

Outstanding Contributions to Logic 9

Geoffrey Hellman  
Roy T. Cook *Editors*

# Hilary Putnam on Logic and Mathematics

 Springer

# **Outstanding Contributions to Logic**

Volume 9

## **Editor-in-chief**

Sven Ove Hansson, Royal Institute of Technology, Stockholm, Sweden

## **Editorial Board**

Marcus Kracht, Universität Bielefeld

Lawrence Moss, Indiana University

Sonja Smets, Universiteit van Amsterdam

Heinrich Wansing, Ruhr-Universität Bochum

More information about this series at <http://www.springer.com/series/10033>

Geoffrey Hellman · Roy T. Cook  
Editors

# Hilary Putnam on Logic and Mathematics

 Springer

*Editors*

Geoffrey Hellman  
Department of Philosophy  
University of Minnesota Twin Cities  
Minneapolis, MN, USA

Roy T. Cook  
Department of Philosophy  
University of Minnesota Twin Cities  
Minneapolis, MN, USA

ISSN 2211-2758 ISSN 2211-2766 (electronic)  
Outstanding Contributions to Logic  
ISBN 978-3-319-96273-3 ISBN 978-3-319-96274-0 (eBook)  
<https://doi.org/10.1007/978-3-319-96274-0>

Library of Congress Control Number: 2018950963

© Springer Nature Switzerland AG 2018

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made. The publisher remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

This Springer imprint is published by the registered company Springer Nature Switzerland AG  
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

# Preface

At its inception, the plan for this volume was to follow the guidelines for volumes in this series honoring the contributions of distinguished logicians, which specify including replies to the essays by the honored logician, a pattern also followed by the Library of Living Philosophers series. In the present instance, Hilary Putnam, though already in his mid-eighties, was still very active, in fact having just completed his replies to all the essays in the LLP volume dedicated to him, which appeared two years ago. Unfortunately, however, Putnam's health then declined, and this led to his passing before he had the chance to write his replies to the essays of the present volume in all but one case, viz. to Tim McCarthy's essay, "Normativity and Mechanism," which reply is included here following McCarthy's paper. Thus, in the spirit of a memorial volume, we solicited and received reminiscences reflecting on the authors' and others' associations with Hilary; we are publishing these here, following the essays.

As brought out in a moving obituary by Martha Nussbaum (published in the *Huffington Post*, 3-14-'16), one has to go back to Aristotle to find a philosopher who has contributed so significantly to so many areas of philosophy as has Hilary Putnam, ranging from technical logic and mathematics, through the philosophy of those vast subjects, to philosophy of physics, especially quantum mechanics, philosophy of language, philosophy of mind and psychology, metaphysics, epistemology, ethics, and philosophy of literature. We believe that the essays in the present volume speak to the breadth and depth of Putnam's work in logic, mathematics proper, philosophy of logic, and philosophy of mathematics.

In addition, it is fitting that we honor Hilary for his greatness as a teacher and adviser (as experienced directly by one of us). His courses in philosophy of science, logic, and set theory were high points for many of us. He had a remarkable gift for conveying the essentials of complex, technical materials, in the classes he taught as well as in his publications, with the effect of strongly encouraging and motivating his students. As an adviser, he set the bar high but within reach, and he showed

genuine appreciation of students' efforts, treating them more like colleagues engaged in joint inquiry than students at the seat of the master. And, in an era when graduate students were expected to address faculty as "Professor X," Hilary insisted that we call him "Hilary." He was not only our esteemed teacher; he became our true friend.

Minneapolis, USA

Geoffrey Hellman  
Roy T. Cook

# Contents

<b>1</b>	<b>Memories of Hilary Putnam</b> . . . . .	1
	Roy T. Cook and Geoffrey Hellman	
<b>2</b>	<b>Bibliography of Hilary Putnam’s Writings in Logic and Mathematics</b> . . . . .	9
<b>Part I Logic and the Philosophy of Logic</b>		
<b>3</b>	<b>Logic, Counterexamples, and Translation</b> . . . . .	17
	Roy T. Cook	
<b>4</b>	<b>Putnam’s Theorem on the Complexity of Models</b> . . . . .	45
	Warren Goldfarb	
<b>5</b>	<b>Extendability and Paradox</b> . . . . .	51
	Geoffrey Hellman and Roy T. Cook	
<b>6</b>	<b>The Metaphysics of the Model-Theoretic Arguments</b> . . . . .	75
	Kate Hodesdon	
<b>7</b>	<b>Normativity and Mechanism</b> . . . . .	93
	Timothy McCarthy	
<b>8</b>	<b>Changing the Subject: Quine, Putnam and Waismann on Meaning-Change, Logic, and Analyticity</b> . . . . .	115
	Stewart Shapiro	
<b>Part II Mathematics, Foundations, and Philosophy of Mathematics</b>		
<b>9</b>	<b>Putnam on Foundations: Models, Modals, Muddles</b> . . . . .	129
	John P. Burgess	
<b>10</b>	<b>Pragmatic Platonism</b> . . . . .	145
	Martin Davis	



**11 Abstraction, Axiomatization and Rigor: Pasch and Hilbert** . . . . . 161  
Michael Detlefsen

**12 Concrete Mathematical Incompleteness:  
Basic Emulation Theory** . . . . . 179  
Harvey M. Friedman

**13 Putnam’s Constructivization Argument** . . . . . 235  
Akihiro Kanamori

**14 Putnam on Mathematics as Modal Logic** . . . . . 249  
Øystein Linnebo

**Index** . . . . . 269

# Editors and Contributors

## About the Editors

**Geoffrey Hellman** received his AB and Ph.D. (1973) from Harvard. Having published widely in analytic philosophy and philosophy of science, he has, since the 1980s, concentrated on philosophy of quantum mechanics and philosophy and foundations of mathematics, where he has, following the lead of his adviser, Hilary Putnam, developed modal-structural interpretations of mathematical theories, including number theory, analysis, and set theory. He has also worked on predicative foundations of arithmetic (with Solomon Feferman) and pluralism in mathematics (with J. L. Bell). In 2007, he was elected as a Fellow of the American Academy of Arts and Sciences.

**Roy T. Cook** is CLA Scholar of the College and John M. Dolan Professor of Philosophy. He is the author of *Key Concepts in Philosophy: Paradoxes* (2013) and *The Yablo Paradox: An Essay on Circularity*, as well as numerous articles and essays in the philosophy of mathematics, the philosophy of logic, and the aesthetics of popular art.

## Contributors

**John P. Burgess** Department of Philosophy, Princeton University, Princeton, NJ, USA

**Roy T. Cook** University of Minnesota, Minneapolis, MN, USA

**Martin Davis** Courant Institute of Mathematical Sciences, New York University, New York, NY, USA; Berkeley, CA, USA

**Michael Detlefsen** University of Notre Dame, Notre Dame, IN, USA

**Harvey M. Friedman** The Ohio State University, Columbus, OH, USA

**Warren Goldfarb** Department of Philosophy, Harvard University, Cambridge, MA, USA

**Geoffrey Hellman** University of Minnesota, Minneapolis, MN, USA

**Kate Hodeson** Department of Philosophy, University of Bristol, Bristol, UK

**Akihiro Kanamori** Boston University, Boston, MA, USA

**Øystein Linnebo** University of Oslo, Oslo, Norway

**Timothy McCarthy** Departments of Philosophy, University of Illinois, Urbana, IL, US

**Stewart Shapiro** The Ohio State University, Columbus, OH, USA

# Chapter 1

## Memories of Hilary Putnam



Roy T. Cook and Geoffrey Hellman

Hilary Whitehall Putnam was many things. He was one of the most important American philosophers of the twentieth century, certainly; a lifelong socialist (and occasional Marxist); a convert to Judaism<sup>1</sup>; a native speaker of French<sup>2</sup>; a childhood “friend” of Samuel Beckett, Ford Maddox Ford, and Luigi Pirandello<sup>3</sup>; and much more. But most important, perhaps, is the fact that Putnam was, according to all that knew him, first-and-foremost four things: an excellent teacher, a great colleague, an unparalleled scholar in both mathematics and philosophy, and, perhaps most importantly, a good friend to those who were lucky enough to count themselves amongst that select group.

It is traditional in volumes such as this – especially volumes appearing relatively soon after the death of their subject – to include a lengthy, and often rather dry and boring, intellectual biography. In the case of Putnam, such an essay would not, in fact, be boring (although, given the richness of Putnam’s life, such an essay might have to be rather lengthy!) But we wanted to do something a little different, and at any rate we knew we couldn’t do a better job at biography than what is already in the gripping intellectual autobiography Putnam wrote for his installment in the Library of Living Philosophers series (Auxier et al. 2015).<sup>4</sup> Instead, we thought we would take a slightly different approach, focusing on the roles mentioned above.

---

<sup>1</sup>Although Putnam’s mother was Jewish, his childhood was secular and he only rediscovered Judaism when his son requested a Bar Mizvah.

<sup>2</sup>Although he was born in America in 1926, his family lived in and around Paris during his early childhood and he only learned English upon their return to the United States in 1933.

<sup>3</sup>Putnam was a childhood friend of these literary luminaries in the sense that, as a young child, he often sat on their laps or interacted with them in other ways when they were visiting his father Samuel Putnam, a translator and writer!

<sup>4</sup>Seriously – if you are interested in a detailed account of both Putnam’s intellectual work and his extremely interesting life, you really should read the autobiographical essay. It’s amazing!

R. T. Cook (✉) · G. Hellman  
University of Minnesota, Minneapolis, MN, USA  
e-mail: roycookparadox@gmail.com

G. Hellman  
e-mail: hellm001@umn.edu

© Springer Nature Switzerland AG 2018

G. Hellman and R. T. Cook (eds.), *Hilary Putnam on Logic and Mathematics*,  
Outstanding Contributions to Logic 9, [https://doi.org/10.1007/978-3-319-96274-0\\_1](https://doi.org/10.1007/978-3-319-96274-0_1)

Actually, we will focus here on Putnam as a teacher, colleague, and friend. No one reading this volume is likely to have doubts regarding Putnam's monumental importance as a mathematician and philosopher. And for the odd bird that does have such doubts, the essays that make up the bulk of this volume will quickly set them straight. But we thought it was worth emphasizing the other, more personal aspects of the positive impact Putnam had on philosophy during his six-plus decade career. And there is no better way to appreciate such positive effects than in the words of those positively affected. Thus, instead of attempting an exhaustive accounting of Putnam's life, we have instead collected a handful of anecdotes that, we think, paint an illuminating portrait of Putnam, the man.

Although there will obviously be overlap in these stories (after all, Putnam, like anyone else, could be more than one of friend, teacher, and colleague at once!), we'll start with some that paint a picture of Putnam the teacher. Warren Goldfarb, who was a student of Putnam's but years later became one of his colleagues at Harvard, notes that although Putnam was far from an easy teacher, he was definitely a teacher who could get students (at least, those who could keep up) excited about the material being taught<sup>5</sup>:

I first encountered Hilary Putnam in the Fall of 1966, when, as a sophomore math concentrator, I decided to take Philosophy 140, the introduction to logic offered at Harvard. That course was Quine's baby. By then he had taught it for 25 years and would continue to teach it for another 10, and he had written *Methods of Logic* to be its textbook. But Quine was at Oxford that year, and so Hilary filled in. It was his second year at Harvard. Now as Quine taught it, the course was considered pretty tough, particularly since it was a required course for the philosophy concentrators, and many of them had no background in mathematical reasoning and were not comfortable with it. Early on in the course, Hilary announced that the teaching of the actual methods of logic in the book *Methods of Logic* – for example, the paraphrase of ordinary language into logical notation, various routines for the assessment of truth-functional and monadic schemata, and the use of natural deduction for quantification theory, particularly skill at constructing deductions – all those were straightforward enough to be handled simply in the weekly sections by the teaching fellows, and he would not lecture on them. In his lectures, he went on, he would discuss other things. And so, starting the third week of the semester in this introductory logic course, Hilary introduced the semantic paradoxes, discussed Tarski's solution of them, and speculated as to whether a non-hierarchical solution was possible and what it might look like (this was ten years before Kripke's work). Hilary then returned to *Methods* to expound the completeness proof, which is usually the very endpoint of this course. But not for Hilary. For then, in the ensuing weeks, he introduced the idea of effectively decidability, using Emil Post's canonical systems rather than Turing machines, and gave a rigorous proof of the undecidability of the halting problem. That's quite an introductory course, in a philosophy department, for a largely non-mathematical audience! The bulk of the students, I have to report, did not fare very happily. But, for me, coming from mathematics, it was a great ride. I decided then and there that logic, in both its technical and philosophical dimensions, was what I wanted to pursue, and before the end of the semester I changed my concentration from math to philosophy. Although I learned a great deal from Hilary through the thirty-five years we were Departmental colleagues, in the end Hilary's greatest influence on me was this: he got me interested.

---

<sup>5</sup>Warren Goldfarb is Walter Beverly Pearson Professor of Modern Mathematics and Mathematical Logic at Harvard University.

Of course, Putnam's impact as a teacher was not limited to those students sitting in his classroom at Harvard – instead, he went out of his way to help students from all over the world and all walks of life. Bahram Assadian illustrates the lengths to which Putnam was willing to go in order to help budding young philosophers<sup>6</sup>:

In 2006, when I was a philosophy undergraduate at the University of Tehran, I was utterly fascinated by Putnam's philosophy of mathematics. I was sympathetic with mathematical structuralism and also struggling, separately, with questions about modality. Putnam's delightful synthesis of structuralism with modality has been one of the most important moments in my philosophical life. Perhaps, the only thing I could do, and I did, to express my excitement was to translate "Mathematics without Foundations" into Persian. His prose style, humour, and framings were sitting very well with Persian, as if a perfect philosophy of mathematics, in every possible sense, has been introduced into it!

With not much hope of receiving a reply, I emailed Putnam to ask some of my questions about his paper. A couple of days later, he replied with his own personal email. He had answered the questions, informed me that he has returned to the philosophy of mathematics, and expressed his hope of writing a paper in the next six months, so that he may have something new to send me. I wrote him again after six months, and he sent me a draft of his "Indispensability Arguments in the Philosophy of Mathematics", which he had read at the 40th Chapel Hill Colloquium in Philosophy in October 2006.

Six months later, I was checking my inbox and came across an email from Putnam. Initially, I felt that a friend has fooled me, but the truth was that Hilary Putnam had sent me his new paper, "Set Theory: Realism, Replacement and Modality", which he had read for the Paul Benacerraf retirement conference in Princeton. I was most honoured and flattered that I have been in Putnam's brain!

Both papers were later published in 2012, in *Philosophy in an Age of Science*. Although when I started my postgraduate studies, it was overshadowed by neo-Fregeanism, Putnam's modal interpretation of mathematics is still very alive in me, and I owe a pleasant debt to his responsibility and generosity for discussing and sharing his philosophy over email with an undergraduate student.

Putnam's role as a teacher and communicator wasn't limited to teaching or talking about philosophy and mathematics, however. On the contrary, he could hold an audience enthralled speaking on a wide variety of topics (no doubt reflecting the wide range of his own experiences). As Michael Lynch notes, such moments not only inspired interest in the topic under discussion, but also inspired the listener to try to live up to Putnam's example<sup>7</sup>:

I remember having dinner with Hilary Putnam when he came to Syracuse University around 1994 to give some lectures. I was a graduate student working on a dissertation on realism and truth. Putnam was my philosophical hero and he was incredibly generous with his time during the week he was there, meeting with me and offering me advice and philosophical wisdom. During this particular dinner he told me about traveling to Mexico when he was 18 and going to Diego Rivera's house and getting invited in for dinner by Frida Kahlo. He told me about his discussions with Einstein. Near the end of the dinner he turned to me and remarked on how lucky he was to have met such great minds, and how grateful he was to

---

<sup>6</sup>A the time of writing this essay Bahram Assadian had just completed his PhD in philosophy at Birkbeck, University of London.

<sup>7</sup>Michael Lynch is Professor of Philosophy and Director of the Humanities Institute at the University of Connecticut - Storrs.

them for their willingness to talk to a young person. I remember thinking then, as I do now, that I was having the exact same experience talking to Hilary. How lucky I was, and am, for having met him. He was one of the greats.

Putnam's interactions with colleagues were marked by a similar kind of generosity and openness, but they were also marked by Putnam's sense of humor and mischievousness, and sometimes by his radical views both with regard to academic philosophy and with regard to other, more pedestrian matters. For example, Martin Davis, with whom Putnam (along with Julia Robinson and Yuri Matiyasevich) solved Hilbert's Tenth Problem, relates the following pair of anecdotes, the first of which involves Matiyasevich<sup>8</sup>:

After the work Julia Robinson, Hilary, and I had done on Hilbert's 10th problem, the 22 year-old Yuri Matiyasevich in Leningrad provided the final piece of the puzzle in January 1970. I met Yuri in Nice that summer, and he met Julia in Bucharest the following year. Learning that Hilary was also at the Congress in Bucharest, Yuri wanted to meet him. Yuri relates: "I was told, 'Don't meet Hilary Putnam; Hilary Putnam is a Maoist.'"

When a carton of my newly published *Computability & Unsolvability* arrived smelling of printer's ink during the summer of 1958 when Hilary and I were working together, I proudly showed them to him. He offered to find an error on any page. Taking up the challenge, I showed him the reverse side of the title page which was almost blank. Hilary pointed to the word "permission": it was misspelled, missing its second "i".

Aki Kanamori tells another story about the same conference at which Matiyasevich was warned to avoid Putnam – one that also involves Putnam's idiosyncratic political ideology<sup>9</sup>:

In the summer of 1971, while I was a research student at Cambridge University, I attended the grand sounding Fourth International Congress for Logic, Methodology and the Philosophy of Science at Bucharest. There, I saw Tarski lecturing in a small amphitheatre and musing about set theory that "old soldiers never die, they just fade away", and Kreisel in a panel discussion suddenly roll up some paper and whack a fat man snoring in the front row on his head. What was just as memorable, however, was a sudden remark by Hilary. Sauntering around the city in the fading afternoon, I chanced into a pub on a side street, set into a century-old building of gray splintering wood. There was a small group from the conference having some white beer, and, connecting names with people, I soon realized that I was chatting with Hilary Putnam about mathematical logic. Suddenly, the doors swung open, and a crew of workers came marching in. They were big, ruddy, and every one of them wore denim overalls, broad and seemingly starched. They went up to the bar and started ordering beers, and the sedate atmosphere became one of commotion and all-around talk. Taking all this in, Hilary exclaimed, "It is so wonderful to see workers coming back from work!" Later I would be told that he was coming out of his Marxist activist phase, but at the time I was impressed less by the naïveté and more by a kind of sophistication of the remark, a sudden clarity of thought and purpose amid the details of mathematical logic. Yes, Hilary would turn from one way of thinking to another, from one subject to the next, but there was always enthusiasm and sophistication on the other side of apparent naïveté.

---

<sup>8</sup>Martin Davis is Emeritus Professor at the Courant Institute of Mathematical Sciences at New York University.

<sup>9</sup>Akihiro Kanamori is Professor of Mathematics at Boston University.

Tim McCarthy, who, like Davis, Goldfarb and Kanamori, also contributed an essay to this volume, shared some recollections that continue in this theme of generosity to colleagues, students, and even random audience members, but also clearly illustrate the care and consideration he extended to his friends (and their food)<sup>10</sup>:

I knew Hilary Putnam in one way or another for almost 40 years; we became good friends in the last 25 years of his life. I first saw him in action at a conference on the occasion of Hempel's retirement from Princeton University, held at Princeton in November, 1975. At this point, I was a third-year graduate student. Many of the luminaries in my own area were present: Quine, Putnam, Davidson, Kripke, and the Princeton contingent including Benacerraf, Lewis, Harman, Rorty and Hempel himself. Even in that company, I thought that weekend, Putnam stood out. He tended to get the better of any argument he embarked on, and was capable of cutting worthy opponents down. But Hilary was also capable of exercising amused restraint in dealing with difficult people, especially in public contexts. As a first-year instructor at Michigan, I helped organize the Tanner Symposium for that year, which included a talk by Putnam to which the public was invited. In the Q&A an Ann Arbor resident asked an utterly unintelligible question that went on for at least three minutes. Hilary stood at the lectern, patiently waiting for the question to come to an end. "Excellent! Excellent!", he replied. "That's the perfect preamble to the question Professor Sklar was going to ask!" Fortunately, Larry Sklar did have a question to ask.

In later years my encounters with Hilary were both philosophical and personal, and frequently included Ruth Anna and my wife Noreen. They also as frequently involved food as a central theme. In the Fall of 1995, Peter Winch and I organized a conference on Wittgenstein in Urbana. Noreen, Peter and I produced several excellent meals for this meeting – a somewhat peculiar emphasis for a conference on a figure who said that he didn't care what he ate as long as it was always the same, and who is reported to have exclaimed "Hot ziggety!" when a peanut butter sandwich was put before him. In any case, one of the culinary high points of the conference came at the conclusion of a dinner at which Noreen produced her signature Tiramisu. Hilary had extracted from her a solemn promise that he was to receive the first piece. When the moment for serving desert arrived, people actually began to pile up at the kitchen door. A woman of her word, my wife cut a sizable serving of cake and presented it to Hilary, much to the consternation of Winch and Stanley Cavell, who were waiting in line. "Why does he get the first slice?", Stanley wanted to know. Noreen replied, "Because he's special!"

Hilary and Ruth Anna were regular visitors to our home in Urbana. One year they were able to join us for Thanksgiving. The two of them spent hours that week playing Upwords with our daughter Johanna. Our turkey preparation that year called for 130 cloves of garlic, peeled – an onerous chore. The day before Thanksgiving, Noreen and I left Hilary and Ruth Anna to run an errand. Returning about an hour later, we found 130 cloves of garlic, peeled and nicely placed into several small dishes, a remarkable occurrence that has ever after in our household been known as the Miracle of the Cloves and the Dishes.

---

<sup>10</sup>Timothy McCarthy is Professor of Linguistics and of Philosophy at the University of Illinois at Urbana Champaign.



Finally, we'll conclude with one last collection of anecdotes that we believe nicely ties all these themes together, but also illustrate Putnam's lifelong connection to France. These brief stories are recounted by Karine Chemla and Bruno Belhoste<sup>11</sup>:

Hilary spent the first years of his life in France, first in a small village and then in Paris, where his father, Samuel, was part of a group that counted many key figures of the artistic scene at the time. Hilary liked to evoke his father's depiction of his life in Paris during these years, in *Paris Was Our Mistress: Memoirs of a Lost & Found Generation* (New York: The Viking Press, 1947). Hilary also liked to recall how, as a kid, he had played on Samuel Beckett's knees, and how the journals and the book that his father had co-edited while in Paris (the literary journals like *This Quarter*, and then *The New Review*, and the anthology of European poetry in English translation titled *The European Caravan: An Anthology of the New Spirit in European Literature*, 1931) had published Beckett's first poems. In relation to this, during one of his trips to Paris in the 2000s, we introduced Hilary to Barbara Bray, who had been one of the closest friends of Beckett for decades, and they exchanged souvenirs about him.

We think that this background explains why Paris occupied a very special place in Hilary's mind and heart, and why he and his wife Ruth Anna made sure to bring each of their grandchildren to Paris when they reached the age of 10. Hilary liked to speak and write French, which in fact had been his first spoken language. He also loved to come with Ruth Anna to Paris, where for years they stayed with us in a most relaxed fashion, often on the way between Israel (where they flew to spend warmer winters) and Boston. Hilary and Ruth Anna were thus regularly with us when Spring broke out, and it was a delight (and a lesson) to see them marveling at the new buds in the chestnut tree in front of our building. However simple the accommodation and the meals we could offer might have been (and in times of renovation of the house, comfort verged towards a minimum), Hilary and Ruth Anna would not mind, and shared what we could offer. This does not mean that Hilary did not know what good food was, and did not like it: he devoured croissants and everything that French cuisine could offer, as he devoured art museums, where he taught us a singular and quite amazing movement of the hand to look at paintings, which he had learnt from his childhood.

Hilary read and thought broadly. His interests had no limit that we could identify. He read about the world at large, and he read publications in any kind of discipline. This included anthropology, where he once enthusiastically brought to our attention a reference to an article of Karine's in Mary Douglas's book *Leviticus as Literature* (Oxford UP, 1999) of which we had not been aware. This was typical of Hilary's generosity and his consideration for others.

Hilary also read a great deal of literature. It was with enthusiasm and a manifest emotion that he visited with us the house of Aunt Leonie in Combray, evoked in the first pages of Marcel Proust's *Remembrance of Things Past*. Later on, he would remember this beautiful sunny day and talk about it for years, like on the last occasion when Karine ever saw him, during a visit she paid Hilary and Ruth Anna in Arlington. Perhaps Hilary's father had been to Combray, and perhaps also Proust's novels had had an important place in his family's readings, when Hilary was a boy.

Over the years, we also witnessed Hilary's concern to make the world a better place. This was evident in his concern to improve "human and social diversity" (we prefer this expression for what is commonly called "cultural diversity") in philosophy – to increase diversity both in the topical concerns of philosophers and in the scholars working in philosophy departments. Hilary was also always ready to lend support to scholars from any part of the world when they were facing a hostile audience, and he was equally friendly with and interested in anybody, independently of the interlocutor's status. Karine experienced this when she met Hilary at the

---

<sup>11</sup>Karine Chemla is a Director of Research at the Centre National de la Recherche Scientifique (CNRS), and Bruno Belhoste is Professor and Director of the Institut d'Histoire Moderne et Contemporaine (IHMC) at Université Paris 1 Panthéon-Sorbonne (Paris 1).

Wissenschaftskolleg in Berlin in 1994, and this was the starting point of the friendship they formed at the time. When Hilary met Bruno a couple of years later, they also quickly realized the values they shared in common and which became the basis of their close friendship. It was easy to love Hilary, though. In addition to his sense of humor and wit, which maintained a sparkling atmosphere around him, his unforgettable smile (even when he may have wanted to criticize) bespoke his true gentleness and fellow-feeling, which naturally drew people to him.

The last year that Hilary and Ruth Anna visited Paris, Ruth Anna was bothered by the effects of her Parkinson's disease, even though Hilary looked after her every minute. We managed a stalemate while we visited the Orsay museum, with Ruth Anna in a wheelchair. When we took them to the airport to leave Paris, we read in Hilary's slightly curved and loaded back in his deep blue rain coat, when he left us, a sense that he might never return. Unfortunately, he was right.

During the planning stages for this volume, Hilary Putnam had generously agreed to write short essays responding to each of the papers, but unfortunately he passed away on March 13, 2016, before such plans could be carried out. This is, of course, a great philosophical loss for the volume you hold in your hands which, however excellent, would have been all the more insightful with Putnam's incisive comments. But it is a much greater *personal* loss for the editors of this volume (one of whom was advised by Putnam in graduate school; the other, although he had admired Putnam from afar, only got to know him personally during the early stages of this project); for the distinguished scholars who contributed essays for this volume on or connected to Putnam's work in mathematics and logic; for the generous friends and colleagues who contributed reminiscences to this essay; and for all the other people whose life was changed for the better through knowing Putnam. He will be deeply missed.

## Reference

Auxier, R., Anderson, D., & Hahn, L. (2015). *The philosophy of Hilary Putnam: library of living philosophers* (Vol. 34). Chicago: Open Court.

**Roy T. Cook** is CLA Scholar of the College and John M. Dolan Professor of Philosophy. He is the author of *Key Concepts in Philosophy: Paradoxes* (2013) and *The Yablo Paradox: An Essay on Circularity*, as well as numerous articles and essays in the philosophy of mathematics, the philosophy of logic, and the aesthetics of popular art.

**Geoffrey Hellman** received his AB and Ph.D. (1973) from Harvard. Having published widely in analytic philosophy and philosophy of science, he has, since the 1980s, concentrated on philosophy of quantum mechanics and philosophy and foundations of mathematics, where he has, following the lead of his adviser, Hilary Putnam, developed modal-structural interpretations of mathematical theories, including number theory, analysis, and set theory. He has also worked on predicative foundations of arithmetic (with Solomon Feferman) and pluralism in mathematics (with J. L. Bell). In 2007, he was elected as a fellow of the American Academy of Arts and Sciences.

## Chapter 2

# Bibliography of Hilary Putnam's Writings in Logic and Mathematics



This bibliography records Hilary Putnam's contributions to mathematics, logic, their philosophy, and related matters. Needless to say, the border between those works that, strictly speaking, address logic and mathematics, and those that address other areas of philosophy, is fuzzy when dealing with a philosopher as systematic and as wide-ranging as Putnam. Thus, we have had to make some decisions – decisions that might not be the same decisions that the reader would have made if given the same task. For example, we have included many (but not all) of the more philosophical papers on truth, but have only included papers on confirmation when they directly address mathematical or logical, rather than scientific, issues and concerns. Nevertheless, we hope that this bibliography will be useful to readers interested in Putnam's extensive and important work in mathematics and logic.

We have given original publication information, and (with the exception of Putnam's PhD dissertation) noted reprints only when they occur in collections of Putnam's essays also included on this list (i.e. those that contain a significant number of papers on mathematics and logic).

- (1951) *The Meaning of the Concept of Probability in Application to Finite Sequences*, PhD Dissertation, University of California – Los Angeles, 1951. Reprinted New York: Garland, 1991; London, Routledge: 2011.
- (1956) “Mathematics and the Existence of Abstract Entities”, *Philosophical Studies* 7(6): 81–88.
- (1957a) “Arithmetic Models for Consistent Formulae of Quantification Theory”, *Journal of Symbolic Logic* 22(1): 110–111.
- (1957b) “Decidability and Essential Undecidability”, *Journal of Symbolic Logic* 22(1): 39–54.
- (1957c) “Eine Unableitbarkeitsbeweismethode für den Intuitionistischen Aussagenkalkül” (w/ George Kreisel), *Archiv für Mathematische Logik und Grundlagenforschung* 3(1–2): 74–78.

- (1957d) "Review of Hughes Leblac", *An Introduction to Deductive Logic, Philosophical Review* 66(4): 551–554.
- (1957e) "Three-Valued Logic", *Philosophical Studies* 8(5): 73–80. Reprinted in (1975a): 166–173.
- (1958a) "Elementary Logic and Foundations of Set Theory", in *Philosophy in the Mid-Century* (Raymond Klibansky, ed.), Florence: La Nuova Italia Editrice: 56–61.
- (1958b) "Feasible Computational Methods in the Propositional Calculus" (w/ Martin Davis), Troy, NY: Rensselaer Polytechnical Institute, Research Division.
- (1958c) "Formalization of the Concept 'About' ", *Philosophy of Science* 25(2): 125–130.
- (1958d) "Reduction of Hilbert's Tenth Problem", (w/ Martin Davis), *Journal of Symbolic Logic* 23(2): 183–187.
- (1959) "Review of Raphael Robinson, "Arithmetical Representation of Recursively Enumerable Sets""", *Journal of Symbolic Logic* 24(2): 170–171.
- (1960a) "A Computing Procedure for Quantification Theory" (w/ Martin Davis), *Journal of the Association of Computing Machinery* 7(3): 201–215.
- (1960b) "An Unsolvable Problem in Number Theory", *Journal of Symbolic Logic* 25(3): 220–232.
- (1960c) "Exact Separation of Recursively Enumerable Sets Within Theories" (w/ Raymond Smullyan), *Proceedings of the American Mathematical Society* 11(4): 574–577.
- (1960d) "Minds and Machines", in *Dimensions of Mind*, (Sydney Hook, ed.), New York: New York University Press: 138–164. Reprinted in (1975b): 362–385.
- (1960e) "Review of Ernest Nagel and James E. Newman, *Gödel's Proof*", *Philosophy of Science* 27(2): 205–207.
- (1961a) "The Decision Problem for Exponential Diophantine Equations" (w/ Martin Davis & Julia Robinson), *Annals of Mathematics* 74(3): 425–436.
- (1961b) "Some Issues in the Theory of Grammar", in *The Structure of Language and Its Mathematical Aspects: Proceedings of a Symposium in Applied Mathematics* (Roman Jakobson ed.), Providence, Rhode Island: American Mathematical Society: 25–42. Reprinted in (1975b): 85–106.
- (1961c) "Uniqueness Ordinals in Higher Constructive Number Classes", in *Essays on the Foundations of Mathematics Dedicated to A. A. Fraenkel on his Seventieth Anniversary* (Yoshua Bar-Hillel et alia, eds.) Jerusalem: The Hebrew University Magnes Press: 190–206.
- (1962a) "Dreaming and 'Depth Grammar' ", in *Analytical Philosophy, First Series* (R. J. Butler, ed.), Oxford: Basil Blackwell: 211–235. Reprinted in (1975b): 304–324.
- (1962b) "On Families of Sets Represented in Theories", *Archiv für Mathematische Logik und Grundlagenforschung* 6(1–2): 66–70.
- (1962c) "Review of Hakan Törnebohm 'On Two Logical Systems Proposed in the Philosophy of Quantum-mechanics' ", *Journal of Symbolic Logic* 27(1): 115.
- (1963a) "A Note on Constructible Sets of Integers", *Notre Dame Journal of Formal Logic* 4(4): 270–273.

- (1963b) "An Examination of Grünbaum's Philosophy of Geometry", in *Philosophy of Science: The Delaware Seminar Volume 2, 1962–1963*, (Bernard Baumrin, ed.), New York: John Wiley: 205–255.
- (1963c) "'Degree of Confirmation' and Inductive Logic", in *The Philosophy of Rudolph Carnap* (Paul A. Schilpp, ed.): 761–783. Reprinted in (1975a): 270–292.
- (1963d) "Diophantine Sets Over Polynomial Rings", (w/ Martin Davis), *Illinois Journal of Mathematics* 7(2): 251–256.
- (1963e) "Probability and Confirmation", *The Voice of America Forum Lectures: Philosophy of Science Series* Number 10: 1–11. Reprinted in (1975a): 293–304.
- (1963f) "Review of Georg Henrik von Wright, *Logical Studies*", *Philosophical Review* 72(2): 242–249.
- (1964a) *Philosophy of Mathematics: Selected Readings* (ed., w/ Paul Benacerraf), Englewood Cliffs, New Jersey: Prentice Hall. 2nd Edition 1983, Cambridge UK: Cambridge University Press.
- (1964b) "On Hierarchies and Systems of Notations", *Proceedings of the American Mathematical Society* 15(1): 44–50.
- (1965a) "Craig's Theorem", *Journal of Philosophy* 62(10): 251–260. Reprinted in (1975a): 228–236.
- (1965b) "On Minimal and Almost-Minimal Systems of Notation" (w/ David Luckham), *Transactions of the American Mathematical Society* 119(1): 86–100.
- (1965c) "On the Notational Independence of Various Hierarchies of Degrees of Unsolvability" (w/ Gustav Hensel), *Journal of Symbolic Logic* 30(1): 69–86.
- (1965d) "Recursively Enumerable Classes and Their Application to Recursive Sequences of Formal Theories", (w/ Marian Boykan Pour-el), *Archiv für Mathematische Logik und Grundlagenforschung* 8(3–4): 104–121.
- (1965e) "Trial and Error Predicates and the Solution to a Problem of Mostowski", *Journal of Symbolic Logic* 30(1): 49–57.
- (1967a) "The Craig Interpolation Lemma" (w/ Burton Dreben), *Notre Dame Journal of Formal Logic* 8(3): 229–233.
- (1967b) "Mathematics without Foundations", *Journal of Philosophy* 64(1): 5–22. Reprinted in (1975a): 43–59. Reprinted in (1983b): 295–313.
- (1967c) "The Thesis that Mathematics is Logic", in *Bertrand Russell: Philosopher of the Century* (Ralph Schoenman ed.), London: Allen & Unwin: 273–303. Reprinted in (1975a): 12–42.
- (1967d) "Time and Physical Geometry", *Journal of Philosophy* 64(8): 240–247. Reprinted in (1975a): 198–205.
- (1968a) "Degrees of Unsolvability of Constructible Sets of Integers", (w/ George Boolos), *Journal of Symbolic Logic* 33(4): 497–513.
- (1968b) "Is Logic Empirical?", in *Boston Studies in the Philosophy of Science* Volume 5 (Robert Cohen & Marx Wartofsky eds.), Dordrecht: D. Reidel: 216–241. Reprinted as "The Logic of Quantum Mechanics" in (1975a): 174–197.
- (1969a) "Normal Models and the Field of  $\Sigma_1$ " (w/ Gustav Hensel) *Fundamentae Mathematicae* 64(2): 231–240.

- (1969b) "A Recursion-theoretic Characterization of the Ramified Analytic Hierarchy", (w/ Gustav Hensel & Richard Boyd), *Transactions of the American Mathematical Society* 141: 37–62.
- (1970) "A Note on the Hyperarithmetical Hierarchy", (w/ Herbert Enderton), *Journal of Symbolic Logic* 35(3): 429–430.
- (1971a) *Philosophy of Logic*, New York: Harper and Row. Reprinted in (1975a), 2nd edition: 323–357.
- (1971b) "An Intrinsic Characterization of the Hierarchy of Constructible Sets of Integers" (w/ Stephen Leeds), in *Logic Colloquium '69* (Robin Grandy & Charles Yates eds.), Amsterdam: North Holland: 311–350.
- (1973) "Recursive Functions and Hierarchies", *American Mathematical Monthly: Supplement: Papers in the Foundations of Mathematics* 80(6): 68–86.
- (1974a) "How to Think Quantum-Logically", *Synthese* 29(1–4): 55–61.
- (1974b) "Solution to a Problem of Gandy's" (w/ Stephen Leeds), *Fundamentae Mathematica* 81(2): 99–106.
- (1974c) "Systems of Notations and the Ramified Analytic Hierarchy" (w/ Joan Lukas), *Journal of Symbolic Logic* 39(2): 243–253.
- (1975a) *Mathematics, Matter, and Method: Philosophical Papers, Volume 1*, Cambridge MA: Cambridge University Press. 2nd Edition 1979, Cambridge UK: Cambridge University Press.
- (1975a) *Mind, Language, and Reality: Philosophical Papers, Volume 2*, Cambridge MA: Cambridge University Press.
- (1975c) "What is Mathematical Truth?", *Historia Mathematica* 2(4): 529–533. Reprinted in (1975a): 60–78.
- (1978) "Quantum Logic, Conditional Probability, and Interference", (w/ Michael Friedman), *Dialectica* 32(3–4): 305–315.
- (1979) "Philosophy of Mathematics: A Report", *Current Research in Philosophy of Science: Proceedings of the P.S.A. Critical Research Problems Conference* (Peter Asquith & Henry Kyburg eds.), East Lansing MI: Philosophy of Science Association: 386–398. Reprinted as "Philosophy of Mathematics: Why Nothing Works" in (1994a): 499–512.
- (1980) "Models and Reality", *Journal of Symbolic Logic* 45(3): 464–482. Reprinted in (1983a): 1–25. Reprinted in (1983b): 421–445.
- (1982) "Peirce the Logician", *Historia Mathematica* 9(3): 290–301. Reprinted in (1990): 252–260.
- (1983) "Vagueness and Alternative Logic", *Erkenntnis* 19(1–3): 297–314. Reprinted in (1983a): 271–285.
- (1984) "Proof and Experience", *Proceedings of the American Philosophical Society* 128(1): 31–34.
- (1983a) *Realism and Reason: Philosophical Papers, Volume 3*, Cambridge MA: Cambridge University Press.
- (1983b) *Philosophy of Mathematics: Selected Readings* (ed., w/ Paul Benacerraf), 2nd Edition 1983, Cambridge UK: Cambridge University Press.

- (1989) "Model Theory and the 'Factuality' of Semantics", in *Reflections on Chomsky* (Alex George ed.), Oxford: Basil Blackwell: 213–232. Reprinted in (1994a): 351–375.
- (1990) *Realism with a Human Face*, James Conant (ed.), Cambridge MA: Cambridge University Press.
- (1991) "Does the Disquotational Theory Really Solve All Philosophical Problems?", *Metaphilosophy* 22(1–2): 1–13. Reprinted as "Does the Disquotational Theory of Truth Really Solve All Philosophical Problems?" in (1994a): 264–278.
- (1992a) "Comments on the Lectures", *Reasoning and the Logic of Things*, Charles Sanders Peirce (Kenneth Ketner ed.), Cambridge: MA: Harvard University Press: 1–54.
- (1992b) "Introduction: The Consequence of Mathematics", *Reasoning and the Logic of Things*, Charles Sanders Peirce (Kenneth Ketner ed.), Cambridge: MA: Harvard University Press: 1–54.
- (1994a) *Words and Life*, James Conant (ed.), Cambridge MA: Harvard University Press.
- (1994b) "Afterthoughts on 'Models and Reality'", *Diálogos* 29(63): 7–39.
- (1994c) "Logic and Psychology", in *The Logical Foundations of Cognition: Vancouver Studies in Cognitive Science* Volume 4 (John Macnamara & Gonzalo Reyes eds.), Oxford: Oxford University Press: 35–42.
- (1994d) "Mathematical Necessity Reconsidered", in *On Quine: New Essays* (Paulo Leonardi & Marco Santambrogio eds.), Cambridge UK: Cambridge University Press: 267–282. Reprinted as "Rethinking Mathematical Necessity" in (1994a): 245–263.
- (1995) "Peirce's Continuum", in *Peirce and Contemporary Thought: Philosophical Inquiries* (Kenneth Ketner ed.), New York: Fordham Press: 1–22.
- (1996) "On Wittgenstein's Philosophy of Mathematics", *Proceedings of the Aristotelian Society* 70 Supplement: 243–264.
- (1997a) "James' Theory of Truth", in *The Cambridge Companion to William James* (Ruth Putnam, ed.), Cambridge UK: Cambridge University Press: 166–185.
- (2000a) "A Note on Wittgenstein's 'Notorious Paragraph' About the Gödel Theorem" (w/ Juliet Floyd) *Journal of Philosophy* 97(11): 624–632.
- (2000b) "Nonstandard Models and Kripke's Proof of the Gödel Theorem", *Notre Dame Journal of Formal Logic* 41(1): 53–58. Reprinted in (2012a): 263–269.
- (2000c) "Paradox Revisited I: Truth", in *Between Logic and Intuition: Essays in Honor of Charles Parsons* (Gila Sher & Richard Tieszen eds.), Cambridge UK: Cambridge University Press: 3–15. Reprinted as "Revisiting the Liar Paradox" in (2012a): 202–215.
- (2000c) "Paradox Revisited II: Sets", in *Between Logic and Intuition: Essays in Honor of Charles Parsons* (Gila Sher & Richard Tieszen eds.), Cambridge UK: Cambridge University Press: 16–26.
- (2001) "Was Wittgenstein Really an Anti-Realist about Mathematics?", *Philosophical Explorations* 4(1): 2–16. An Expanded version of (1996). Reprinted in (2012a): 495–513.

- (2005) "James on Truth (Again)", in *William James and the Varieties of Religious Experience: A Centenary Celebration* (Jeremy Carrette ed.), London & New York: Routledge: 172–182.
- (2006a) "After Gödel", *Logic Journal of the IGPL* 14(5): 745–759. Reprinted in (2012a): 256–262.
- (2006b) "Bays, Steiner, and Wittgenstein's 'Notorious' Paragraph about the Gödel Theorem" (w/ Juliet Floyd), *Journal of Philosophy* 103(2): 101–110.
- (2007) "Wittgenstein and the Real Numbers", in *Wittgenstein and the Moral Life* (Alice Crary ed.), Cambridge MA: MIT Press: 235–250. Reprinted in (2012a): 442–457.
- (2008) "A Note on Steiner on Wittgenstein, Gödel, and Tarski", *Proceedings and Addresses of the American Philosophical Association* 82(2): 101–115.
- (2010) "Between Dolev and Dummett: Some Comments on Antirealism, Presentism, and Bivalence", *International Journal of Philosophical Studies* 18(1): 91–96.
- (2011) "The Gödel Theorem and Human Nature", in *Kurt Gödel and the Foundations of Mathematics: Horizons of Truth* (Matthias Baaz, ed.) Cambridge UK: Cambridge University Press. Reprinted in (2012a): 239–255.
- (2012a) *Philosophy in an Age of Science: Physics, Mathematics, and Skepticism*, (Mario De Caro & David MacArthur eds.), Cambridge MA: Cambridge University Press.
- (2012b) "On Mathematics, Realism, and Ethics", *Harvard Review of Philosophy* 18(1): 143–160.



**Part I**  
**Logic and the Philosophy of Logic**

# Chapter 3

## Logic, Counterexamples, and Translation



Roy T. Cook

**Abstract** In “Is Logic Empirical” (Putnam 1968), Putnam formulates an empirical argument against classical logic—in particular, an apparent counterexample to the distributivity laws. He argues further that this argument is also an argument in favor of quantum logic. Here we challenge this second conclusion, arguing instead that counterexamples in logic are counterexamples not to particular inferences, but to logics as a whole. The key insight underlying this argument is that what counts as a legitimate translation from natural language to formal language is dependent on the background logic being assumed. Hence, in the face of a counterexample, one can move to a logic that fails to validate the inference seemingly counter-instanced, or one can move to a logic where the best translation of the natural language claims involved in the counterexample are no longer best translated as an instance of the inference in question.

### 3.1 Introduction

In “Is Logic Empirical?” (Putnam 1968), Hilary Putnam formulates a now-famous, *empirical* argument against classical logic.<sup>1</sup> The argument hinges on the fact that quantum mechanics seems to provide examples where at least one of the standard distributivity laws:

$$\begin{aligned}A \wedge (B_1 \vee B_2) &\not\vdash_{\mathbf{C}} (A \wedge B_1) \vee (A \wedge B_2) \\A \vee (B_1 \wedge B_2) &\not\vdash_{\mathbf{C}} (A \vee B_1) \wedge (A \vee B_2)\end{aligned}$$

---

<sup>1</sup>Although arguments for the in-principle possibility of purely empirical challenges to the correctness of a logic – classical or otherwise – go back to Quine (1951), Putnam was amongst the first to provide a serious putative example of such an empirical challenge.

R. T. Cook (✉)  
University of Minnesota, Minneapolis, MN, USA  
e-mail: roycookparadox@gmail.com

(in particular, the left-to-right direction of the first equivalence) fails. As a result, Putnam suggests we ought to abandon classical logic, and adopt quantum logic as the correct account of logical consequence.

The purpose of the present essay is not to challenge the correctness of Putnam's conclusion – that is, that quantum logic is the one, true, correct account of logical consequence.<sup>2</sup> Nothing in the sections to follow implies that Putnam got that bit *wrong*.<sup>3</sup> Instead, the worry that I will develop here concerns the methodology by which that conclusion was reached. In short, Putnam has shown, at best, that quantum logic is one amongst a number of logics that we might adopt in the face of the recalcitrant evidence (seemingly) provided by quantum mechanics.

Thus, I am going to grant without argument or critical examination many of the claims that Putnam spends a great deal of ink discussing – claims that have also played central roles in much of the discussion of “Is Logic Empirical?”. In particular, I will assume without argument that:

1. Our account of the correct logic(s) should be sensitive to the empirical evidence provided by our best science.
2. Quantum mechanics provides sufficient evidence for abandoning classical logic – that is, it provides a counterexample to classical logic.

It is perhaps worth noting, as a matter of autobiographical detail, that I am extremely sympathetic to the first claim – and to the more general idea that logical theorizing should be attentive to all sorts of considerations in addition to those associated with the armchair. The second claim strikes me as somewhat more questionable, however.<sup>4</sup> But, since we have other logical fish to methodologically fry, I shall assume both claims throughout what follows.

---

<sup>2</sup>Of course, I personally don't believe that there is *one* correct logic (i.e. I am a logical pluralist of some sort), nor do even I believe that quantum logic is a plausible candidate for being one of the multitude of 'correct' or 'best' logics – see Cook (2014). The point is that nothing in the present paper depends on these further views.

<sup>3</sup>There are, of course, a number of extant criticisms of his argument along these lines. There are four primary themes running through such criticisms. First, there are objections to the role that realism plays in Putnam's argument – for a prominent example of this sort, see Dummett (1976). Second, there are objections to Putnam's presentation of the physics, the logic, or the connections between the two – for a recent example of this sort, see Maudlin (2005). Third, there are objections to the claim that formal quantum logic – that is, the propositional/first-order theory obtained via constructing a semantics in terms of the lattice of 'quantum propositions' – blocks the problematic inferences anyway. Gardner (1971) and Gibbons (1987) are notable examples of this approach. Finally, there is the claim – forcefully argued for in Hellman (1980) – that the language and logic within which quantum mechanics is formulated is fully classical, and hence shifting to a different logic once these puzzles arise (regardless of whether, *pace* Gardner (1971) and Gibbons (1987), such a logic actually blocks the problematic inferences) amounts to ignoring the problem rather than addressing it. The present essay will address none of these specific concerns.

<sup>4</sup>Of course, a fully sufficient examination of whether or not quantum mechanics really does provide counterexamples to classical logic would require a significantly deeper understanding of the relevant science than I possess. Fortunately, as I have already emphasized, the point of this paper, which really concerns the methodology of logic rather than that of science, does not depend on answering this distinct question.

Accepting, even merely for the sake of argument, that quantum mechanics provides a counterexample to classical logic is one thing, however. Understanding what such a counterexample might amount to, however, and in particular, what such a counterexample tells us about the identity of the correct logic (or logics), is something else. The main goal of this essay is to provide an examination of the structure and methodology of Putnam's argument for logical revision that is a good bit more careful and more complete than previous treatments. The previous sentence might seem a bit overstated if one does not attend to the wording carefully: as already emphasized, the point here is not to determine whether or not Putnam is right about quantum logic being the one true correct logic – a point that *has* been debated extensively in the literature. Instead, the focus here will be on the structure of Putnam's argument, and arguments like it, that take apparent counterexamples to classical logic as premises and arrive at revisions to classical logic as conclusion. In short, the topic at issue here involves the *general strategy* underlying Putnam's argument, and arguments like it. I am only interested (in the present essay, at least) on examining the methodology of logical revision, and not on the particular logic one might arrive at by correctly applying such methodology.<sup>5</sup>

Now that we are clear about what we are going to grant to Putnam (and, more generally, to philosophers, mathematicians, and logicians engaged in relevantly similar arguments for logical revision based on apparent counterexamples – empirical or not – to inferences previously accepted as valid), it is time to clearly identify where, exactly, we are going to disagree with Putnam. As already noted, we are going to grant for the sake of argument that quantum mechanics provides a counterexample to classical logic. What will be denied, however, is the following third claim:

3. Quantum mechanics provides sufficient evidence for abandoning the laws of distributivity – that is, it provides a counterexample to one of the laws of distributivity.

At first glance this third claim might seem nearly synonymous, in the present context, to the second claim above. After all, isn't the point of Putnam's argument to show that classical logic is not the correct logic (at least, for reasoning about the quantum realm) by showing that one of the distributivity laws fails when reasoning about quantum mechanics?

In fact, Putnam has not provided a counterexample to the distributivity laws. The reason, as we shall see, is simple (although teasing out the subtleties involved will take some work, of course): it is, in fact, *impossible* to provide a counterexample to any logical law *tout court*. Rather, counterexamples such as Putnam's quantum mechanical example do not show that individual inferences are invalid – rather, they show that individual inferences *understood from the perspective of a particular background logic* are invalid. As a result, it is not the individual inference that is impugned by a counterexample, but the background logic against which this inference is judged.

---

<sup>5</sup>Of course, it goes without saying that any novel conclusions we arrive at with respect to the correct methodology for dealing with purported counterexamples to our favored logics will have very real consequences for what candidate logics we might take seriously as correct or (if one has pluralist leanings) legitimate. The point is merely that we are not focusing here on answering the latter question.