit in the library of the nearby university in order to go through the latest publications. How should they even take note of them?

Even if we assume that (some) highly interested teachers were fond of falling back upon external suggestions from the research sector, do not such teachers need to struggle with the belief that researcher often lack broad practical teaching experience? Following the author's observations, teachers are often skeptical towards well-meant recommendations by researchers. A renowned scientist from the USA has confirmed to the author:

> … but they resonate with my experience in the US—there is, in my opinion (and as a gross abstraction) a gulf between content-focused researchers and policy-related researchers/practitioners.

Those content-focused researchers who have "lived" in schools for some time may be somewhat realistic (I hope to count myself among them), but for the most part, the content-focused and policy communities seem to live separate realities. This causes difficulties in both directions—a neglect of school realities on the part of content-focused researchers, which is as you describe, and a neglect of content-based necessities on the part of most policy people.

Further arguments cannot be neglected: Are not teachers closely bound to the curricular teaching guidelines in most countries? Besides, in most schools (recommended) consultations take place among the group of colleagues when specific contents in parallel courses are taught by different teachers. How should one single teacher step out of line just because he or she has been recommended a modification of lessons by a researcher?

1.5 Research and Practice

This conceptual couple highlights a central task being in store for research: Influencing the practice with newly gained insights. Admittedly, this conceptual couple raises a lot of questions which are not answered easily. Berliner (2002) refers to this dilemma when describing *Education Research as the Hardest Science of All*.

Many colleagues agree with the author in admitting that answering a research question is far easier than using the gained insights as implications for actual teaching. We have not realized this only yesterday, but this insight is in fact about as old as the attempt to improve teaching. Writing about this in further detail would surely fill dozens of pages. At this point we refer to a recently published special issue of the Journal for Research in Mathematics Education and the article (Cai et al., 2017):

> In our May editorial (Cai et al., 2017), we argued that a promising way of closing the gap between research and practice is for researchers to develop and test sequences of learning opportunities, at a grain size useful to teachers, that help students move toward well-defined learning goals. We wish to take this argument one step further. If researchers choose to focus on learning opportunities as a way to produce usable knowledge for teachers, we argue that they could increase their impact on practice even further by integrating the implementation of these learning opportunities into their research.

1.6 Continuous Professional Development of Teachers

The author believes in having found a further "blank spot" in relation to researchers. This topic, however, can only shortly be touched on. It is to be judged favorably that this obligation for teachers is becoming more evident and indisputable within the scientific community. It is B. Rösken's (2011) credit who, in her PhD thesis, highlighted the fact that continuous professional development of teachers is loaded with various beliefs of which adequacy often needs to be questioned. Furthermore, the author points to the work by Timperley, Wilson, Barrar, and Fung (2008) which underlines that in order to be successful it is necessary to question and contrast many of the beliefs uttered by the teacher clientele.

Especially the political side and sometimes also the research side occasionally make the suggestion that it would merely (?) take an investment in further education in order to liberate the tedious deficits in greater areas of teaching methodology.

In doing so, they ignore that there are various conditional factors that need to be influenced positively in order to guarantee change. But how does an averagely engaged teacher learn? When it comes to adult learning, the respective individuals are often occupied with the question: Does this expenditure of energy and time really pay off? It takes massive efforts of motivation from the parts of teacher educators. We have to keep in mind that the introduction of a new curriculum results—among other expenses—in the fact that many of the teachers' teaching transcripts become outdated. Many of the documents designed for teaching lessons need massive revision or have simply become invalid. Do researchers have this in mind when propagating ad hoc curricular changes? Are the teachers who need to be taught ready for this?

1.7 Final Remarks

It should have been pointed out that in research literature dealing with beliefs, researchers' beliefs are often neglected. This may be due to the assumption that researchers should not be accused of having beliefs in the first place. Beliefs are regarded as features of subordinate teachers, students, parents, educational administrators and further stakeholders, but not as features of researchers. In research literature, this lack of self-reflection is hardly ever mentioned. We believe that this can be regarded as a "blank spot." This circumstance is tragic since researchers have to be seen as important players in terms of educational change. Especially the school sector requires the important educational agents to cooperate on equal terms. Given this background, this work is supposed to encourage a detailed stocktaking. The author believes that it appears inevitable to refer to the work by Abelson (1979, 1986). Despite its year of publication, it is still a good read as it describes beliefs as possessions and warns that the costs associated with the adoption of beliefs should not be lost sight of.

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Part I
Pupils' and Students' Views and Beliefs of Mathematics

Chapter 2
Engagement in Mathematics MOOC Forums

Chiara Andrà and Elisabetta Repossi

Abstract The research focuses on mathematics MOOC discussion forums, how affective instances emerge from written interactions and how they can be measured. Interactionist research, as well as the intertwining of affective and cognitive components in students' interactions, represents the theoretical background of our investigations. In particular, we refer to engagement as the main affective element in discussion forums. The affective lens is paired with network analysis to examine how and to what extent forums may represent an occasion for a deeper understanding of mathematics for the students. This paper reports on a pilot phase of the research and considers two examples of discussion forums that involved around ten students each. The findings from a small scale analysis serve as a basis for first, general conclusions.

2.1 Introduction

Interactional research does not only postulate the intrinsically social nature of learning (e.g. Ernest, 1998) but also provide evidence that both *cognitive* and *affective* aspects of students' interaction play a role in mathematical understanding. Lave (1988) maintains that "developing an *identity* as a member of a community and becoming *knowledgeably skilful* are parts of the same process" (p. 65). Goos (2004) observes that community is essential to both the development of *a sense of belonging* and to the students' *active participation*. Roth and Radford (2011) further stress that every *idea* contains an *affective attitude* towards the piece of reality the idea refers to, and hence propose that each activity is made of both the conscious awareness and the emotion of each individual engaged in it.

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When we are engaged with others in social interactions, we do not share our ideas only through utterances, but we also share our emotions: simulation theories (e.g. Gallese, Eagle, & Migone, 2007) refer to mirror neuronal circuits to suggest that, in order to recognise an interlocutor's emotion, we experience that emotion ourselves. Vertegaal, van der Veer, and Vons (2000) make a strong link between the amount of eye contact people give and receive to their degree of participation in group communications. Hence, Goos' (2004) sense of belonging and active participation of the students in a group can be further characterised by exchange of glances, mirroring gestures and echoing emotions. Furthermore, with Roth and Radford (2011), we can say that the students' identity develops during the interaction as part of the emotionally intense and embodied process of understanding, and the flow of glances contributes both to the development of their identities and their becoming knowledgeably skilful.

To transfer all these considerations into the context of MOOC is all but straightforward: if we maintain that mathematical understanding is unavoidably interactional, Naidu (in press) observed that most contemporary MOOCs have tended to adopt a predominantly content-specific approach to teaching and learning with little or no regard to the value of promoting and supporting a rich set of interactions between and among students and their teachers about the subject matter. If we maintain that learning is made of an amalgam of cognitive, social and affective components, and that for learning to take place the interlocutors should establish a sense of belonging at cognitive, social and emotional levels by sharing not only the ideas, but also the emotions that come with these ideas, and if eye-contact plays a crucial role in such a sharing, we can question how all this is possible in MOOCs. Many MOOCs, however, provide discussion forums parallel to the video contents and one of their major purposes is to allow the students to engage in an exploration of their ideas to develop their knowledge and understanding of the subject (Zhang, Skryabin, & Song, 2016). A promising approach for the analysis of the dynamics of such free-flowing discussion forums is network analysis, which enables insights into the different roles the interactors can take, namely creating, maintaining or terminating ties (Snijders, van de Bunt, & Steglich, 2010). Our understanding of Snijders et al.'s roles is as follows: in a creative tie, a student poses a new question or problem in the forum. In a maintaining tie, a student replies and opens the possibility to be replied, while in a terminating tie a student posts an answer which does not prompt the others to intervene.

In this paper, we focus on how students develop their *knowledge* and deepen their *understanding* in mathematics, in relation with their *engagement* in discussion forums by first building and then analysing the network of their interactions. Our theoretical framework, thus, consists in Goldin's (2017) understanding of engagement, while our methodology is built around the construction of a network in order to resort to standard mathematical tools for network analysis, paired with an analysis of the affective dimension (engagement). The research question reads as follows: what does the intertwining of network analysis and engagement structure add to our understanding of MOOC discussion forums?

2.2 Engagement

Engagement is considered as fundamental to learning outcomes in general and to students' interactions in particular: Davis (1996), for example, argues that for a true dialogue to take place the interlocutors need to be *willing to engage* in the conversation. According to Goldin (2017), engagement can be characterised by *motivating desires*, namely by *the reasons for engagement*. Gerald Goldin and his colleagues identify a list of desires, but in this paper we recall and adapt the ones that emerged in discussion forums: *Get The Job Done* (the desire to complete an assigned task), *Look How Smart I Am* (the desire to exhibit one's mathematical ability, and have it recognised or acknowledged), *Check This Out* (the desire to control whether a computation is correct), *I'm Really Into This* (the desire to enter and maintain the experience of doing mathematics), *Let Me Teach You* (the desire to explain a mathematical procedure or concept to another student), *Help Me* (the desire to obtain help or support in solving a mathematical problem or understanding the mathematics), *Value Me* (the desire to be held highly in the opinion or caring of other students or teacher), and *Stop The Class* (the desire to interrupt the ongoing mathematical activity of others in the class).

According to Goldin (2017) an engagement structure consists not only of a motivating desire, but also of behaviours and social interactions, thoughts, emotions, which interact dynamically. Most of the motivating desires identified have some explicit *social* aspect (e.g. belonging, recognition, respect, equity, generosity). Some of the motivating desires involve *approach* goals, while others involve *avoidance* goals. Most importantly for a discussion forum, many of the motivating desires tend to *productive mathematical* engagement (Goldin, 2017).

Goldin observes that to infer a student's motivating desire is all but simple and different tools entail different limitations. In analysing a MOOC forum, the limitations seem to be even more, given that we have to resort only to written words. Moreover, Goldin argues that not always a unique motivating desire guides a student's response, given the complexity of engagement. Hence, a student's post seems to be susceptible to more than one interpretation about its motivating desire. However, we claim that some clues in the statements may help us revealing the main motivating desire that is guiding a student's response in the discussion forum.

2.3 Methodology

As stated in the previous session, we try to infer the motivating desires that move the students in interaction forums, and we plug this lens of analysis onto a network that is built from the discussion flow of two forums.

2.3.1 The Tasks, the Participants and the Context

The data for this study come from a blended course that has taken place on January–February 2017. It involved 30 students from grades 12 and 13 (16–18 years old), who attended a math course aimed at strengthening their mathematical knowledge that is necessary for the transition to university mathematics. The students attended six traditional math lessons at the Polytechnic of Milan, on a weekly basis: the lessons paid specific attention to the conceptual understanding of mathematics, how the main mathematical ideas arose historically and how these connect to the most common algorithms in calculus. Between one lesson and the following one, the students had to attend a "week" on a MOOC course, which recaps the main concepts and focuses on the procedural aspects of the mathematical ideas the students have been exposed to in the traditional lessons. Parallel to this, every evening a tutor (the second author of this paper) posted a task on the MOOC discussion forum, intended to enhance the students' conceptual understanding. The students were invited to interact in solving the task. Among the 30 tasks posted, we select the following two ones.

Task A: compute the perimeter and the area of the triangle ABC, where A(2,0), B(8,1) and C(4,5).

Task B: consider the points A(3,2) and B(9,2). The point C varies on the straight line $y = 5$. How does the area of the triangle ABC varies with C? How does the perimeter?

As regards task A, we can see that it is rather a routine exercise and we expect that the students' interactions would be on the results and/or the way to compute them.

As regards task B, the points A and B lie on the horizontal line $y = 2$, hence the area of ABC does not change when C varies on the horizontal line $y = 5$, since its basis remains AB and it height remains equal to 3. The perimeter, indeed, changes. We can notice that task B has a conceptual nature, since it prompts the students to reason, discuss and generalise about the properties of areas and perimeters.

We analyse the motivating desires that drive the students' comments and in particular which ones lead to creating/maintaining and which ones lead to terminating ties.

2.3.2 Network Analysis

Network analysis is a mathematical tool that features a network as made of *nodes* and *links* between two nodes. In case of MOOC forums, the nodes can be thought of as the participants and a link as a participant's reply to another one's post. If a person replies more than once to another person, the link can be counted more than once, namely the network can be *weighted*. If we want to distinguish the case when A replies to B to the case when it is B that replies to A, the network can be *directed*.

In our situation, a network represents the interactions between participants within a forum discussion. In order to recognise the role played by each person inside the discussion, or better, its *centrality* inside the network, it is possible to analyse a node's *degree,* that is, the more links arrive and depart from a node, the higher its *degree*. In our study, we represent the degree with the radius of the circle: the bigger the radius, the higher the degree. The colour of the nodes denotes the *in-degree,* that is the number of links that reach this node: the lighter the colour, the higher the *in-degree*. So a big node in light blue means that the person receives many replies to her posts. A big node in dark blue means that the person makes many comments. The colours of the links correspond to the colour of the node the comment is made to.

From network analysis, we draw on Zhang et al.'s (2016) study, which focuses on reciprocity, transitivity and preferential attachment in a MOOC discussion forum, and aims at explaining how these three network-effects could be used as metrics to inform the design of a better social learning environment.

Reciprocity refers to a communicative relationship in which a conversation is paired up with a returned flow. Research has shown that it is important that participants use the forum not only to express their own ideas and thoughts but also to interact with others by responding to their messages (Arvaja, Rasku-Puttonen, Häkkinen, & Eteläpelto, 2003). Reciprocal interaction is considered as a vitally important part of sharing the cognitive processes at a social level (Resnick, Levine, & Teasley, 1993). The network of the discussion forum can, thus, be characterised by the number of reciprocal interactions.

A *transitive relationship*, in which A connects to B, B connects to C, and A also connects to C, may be more conducive to social learning, as participants are more likely to receive stimuli from multiple peers as the desired information diffuses through a network (Centola, 2010; Todo, Matous, & Mojo, 2015). Hence, the network can be characterised by the number of transitive interactions.

Preferential attachment represents the tendency of heavily connected nodes to receive more connections in a network. That is, if a new participant contributes to the forum, the probability of replying to or being replied by another participant would be proportional to her degree. Initially random variations, such as a participant having started to contribute earlier than others, are increasingly enlarged, thus greatly amplifying differences among participants. Network centralisation is a measure of how unevenly centrality is distributed in a network (Scott, 2000). Centrality relates to the importance or power of a participant in a network. Highly centralised networks appear to be conducive to the efficient transmission of information (Crona & Bodin, 2006), as the central participants play an important role in delivering messages. But central participants can manipulate the communications in networks, and thus, centralised networks are not likely to enable optimum levels of intellectual exchange because of the high imbalances of power in such settings (Leavitt, 1951). Furthermore, learning processes are more likely to collapse if a central participant leaves the networks (Nicolini & Ocenasek, 1998). Hence, the network can be characterised by its even centrality, and in particular we can focus on the degree and in-degree of each participant.

To build the networks and to compute the measures of centrality we have used the open source software Gephi (Bastian, Heymann, & Jacomy, 2009).

2.4 Data Analysis

Figure 2.1 (left) shows the discussion network around task A, which unfolds as follows:

SD	Distance between two points in the Cartesian plane: square root of [(x2−x1)^2 + (y2−y1)^2]. So AB = square root of 37 = 6.1 BC = square root of 16 = 4 AC = square root of 5 = 2.2 Perimeter: 6.1 + 2.2 + 4 = 12.3. Area = square root of [P/2 × (P/2-AB) × (P/2-BC) × (P/2-AC)] = square root of [6.15 × (6.15–6.1) × (6.15–4) × (6.15–2.2)] = 1.6
AJ	Why do you say that BC is the square root of 16? If you compute better, you find out that it is the square root of (16 + 16), that is the square root of 32.
ALC	To find AB: square root of [(2–8)^2 + (0–1)^2] = 6.1 To find AC: square root of [(2–4)^2 + (0–5)^2] = 5.3 To find BC: square root of [(8–4)^2 + (1–5)^2] = 5.6 2p = 6.1 + 5.3 + 5.6 = 17 cm. To find the area when the sides are known: 1/2 × 17 = 8.5 cm square root of [8.5(8.5–6.1) (8.5–5.3)(8.5–5.6)] = 13.7 cm^2
IC	I got different results. AB = 6.1, BC = 16 and CA = 5.3 …I have computed them putting always before x2 and y2. As a consequence, *p* = 27.4 and *A* = 13.22. Before computing the area I have found AB's median point and then the height CH = 4.33 with Pythagora's theorem then I have used the results to compute the area …Why we got different results?
FI	Isn't that you have confused the median with the height: to pass through the median point is a property of the median, not of the height. For the perimeter you have put BC = 16 when actually is it 4 times the square root of 2

SD opens the conversation and recalls the general formula to compute the distance between two points on the Cartesian plane, then she applies the formulas to the given points and computes the area and the perimeter. AJ replies to her, correcting a computation: the length of BC is not the square root of 16, but the square root of 32. We infer that her motivating desire is *Let me teach you*. ALC posts an independent post with his computations. While AJ's comment can be seen as a maintaining tie, ALC's one can be seen as a terminating tie and his motivating desire can be inferred to be *Get the job done*. IC intervenes and says that her results are different

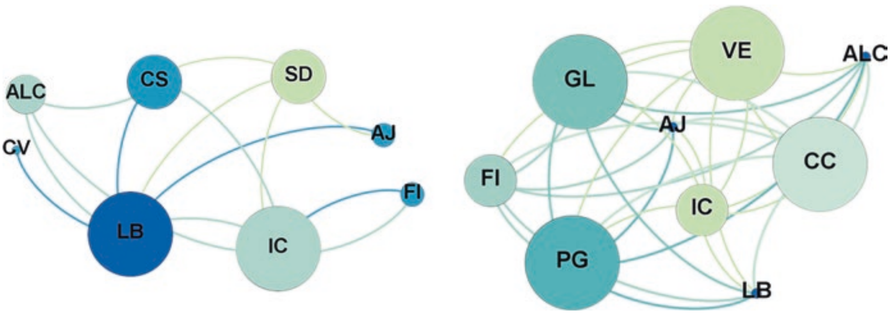

Fig. 2.1 The network for the forum discussion about task A (left) and B (right)

from her mates' ones, hence a link is established from IC to SD and to ALC in the network. IC's post has a maintaining purpose and we also infer that her motivating desire is *Help me*. FI provides her with an explanation, in a way that reveals *Check this out* as motivating desire, and a terminating tie.

Why from AJ's post we infer that her motivating desire is *Let me teach you*, and from FI's one we infer *Check this out*? AJ writes: "if you make the computations accurately, you'll find out that it is the square root of 16 + 16, not 16". AJ seems to be willing to teach SD. FI, instead, writes: "isn't that you have confused the median with the height?". FI's post has a dubitative nature, suggesting IC to check her results but also being quite sure that he is right.

IC replies to FI with a terminating tie, saying: "You're right, thanks!" We interpret her motivating desire as *Get the job done*. We can also see that a reciprocal interaction is established between FI and IC, since they reply to each other. Furthermore, given that IC posts a question to SD and to ALC, and given that FI replies to IC's question, we can also say that a transitive relationship is established from FI to IC to SD and ALC. The discussion goes on:

CS	I got a different result for the area. The sides are the same AC = sqrt29, CB = sqrt32, AB = sqrt37. To find the height CH, I have used the formula to find the distance between a point and a straight line on the Cartesian plane. The straight line on which the segment AB lays is −1/6x + y + 1/3 = 0. The distance between C and the straight line is |1/6 × 4 + 1 × 5 + 1/3|/sqrt 1/6^2 + 1^2 = 36/sqrt37. Hence the area is sqrt37 × 36/37 × 1/2 = 18

CS's post establishes a link to IC, to ALC and to SD by replying to their posts. We infer that her motivating desire is *Help me*, and hers is a maintaining tie, but nobody replies. Instead, AJ and CV post their solutions with no reference to the previous posts. These look like terminating ties. The motivating desires of these two students seem to be: *Look how smart I am* for AJ, and *Get the job done* for CV. Finally, LB's post seems to be a terminating tie and her motivating desire seems to be *Value me*.

The network in Fig. 2.1 (left) has eight nodes: the highest degree is associated to nodes IC and LB, but the former's one is given by many links towards the node, while the latter one is the result of many links going out from the node. IC, in fact, appears in the discussion quite early and poses a question, hence she got responded by some students; LB's post, conversely, is the last one in the discussion: she mentions and replies to the posts of her mates, but she gets no answer. We interpret this phenomenon as a case of *preferential attachment*: participants having started to contribute earlier than others receive more comments to their posts. The same holds for the other three nodes that have a quite high degree: SD and ALC, who show up in the first two interventions, receive many links, while CS's degree is determined by going-out-from-the-node links. A relationship of *reciprocity* is established between the nodes IC and FI, and *transitivity* for FI -> IC -> SD and for FI -> IC -> ALC. We can also see that in this network there are three maintaining and six terminating ties. The motivating desires associated to the maintaining ties are: *Let me teach you* in one case, and *Help me* in the other two cases. We have further observed, however, that only one of these maintaining ties receives a reply: IC's one. Why? We notice that her post comes quite early in the conversation and her desire is to get help. Coming late in the discussion with a desire of getting helped, or coming early with a desire to teach seems not to attract a reply in this discussion. For the terminating ties, *Get the job done* is the motivating desire associated to three cases, while *Check this out, Look how smart I am* and *Value me* characterise the other three cases.

Figure 2.1 (right) shows the network of the discussion around task B. Nine students intervene in the discussion and the network seems much more connected.

IC	The area remains constant because basis and height remain constant. The perimeter varies with a symmetry around $x = 6$. Right?
VE	I agree: the area is constant because the basis is so (the segment AB remains fixed) and the height is so (because, even if C varies, it is always a point on the straight line that is parallel to the segment AB). The perimeter varies and increases as C gets far and far (either to the left or to the right) from the position (6, 5). I was thinking that, if the point C tends to infinity, the area would remain the same, but would the perimeter tend to infinity?

IC's opening is quite different from the opening of the previous discussion: while SD is assertive, IC here ends with a question. Also in the previous discussion, however, IC intervened with a question and it is possible that her style of being into a discussion entails being interrogative rather than assertive. VE's post results to be a creative tie since she poses a new question: "if the point C tended to infinity, the area would remain the same, but would the perimeter tend to infinity?" The motivating desire seem to be *I am really into this*. CC replies to the first post saying that she agrees, and to the second post saying that to her the perimeter cannot tend to infinity since it is a geometrical object. The motivating desire seem to be *I am really into this*, but this is a maintaining tie. The discussion goes on, with FI that writes a long post to provide an argumentation for CC's observation, and it links to all the previous posts. It ends with a question ("how can the sides of a triangle be infinite?"), hence it is a maintaining tie and the motivating desire seems to be *I am really into this*. GL and PG intervene, saying that they agree: these are terminating ties and the former one is characterised by *Value me* as motivating desire, since it shortly explains why there's agreement and then it goes on saying "one can notice that the triangle's shape will be more and more stretched when C goes further and the angle in A will get closer to 180°, never reaching this value". The latter one can be seen as another case of *I am really into this*, since PG provides a long argumentation to sustain the other students' point of view. A terminating tie comes from LB's comment: "I do not know what to add to the discussion" and her motivating desire seems to be *Stop the class.* The same features can be assigned to ALC's post, which says "I think that the given responses are exhaustive". The last post comes from AJ, who says that she believes there's not so much to add to the others' posts, but she