Propositional Logics

The Semantic Foundations of Logic

THIRD EDITION

Richard L. Epstein

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Dedicated to

Peter Eggenberger
Harold Mann
and
Benson Mates

with gratitude for their encouragement and guidance

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Preface

If logic is the right way to reason, why are there so many logics?

Viewing logics as formalizations of how we do or should reason, we can find a structural and conceptual unity based on common assumptions about the relation of language, reasoning, and the world. What we pay attention to in reasoning determines which logic is appropriate.

In order for you to understand this I have retraced my steps: from the concrete to the abstract, from examples to general theory, and then to reflections on the significance of the work. In doing so I have had to begin at the beginning: What is logic? What is a proposition? What is a connective? If much seems too well known to be of interest, then plunge ahead. The chapters can be read more or less independently, which explains the occasional repetitions.

Chapter I is devoted to assumptions about the nature of propositions and what forms of propositions we will study. In Chapter II we then have the simplest symbolic model of reasoning we can devise given those assumptions. In classical logic a proposition is abstracted to only its truth-value and its form, relative to the propositional connectives. This provides a standard of reference for other logics.

In Chapter II I also present a Hilbert-style formalization of the notion of proof and syntactic deduction that I use throughout the book. The metalogical investigations that I concentrate on concern the relation between the semantic and syntactic notions of consequence, and whether or how those can be represented in terms of theorems or valid formulas by means of a deduction theorem.

Chapter III sets out the simplest example of a logic that incorporates some aspect of propositions other than truth-value into the semantic analysis. Taking the subject-matter of a proposition to be a primitive notion, we get the archetype of how to incorporate differing aspects of propositions into semantics.

Following Chapter III is a Summary and Overview which serves as an introduction to all that follows. The succeeding chapters present examples of many different logics based on differing semantic intuitions all of which can be understood within a general framework that is presented in Chapter IX. That framework arises from the view that each logic, except for classical logic, incorporates into the semantics some aspect of propositions other than truth-value and form. Each logic analyzes an 'if . . . then . . .' proposition classically if the aspects of antecedent and consequent are appropriately connected, while rejecting the proposition otherwise. As we vary the aspect, we vary the logic. I have argued for this bivalent falsity-default analysis of semantics throughout this volume, in part by presenting a wide variety of logics in that form, and I have used that analysis further in *Epstein, 1992* and *Epstein, 2012A.*

The general form of semantics is not intended to replace other semantics. For example, under certain assumptions possible-world semantics are a good explanation of the ideas of modal logics. But providing uniform semantics that are in reasonable conformity with the ideas on which various logics are based allows for comparisons and gives us a uniform way in which to approach the sometimes overwhelming multiplicity of logics.

In particular, the overview of the general framework allows Stanisław Krajewski and I to consider in Chapter X the extent to which one logic or way of seeing the world can be reduced to another. We present a general theory of translations and try to characterize what we mean when we say that a translation preserves meaning.

The semantic framework I set out in Chapter IX is a very weak general form of logic that becomes usable only upon the choice of which aspect of propositions we deem significant. But then is logic relative to the logician? Or does a notion of necessary truth lie in this general framework? In Chapter XI I discuss how our agreements about how we reason determine our notion of objectivity.

Throughout I have tried to find and then make explicit those assumptions on which our reasoning and logic are based. I have repeated the statement of certain of those assumptions in different places, partly because I want the chapters to be as self-contained as possible but also because it is important to see those assumptions and agreements in different contexts and applied differently to be able to grasp their plausibility and pervasiveness.

What I am doing here can be seen as founding logic in ordinary language and reasoning. When nonconstructive assumptions are used to apply mathematics to logic to prove theorems about our formalizations we can see precisely where they are needed. Those assumptions I treat as abstractions from experience. However, they need not be viewed that way, and I have attempted to provide alternate readings of the technical work based on the view that abstract things such as propositions are as real or more real than the objects we daily encounter. Most of the discussion of these matters is in Chapter I and in the development of classical logic in Chapter II, particularly Section II.G. In Chapter IX I point out specific nonconstructive, infinitistic abstractions of the semantics that we usually make in pursuing metalogical investigations. This general approach to modeling and theories is explored more fully in my essays in *Reasoning in Science and Mathematics*, while the issue of whether logic is prescriptive or descriptive is explored in my book *Prescriptive Reasoning.*

I have included many exercises, some of them routine, many requiring considerable thought, and some which are open questions (marked ‘Open’).

Depending on the choice of which are assigned, this book can serve as a text in an undergraduate course, a text for a graduate course, or as the basis for research.

There are important subjects in the study of propositional logics that I do not deal with here. I have not discussed the algebraic analyses of propositional logics, for which you can consult *Rasiowa, 1974* and *Blok and Pigozzi, 1989.* I have made no attempt to connect this work with the categorial interpretation of logic, for which you can consult *Goldblatt, 1979.* Nor have I dealt with other approaches to the notion of proof in propositional logics. And there are many propositional logics I have not discussed, quite a few of which are surveyed also in *Marciszewski, 1981* as well as in *Haack, 1974,* and *Gabbay and Guenthner, 1989,* which also discuss philosophical issues.

This is not the story of all propositional logics. But I hope to have done enough to convince you that it is a good story of many logics that brings a kind of unity to them.

> In the discussions of the wise there is found unrolling and rolling up,
> convincing and conceding; agreements and disagreements are reached.
> And in all that the wise suffer no disturbance. —Nagasena

Come, let us reason together.

Preface to the third edition

In 1992 I was asked to publish *Predicate Logic*, the second volume of this series *The Semantic Foundations of Logic*. I suggested also doing a second edition of *Propositional Logics*. There were a few corrections that colleagues had pointed out, and I thought I could clean up the text a bit. It turned out that a lot of corrections were needed, both to the technical work and the exposition. For that edition I revised the entire text, with more changes than I could easily list here. Among the most significant are the correction or simplification of many axiomatizations, the addition of examples of formalization of ordinary reasoning, and the addition of exercises to make the text more suitable for individual or classroom use.

In 2011 Esperanza Buitrago-Díaz came to the Advanced Reasoning Forum at Dogshine as an ARF Student Fellow to work through the second edition of this text with me. Her questions and comments, difficulties and insights led me to prepare this new edition. The most notable differences from the second edition are:

- The chapter on the general framework now follows the development of the examples of logics rather than preceding them.

- In the chapter on modal logics the logic of logical necessity is developed before accessibility relations are introduced
- In the chapter on paraconsistent logics a new approach to paraconsistency is introduced by modifying the notion of semantic consequence.

In my recent studies I have tried to place formal logic in the larger context of a general theory of inference. The first presentation of that was in my *Five Ways of Saying "Therefore"*. The mature version can be found in my series of books *Essays on Logic as the Art of Reasoning Well*. It would have been too large a project to modify this text to fully take account of that work, although I have made some changes in Chapters I and II to reflect those ideas.

There is, after all, no end but only a continual beginning.

Acknowledgements

The story of this book began in Wellington, New Zealand. Working with the logicians there in 1977, Douglas Walton and I developed relatedness logic.

In 1978 I met Niels Egmont Christensen, who led me to see that a variation on relatedness logic could model his ideas on analytic implication. Later that year at Iowa State University I began to question what was the "right" logic. For three years there Roger Maddux challenged me and helped me to technically clarify my intuitions. He and Donald Pigozzi introduced me to nonclassical logics and the algebras of them. In 1980, Roger Maddux, Douglas Walton, and I wrote a monograph that contained the basis of much of the technical work of Chapters IV–VII. In 1980 and 1982 I gave lectures on propositional and predicate logics where Howard Blair, William Robinson, and later Gary Iseminger challenged me to explain my philosophical assumptions. I am grateful to Dan Zaffarano, Dean of Research at Iowa State University, for providing ample time for that research.

In 1981 I visited the University of Warsaw on an exchange sponsored by the U.S. National Academy of Sciences and the Polish Academy of Sciences. There I met and began collaborating with Stanisław Krajewski, whose insights led me to clarify the relationship between formal languages and the languages we speak. Part of our joint work is the chapter on translations between logics, which was influenced by discussions with L. Szczerba.

In 1982 I moved to Berkeley where it was my good fortune to meet Peter Eggenberger and Benson Mates. They are fine teachers. Listening to my early inchoate ideas, reading my confused analyses, guiding my reading in philosophy, they helped shape this volume through the many discussions I had with them.

In 1984 I lectured to a group of Brazilians at Berkeley on what was still a series of separate papers. It was through the urgings of Walter Carnielli that I made the decision to turn those papers into a book and, finally, to publish the work on propositional logics as a separate volume. Much of the form and outline of this volume was developed in discussions with him. He, Newton da Costa, and Itala D'Ottaviano read versions of several of the chapters, and in 1986 Karl Henderscheid read a draft of the entire volume. Their questions and criticisms substantially improved the exposition.

In 1985 the Fundação de Amparo à Pesquisa do Estado do São Paulo provided me with a grant to visit Brazil and lecture at the VII Latin American Symposium on Mathematical Logic. That lecture was published in the proceedings of the conference as *Epstein, 1988,* and parts of it are reprinted in Chapter IX with permission of the American Mathematical Society.

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In 1987 the Fulbright Foundation awarded me a fellowship to lecture and do research at the Center for Logic, Epistemology and History of Science at the University of Campinas and at the Universidade Federal da Paraíba in Brazil. That visit gave me an opportunity to collaborate with Walter Carnielli and Itala D'Ottaviano and resulted in Chapters VIII.A, B and IX.G. Part of Chapter VIII appeared in *Reports on Mathematical Logic, 22* and is reprinted here with permission.

In 1987 I also visited the University of Auckland and met Stanisław Surma who gave me many useful suggestions for the book. Later David Gross helped me with the presentation of Chapter I . The first edition was then published in 1990.

For preparing the second edition, P. H. Rodenburg, Piergiorgio Odifreddi, Andrew Irvine, Branden Fitelson, and W. Carnielli pointed out errors and suggested improvements to the first edition. Arnon Avron gave a close reading of the text, resulting in suggestions for corrections. The late George Hughes was, as always, a great source of comment and encouragement. David Isles used revised versions of Chapters I–IV in his classes, and made many useful comments. Walter Carnielli helped me with the research on valid deductions in Chapter IX.G, which appeared previously in *Reports on Mathematical Logic, 26* and is reprinted here with permission.

For this third edition, Esperanza Buitrago-Díaz made many corrections and suggestions for improving the book. Alexandre Korolev also commented on the penultimate draft. The new approach to paraconsistency taking account of the content of propositions first appeared in *Logique et Analyse,* vol. 189–192, and is reprinted here with permission.

And over many years my sometime editor Peter Adams has encouraged me and helped me to see my works into publication.

To all these people, and any others I have inadvertently forgotten, I am most grateful. Much that is good in this book is due to them. The mistakes and confusions are mine alone. It is with great pleasure I thank them here.

I The Basic Assumptions of Propositional Logic

A. What is Logic?

Logic is the study of how to reason, how to deduce from hypotheses, how to demonstrate. As presented here, logic is concerned with providing symbolic models of acceptable reasoning.

What do we mean by 'acceptable'? Is logic concerned only with the psychology of how people reason, setting out pragmatic standards? I, or you and I together, can reflect on our rules for reasoning but those cover only very simple cases. We are led, therefore, to formal systems, devised to reflect, model, guide, and/or abstract from our native ability to reason. These formal systems are based on our understanding of certain notions such as truth and reference, and those in turn seem to be dependent on (*i*) how we understand the world and (*ii*) how the world really is.

But is there any difference between (*i*) and (*ii*)? And if so, is it a difference we perceive and can take into account? We must ask these questions in doing logic, for they concern how we will account for objectivity in our work and to what extent we shall see our systems as prescriptive, not just a model of what is, but what should be done.

B. Propositions

Let's begin by asking what objects, what things we are going to study in logic.

1. Sentences, propositions, and truth

When we argue, when we prove, we do so in a language. And we seem to be able to confine ourselves to *declarative sentences* in our reasoning.

For our purposes here I will assume that what a sentence is and what a declarative sentence is are well enough understood by us to be taken as *primitive,* that is, undefined in terms of any other fundamental notions or concepts. Disagreements about some particular examples may arise and need to be resolved, but our common understanding of what a declarative sentence is will generally suffice.

So we begin with sentences, written (or uttered) concatenations of inscriptions (or sounds). To study these we may ignore certain aspects, such as what color ink they are written in, leaving ourselves only certain features of sentences to consider in reasoning. The most important of these aspects for logic are called *truth* and *falsity.*

I will not try to explain truth and falsity here. In general we understand well enough what it means for a simple sentence such as 'Ralph is a dog' to be taken as true or to be taken as false. For such sentences we can regard truth as a primitive notion, one we understand how to use in most applications, while falsity we can understand as the opposite of truth, the not-true. Our goal, then, is to formalize truth and falsity in more complex and controversial situations, leading us, according to various conceptions of truth, to various formal logics.

Which declarative sentences are true or false, that is, have a *truth-value*? Some, it would seem, are too ambiguous, such as 'I am half-seated', or nonsensical, such as '7 is divisible by lightbulbs'. But if only sentences that are completely objective, precise, and unambiguous are true or false, then 'Strawberries are red' can be neither true nor false: Which strawberries? What hue of red? Measured by what instrument or person? And then we couldn't analyze the following:

(1) If strawberries are red, then some colorblind people cannot see strawberries among their leaves
Strawberries are red
Therefore:
Some colorblind people cannot see strawberries among their leaves

Surely this is an example of acceptable reasoning, reasoning that is important for us to formalize, for this is how we actually reason. And yet, I suspect, any attempt to make the sentences in (1) fully precise will fail. At best we can redefine terms, using others that may be less vague. But always we have to rely on our common understanding. What we need in order to justify our example as acceptable reasoning is that we may treat 'Strawberries are red' and the other two sentences in the example as if they have truth-values, not that they are completely precise. All declarative sentences, except perhaps those in highly technical work such as mathematics, are in some way imprecise. This imprecision is an essential component of communication, for no two persons can have exactly the same thoughts or perceptions and hence must understand every linguistic act somewhat differently.

It is sufficient for our purposes in logic to ask whether we can agree that a particular sentence, or class of sentences as in a formal language, is declarative and whether it is appropriate for us to hypothesize a truth-value for it. If we cannot agree that a certain sentence such as 'The King of France is bald' has a truth-value, then we cannot reason together using it. This does not mean that we adopt different logics or that logic is psychological; it only means that we differ on certain cases. The assumption that we agree that a sentence has a truth-value, that the imprecision of the sentence is inessential, is always there, even if not explicit.

But then is truth agreement? The word 'agreement' may be too strong, and 'convention' even worse. Almost all our conventions, agreements, assumptions are implicit, tacit. They needn't be conscious or voluntary. Many of them may be due to physiological, psychological, or perhaps metaphysical reasons: for the most part we shall never know. Agreements are manifested in lack of disagreement and in that we communicate. To be able to see we have made, or been forced into, or simply have an agreement is to be challenged on it. In Chapter XI we'll consider further the notion of agreement and how it relates to an explanation of the objectivity of logic.

My goal in this series of books is to find, or perhaps devise agreements upon which to found logic, agreements sufficiently fundamental and universal to account for not just one logic, but many, perhaps all logics. The agreement with which I begin summarizes our discussion to this point.

Propositions A *proposition* is a written or uttered sentence that is declarative and that we agree to view as being either true or false, but not both.

Again, our agreements need not be explicit. For example, if I say 'Cats are nasty' and you disagree with me, then I know that you consider that sentence to be a proposition, even if we haven't explicitly said that. From now on I will often say a proposition has a truth-value, since we've agreed to view it as if it does, though we need not agree on which truth-value it has.

But how can I say that this definition is fundamental when many logics have been based on very different conceptions of propositions?

2. Other views of propositions

Consider one such view: what is true or false is not the sentence, but the "meaning" or "thought" expressed by the sentence. Thus 'Ralph is a dog' is not a proposition; it expresses one, the very same one expressed by 'Ralph is a domestic canine'.

Platonists take this one step further. A *platonist,* as I shall use the term, is someone who believes that there are abstract objects not perceptible to our senses that exist independently of us. Such objects can be perceived by us only through our intellect. The independence and timeless existence of such objects account for objectivity in logic and mathematics. In particular, propositions are abstract objects, and a proposition *is* true or *is* false, though not both, independently of our even knowing of its existence. Thus the following, if uttered at the same time and place, all express or stand for the same abstract proposition:

(2) It is raining
Pada deszcz
Il pleut

It is argued that the word 'true' can only be properly applied to things that cannot be seen, heard, or touched. Sentences are understood to "express" or "represent" or "participate in" such propositions.

Those who take abstract propositions as the basis of logic argue that we cannot answer precisely the questions: What is a sentence? What constitutes a use of a sentence? When has one been used assertively or even put forward for discussion? These questions, they say, can and should be avoided by taking things inflexible, rigid, timeless as propositions. But then we have the no less difficult questions: How do we use logic? What is the relation of these formal theories of mathematical symbols to our arguments, discussions, and search for truth? How can we tell if this utterance is an instance of that abstract proposition? It's not that taking utterances of sentences as propositions raises questions that can be avoided. For example, were we to confine logic to the study of abstract propositions, argument (1) would be defective: the sentences there could not be taken to express propositions because of their lack of precision.

Williamson, 1968 compares several other views, too, of what kind of thing a proposition is from a viewpoint similar to mine. Most notably, Gottlob Frege has taken the thought of a sentence to be what is true or false. I find it difficult to understand how two people can have the same thought, which is in any case not a material thing, so I will direct you to *Frege, 1918,* for his explanation. In the chapters that follow I will consider arguments that there are not two truth-values, but many, or that it makes no sense to classify a proposition as true or false, only as assertible or not assertible. See also my essay 'Truth and Reasoning' in *Epstein, 2012B.*

But in the end the platonist, as well as the person who thinks a proposition is the meaning of a sentence or a thought, reason in language, using declarative sentences that they call 'representatives' or 'expressions' of propositions. Can we not reason together by concentrating on these sentences?

For me to reason with one who understands propositions differently it is not necessary that I believe in abstract propositions or thoughts or meanings. It is enough that we agree that certain sentences are—or from his viewpoint represent—propositions. Whether such a sentence expresses a true proposition or a false proposition is as doubtful to him as whether, from my view, it is true or is false. From my perspective, the platonist conception of logic is an idealization and abstraction from experience; from his perspective I mistake the effect for the cause, the world of becoming for the reality of abstract objects. But we can and do reason together using sentences, and to that extent my definition of 'proposition' can serve him, though he might prefer another word for it. Then in constructing a particular logic we can take other views of propositions into account as added weight to the significance of the word 'proposition'.

C. Words and Propositions as Types

Suppose now that we are having a discussion. An implicit assumption that underlies our talk is that words will continue to be used in the same way, or, if you prefer, that the meanings and references of the words we use won't vary. This assumption is so embedded in our use of language that it's hard to think of a word except as a *type,* that is, as a representative of inscriptions that look the same and utterances that sound the same. I do not know how to make precise what we mean by 'look the same' or 'sound the same'. But we know well enough in writing and conversation what it means for two inscriptions or utterances to be *equiform.* And so we can make the following agreement.

Words are Types We will assume that throughout any particular discussion equiform words will have the same properties of interest to logic. We therefore identify them and treat them as the same word. Briefly, *a word is a type.*

This assumption, while useful, rules out many sentences we can and do normally reason with quite well. For example:

Rose rose and picked a rose

If we subscribe to the assumption that words are types, we shall have to distinguish the three equiform inscriptions in this sentence. We can use some device such as 'Rose$_1$ rose$_2$ and picked a rose$_3$' or 'Rose$_{\text{name}}$ rose$_{\text{verb}}$ and picked a rose$_{\text{noun}}$'.

Further, if we accept this agreement, we must also avoid words such as 'I', 'my', 'now', or 'this', whose meaning or reference depends on the circumstances of their use. Such words, called *indexicals,* play an important role in reasoning, yet our demand that words be types requires that they be replaced by words we can treat as uniform in meaning or reference throughout a discussion, such as 'Richard L. Epstein', 'Richard L. Epstein's', 'March 9th, 1991', and so on.

Now suppose I write down a sentence that we take to be a proposition:

Socrates was Athenian

Later I want to use that sentence in an argument, say:

If Socrates was Athenian, then Socrates was Greek
Socrates was Athenian
Therefore, . . .

But we have two distinct sentences, since sentences are inscriptions. How are we to proceed?

Since words are types, we can argue that these two equiform sentences should both be true or both false. It doesn't matter to us where they're placed on the paper, or who said them, or when they were uttered. Their properties for logic depend only on what words (and punctuation) appear in them in what order. Any property that differentiates them isn't of concern to reasoning.

We couldn't make this argument were we to allow indexicals in our reasoning. If first I say 'I am over 6 feet tall', and then you say 'I am over 6 feet tall', we would not be justified in assuming that these two utterances have the same properties of concern to logic. Yet formalized versions of self-referential sentences such as '*a* is false', where the letter '*a*' names the last quoted sentence, can introduce a form of indexicality that leads us to classify equiform sentences differently (see, for example, *Epstein, 1992* or Chapter 22 of *Epstein, 2006*). Avoiding such problem sentences for now, let us make the following assumption to simplify our work.

Propositions are Types In the course of any discussion in which we use logic we will consider a sentence to be a proposition only if any other sentence or phrase that is composed of the same words in the same order can be assumed to have the same properties of concern to logic during that discussion. We therefore identify equiform sentences or phrases and treat them as the same sentence. Briefly, *a proposition is a type.*

It is important to identify both sentences and phrases, for in argument (1) above we want to identify the phrase 'strawberries are red' in the first sentence with the second sentence.

The device I just used of putting *single quotation marks* around a word or phrase is a way of naming that word or phrase, or any linguistic unit. We need some such convention because confusion can arise if it's not clear whether a word or phrase is being used *as* a word or phrase, as when I say 'The Taj Mahal has eleven letters', where I don't mean the building has eleven letters, but that the phrase does. When we use this device we'll say that we have *mentioned* the word or phrase that is in quotation marks, and the entire inscription including the quotes is a *quotation name* of the word or phrase. Otherwise, we simply *use* the word or phrase, as we normally do. We are justified in using quotation names because we have agreed to view words and propositions as types. Mentioning a linguistic unit can also be done by italicizing or putting the phrase in display format.

I use these devices for mentioning linguistic units with some reluctance, for there is not always a clear distinction between using a word and mentioning it. Moreover, when we write 'and' do we mean a string of symbols or the word with all its aspects? If we mean the word, then when we write 'Ralph is a dog' do we mean those words in that order, or do we mean the proposition? The linguistic unit intended must be inferred from the context, and sometimes it's not even clear to the user of the convention.

I will also use single quotation marks for quoting direct speech.

The device of enclosing a word or phrase in *double quotation marks* is equivalent to a wink or a nod in conversation, a nudge in the ribs indicating that I'm not to be taken literally, or that I don't really subscribe to what I'm saying. Double quotes are called *scare quotes,* and they allow me to get away with "murder".

D. Propositions in English

It will be hard for us to agree that a particular sentence is a proposition if we are speaking different languages. Therefore, throughout this book I will deal only with propositions in English or some formalized version of English.

Some argue that since modern logic is done by people speaking many different languages, it should not be considered so closely connected to one language as I draw it in this volume. Abstract or mathematical notions such as function and object suffice. But if logic does not grow out of reasoning as we do it in our daily lives, how are we to use it? And how are we to justify the methods of reasoning our logic endorses? I start with what we have—reasoning in English —and look for abstractions and idealizations that I hope can serve speakers of many different languages.

Exercises for Sections A–D

1. Give an example of formal modeling that is prescriptive in a discipline other than logic. Give another example that is descriptive.

2. a. Which of the following are declarative sentences?
   Ralph is a dog
   I am 2 meters tall
   Is any politician not corrupt?
   Power corrupts
   Feed Ralph