Richard J. Morris *Editor*

Mathematical Modelling in Plant Biology

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Richard J. Morris
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John Innes Centre
Norwich, Norfolk, UK

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Preface

Plant biology is a rich, fascinating and rewarding playground for those with an interest in modelling and simulation.

Plants are constantly processing information, computing and adapting to their surroundings. This notion of information processing is becoming increasingly important in biology as is the appreciation of physical and engineering approaches for understanding these processes and how they are manifested in form and function. Our basic premise is that whatever plants are doing, they are doing within the laws of physics. Physical approaches using the established language of mathematics and computation are therefore key research tools for unravelling the biology of plants. For instance, plant growth can be viewed as a mechanical problem in which plants exploit hydraulics and material properties to determine their shape. Photosynthesis can be treated as a quantum mechanical problem, whereby photons are captured and their energy used to catalyse chemical processes that convert carbon dioxide to sugar. Plants use diffusible particles and electrical waves for transmitting information. Plants generate pressure gradients that drive fluid flow through small elastic tubes to transport nutrients. The list goes on.

Perhaps with the exception of the theory of evolution, biology may seem to currently lack the unifying laws of physics, the axiomatic nature of mathematics or the abstractions of computer science, thus giving rise to the usual clichés. The ‘hard’ sciences are typically viewed as abstract, reductionist, mathematical and quantitative. For some theories, such as quantum electrodynamics or special relativity, astonishing levels of precision have been reported, achieving better than ten significant digits. Such precision (and accuracy) can induce the idea of these theories being ‘exact’. On the other hand, ‘soft’ sciences have a reputation of being more descriptive and qualitative. Biological systems perform tasks such as self-repair and reproduction that fit less readily within existing physical and engineering frameworks. The science of living systems can appear ‘messy’ and therein lies the challenge. It is ‘easy’ to achieve robust computation using well-characterised, virtually error-free components, but how does biology with its noisy, fluctuating systems manage to carry out tasks such as reproducing cells and whole organisms so robustly? Biology is full of such ‘hard’ problems.

More important than this artificial division into disciplines is the approach. Unlike in physics where the question of 'function' or 'utility' of something (why questions) makes little sense, in biology thinking about function can help place observations in an evolutionary context and enhance our understanding. Thinking about problems in terms of cause and effect (how questions) is what leads to mechanistic insights which in turn helps understand evolutionary innovation, and this is the spirit of the chapters presented in this book. Modelling and simulation have a key role to play in unravelling mechanism and trying to discern cause and effect but also in making sense of evolutionary changes. Efficient models can compress a lot of data and knowledge into a few rules or equations. Despite all models being wrong at some level, they offer powerful tools to synthesise data and ideas, to generate and evaluate hypotheses and to make predictions. Importantly, useful models are falsifiable (which is when we learn the most about a system). Predictions can be used to guide new experiments and to validate, falsify or define the application boundaries of a model.

This book aims to provide a mix of introductory chapters and latest state-of-the-art research overviews that place key questions in plant biology within a cause-and-effect framework, thereby drawing on relevant physical, mathematical and computational approaches. Modern plant biology requires an increasing and diverse set of skills and seamlessly blends them into an integrative, interdisciplinary approach. All chapters are written by leading experts who are driving such interdisciplinary developments. The chapters are not meant to be exhaustive but to give a flavour of some of the current problems and to provide some background upon which can be built to develop a solid foundation for research in the area. The book is aimed at physicists, mathematicians, computer scientists and engineers, whom we hope to excite with the challenges and opportunities in plant biology but also the increasing number of mathematically skilled biologists with an interest in modelling and simulation as a means to understand biology. For others there are likely better and more suitable introductions.

For those without much biology background, there are many truly excellent text books. A fantastic and stimulating up-to-date classic is *Molecular Biology of the Cell* by Bruce Alberts, Alexander Johnson, Julian Lewis, Martin Raff, Keith Roberts and Peter Walter. Some basic understanding of DNA, genes and proteins can readily be developed from online resources. For the current book, the prerequisites in terms of plant biology are limited, many of which are explained in the individual chapters. Important concepts include the following: plants consist of cells that are surrounded by a cell wall and are therefore not mobile; plant cells can build substantial pressures within them through osmosis, which is the exchange of water driven by the chemical potential that arises from the concentration difference on solutes; plant cells have various components (proteins) that can transport ions across membranes, giving rise to concentration differences and therefore voltages which can be exploited for signalling. Personal favourites for more in-depth studies include *Plant Biomechanics* by Karl Niklas, *An Introduction to Systems Biology* by Uri Alon and *Information Theory, Inference and Learning Algorithms* by David MacKay.

Given that human life is sustained by plants in that they influence our atmospheric composition, providing us with oxygen to breath, form important ecosystems, contribute a substantial part of our diet and calorific intake and provide natural products and medicines, understanding the biology, physics and computation of plants is perhaps one of the most relevant challenges of our time.

As I hope will become apparent from this book, plant biology is a 'hard' science. It extensively uses quantitative data, physical theories and mathematical and computational modelling and is becoming increasingly predictive. Furthermore, plant biology is great fun.

Chapter 1 introduces physical models of plant morphogenesis. The theory of forces, stress and strain is explained with selected case studies. Chapter 2 summarises the basic mathematical approaches to fluid transport in plants. Fluid dynamics plays an important role in the transport of nutrients, growth and long-distance signalling. Chapter 3 describes how we can use physical models to understand ion channels. Much of the information processing carried out by plants uses changes in ionic concentrations to transmit signals that activate responses to environmental challenges. Chapter 4 introduces plant microtubules and mathematical and computational techniques for modelling their behaviour. Microtubules are dynamic entities that play a key role in determining plant cell shape and function. From here onwards we are exposed to problems of organisation over multiple spatial and temporal scales—a reoccurring theme and challenge in approaches to modelling in biology. Chapter 5 takes a closer look at how cell shape changes as a function of ion channel activity on the example of guard cells, thereby integrating transport processes with macroscopic function. In Chap. 6, an overview is provided of the most recent developments in single-cell approaches for understanding morphogenesis, particularly in terms of image processing, quantitative data analysis and computational modelling techniques. Chapter 7 goes beyond single cells and tackles approaches for describing collections of cells, tissues, their interactions, growth and division. Chapter 8 takes an abstract computational approach to plant development with the development of L-systems. L-systems offer powerful tools for studying plant development at different levels from reactions to whole plant behaviour. Chapter 9 describes recent results on the important trait of flowering time to move up further in scales of synthesising knowledge. This chapter considers gene networks, phenology and evolution. Chapter 10 takes the scale of modelling one important step further and investigates the lifestyle strategy of plants in natural environment on the example of seed banks. Together these chapters are exemplars of how plant science is developing and the inherent challenges of bridging scales between micro-mechanisms through cells to whole plant behaviour and populations of plants in a changing environment. There is clearly no shortage of really exciting and highly relevant challenges ahead for which computational approaches will have a key role to play.

There are many exciting computational developments in plant biology, and hopefully future editions of this book can be extended to include further chapters that display the power of interdisciplinary journeys into the processes and mechanisms of plants.

Norwich, UK Richard J. Morris
April 2018

Contents

Contributors

Milad Adibi Max Planck Institute for Plant Breeding Research, Köln, Germany

Michael R. Blatt Laboratory of Plant Physiology and Biophysics, University of Glasgow, Glasgow, UK

M. G. Blyth School of Mathematics, University of East Anglia, Norwich, UK

Arezki Boudaoud Laboratoire Reproduction et Développement des Plantes, Univ Lyon, ENS de Lyon, UCB Lyon 1, CNRS, INRA, Lyon, France

Mikolaj Cieslak Department of Computer Science, University of Calgary, Calgary, AB, Canada

Eva E. Deinum Biometris, Wageningen University & Research, Wageningen, The Netherlands

Mathilde Dumond Laboratoire Reproduction et Développement des Plantes, Univ Lyon, ENS de Lyon, UCB Lyon 1, CNRS, INRA, Lyon, France

Pascal Ferraro Department of Computer Science, University of Calgary, Calgary, AB, Canada

Pau Formosa-Jordan Sainsbury Laboratory, University of Cambridge, Cambridge, UK

Jim Hanan Centre for Horticultural Science, University of Queensland, Brisbane, QLD, Australia

Adrian Hills Laboratory of Plant Physiology and Biophysics, University of Glasgow, Glasgow, UK

Henrik Jönsson Sainsbury Laboratory, University of Cambridge, Cambridge, UK

Computational Biology and Biological Physics, Lund University, Lund, Sweden

Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge, UK

R. J. Morris Computational and Systems Biology, John Innes Centre, Norwich, Norfolk, UK

Gabriella Mosca Max Planck Institute for Plant Breeding Research, Köln, Germany

Bela M. Mulder Living Matter Department, Institute AMOLF, Amsterdam, The Netherlands

Department of Plant Science, Wageningen University & Research, Wageningen, The Netherlands

Maria Papanatsiou Laboratory of Plant Physiology and Biophysics, University of Glasgow, Glasgow, UK

Przemyslaw Prusinkiewicz Department of Computer Science, University of Calgary, Calgary, AB, Canada

Adam Runions Max Planck Institute for Plant Breeding Research, Köln, Germany

Aleksandra Sapala Max Planck Institute for Plant Breeding Research, Köln, Germany

Akiko Satake Department of Biology, Faculty of Science, Kyushu University, Fukuoka, Japan

Richard S. Smith Max Planck Institute for Plant Breeding Research, Köln, Germany

Soeren Strauss Max Planck Institute for Plant Breeding Research, Köln, Germany

J. Tabak University of Exeter Medical School, University of Exeter, Exeter, UK

José Teles Sainsbury Laboratory, University of Cambridge, Cambridge, UK

Aurélien Tellier Section of Population Genetics, Technical University of Munich, Freising, Germany

K. Tsaneva-Atanasova Department of Mathematics and Living Systems Institute, University of Exeter, Exeter, UK

K. C. A. Wedgwood Department of Mathematics and Living Systems Institute, University of Exeter, Exeter, UK

Daniel Živković Section of Population Genetics, Technical University of Munich, Freising, Germany

About the Editor

Richard J. Morris Richard's research aims to shed light on the physics of information processing in plants. He completed a degree in Mechanical Engineering at the age of 19 before obtaining a BSc in Physics and then an MSc in Theoretical Physics in 1996 from the Erzherzog University of Graz, Austria. He won an EMBL fellowship to carry out his PhD research at the European Molecular Biology Laboratory (EMBL) in the field of computational protein crystallography with Dr Victor Lamzin. After completing his PhD in 2000, Richard joined the group of Dr Gerard Bricogne (MRC-LMB Cambridge and Global Phasing Ltd.) to work on Bayesian approaches for protein structure solution. Richard then joined the group of Prof Dame Janet Thornton, FRS, at the European Bioinformatics Institute (EMBL-EBI) in 2002, where he developed novel shape mathematics for protein function prediction. In 2005, Richard was recruited to the bioinformatics group at the John Innes Centre (JIC) as a tenure-track project leader. Richard played a key role in building up computational biology at JIC. He became Head of the Department for Computational and Systems Biology in 2010. In 2013 he took on the role of institute strategic programme leader as an associate director. He is active in promoting quantitative, and in particular physical, approaches to plant biology and in training the next generation in mathematical modelling and computational methods.

Chapter 1
Physical Models of Plant Morphogenesis

Mathilde Dumond and Arezki Boudaoud

Abstract Biological form is closely associated with function. Yet, despite much progress in developmental biology, we are still far from understanding how organs grow and reach their final size and shape, through a process known as morphogenesis. Morphogenesis is associated with a variety of cellular scale phenomena such as cell expansion, cell proliferation, and cell differentiation. These processes occur within the thousands to billions of cells that yield a well-defined organ. How these phenomena are coordinated over time and space to shape a consistent and reproducible organ or organism is still an open question. In this chapter, we focus on physical models of morphogenesis. We first introduce quantitative descriptions of growth. We then expand on mechanical models of growth; we review types of models and we discuss case studies where such models were used.

1.1 Describing Morphogenesis

To better understand morphogenesis and reliably compare models to experiments, qualitative observations are not sufficient and quantitative measurements are necessary. From an analytical viewpoint, morphogenesis can be dissected as the combination of a small set of elementary transformations. The final shape of an organ results from the integration throughout time of growth. Growth can be decomposed into three parameters: growth rate (differential of area over time), growth anisotropy (ratio between the maximal and the minimal principal directions of growth), and maximal growth direction (see Fig. 1.1) [12, 18, 27, 65]. Formally, growth is a tensor that can be defined similarly to the strain tensor in continuum mechanics. Consider a generic material point of coordinates (x_1, x_2, x_3); it is displaced by growth to $(x_1 + u_1, x_2 + u_2, x_3 + u_3)$, (u_1, u_2, u_3) being the displacement field. The growth tensor is then:

M. Dumond · A. Boudaoud (✉)
Laboratoire Reproduction et Développement des Plantes, Univ Lyon, ENS de Lyon, UCB Lyon 1, CNRS, INRA, Lyon, France
e-mail: arezki.boudaoud@ens-lyon.fr

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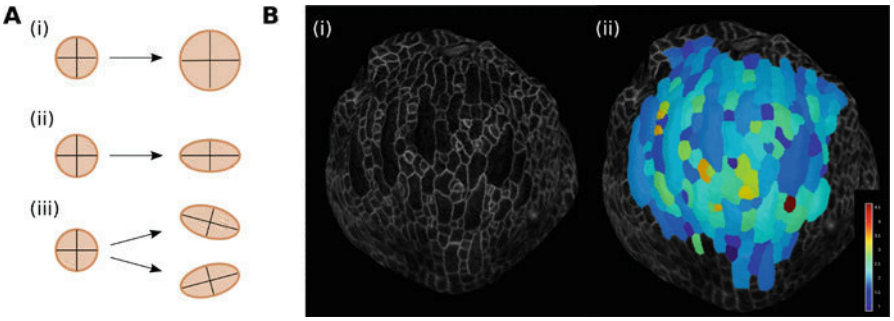

Fig. 1.1 Quantifying morphogenesis. (**a**) Growth can be assessed by monitoring a circle drawn on the tissue (a sphere in 3D): (i) if the circle remains a circle, growth is isotropic; (ii) if the circle becomes an ellipse, growth is anisotropic (growth rate can be deduced from the ratio of surfaces between the two time points; growth anisotropy derives from the ratio in length of the ellipse axes); (iii) growth direction corresponds to the direction of the great axis of the ellipse. (**b**) Example of a software package developed to quantify growth: (i) image of an *Arabidopsis thaliana* sepal with the membrane tagged with a fluorescent molecule; (ii) growth rates quantified over each cell for a 24h interval using MorphoGraphX [5] (the color scale corresponds to the ratio in cell area between the two time points)

$$g_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_i}{\partial x_j} + \sum_m \frac{\partial u_i}{\partial x_m}\frac{\partial u_j}{\partial x_m}\right). \tag{1.1}$$

Note that this definition implicitly assumes that the elastic strain tensor (due to internal or external forces exerted on the growing body) is negligible. Other slightly different definitions, or sometimes rates (time derivatives), are also used. Such quantifications enable the conversion of successive images of organs into quantitative data to perform statistical analyses and to compare models to experiments in a systematic manner.

1.1.1 Quantifying Cell Growth

The starting point is two- or three-dimensional images of tissues or organs showing cell contours, for instance tagged with a fluorescent protein when using confocal microscopy. Quantitative measurements have been facilitated by the development of software that segment cells from such images and measure their growth parameters semi-automatically, in 2D [5], or in 3D [29]. Such software has been used to extract and characterize cell shapes [47, 53, 64], cell growth [15, 37, 42, 66], and to compare mutants to wild-type growth [40, 69]. These cell-based quantifications have been performed so far for rather small organs (less than a thousand of cells). The growth patterns of bigger organs such as older leafs and flowers are often measured at the supracellular level using a continuous description.

1.1.2 Quantifying Organ Growth

The growth patterns of large organs are measured with methods such as landmark analysis and clonal analysis. It is possible to use landmarks on an organ, and measure the relative displacements of the landmarks over time. Early studies considered either natural landmarks such as the vein intersections in leaves [48] or ink-drawn landmarks—grids [3] or set of points [33, 55]—restricting these approaches to relatively older leaves. More recent studies used fluorescent microparticles deposited on leaves [57, 60].

It is also possible to perform clonal analysis, which consists in labelling groups of cells or single cells by expressing a specific heritable marker, and observing their descendants. When a notable fraction of the cells is labelled without any neighbor marked, this enables to define growth rate, anisotropy, and direction at the supracellular level [56]. Because the tissue deforms during growth, the interpretation of clonal analysis requires the use of a model to account for the advection of material points by growth [59].

Finally, two studies have shown that measuring the leaf contour change over time was sufficient to use conformal maps to roughly predict the displacement field of all points inside the leaf [1, 52], because the leaf remains flat all over its development, and because its growth is roughly isotropic at later stages of leaf morphogenesis.

1.2 Forces in Plants

Plant cells are surrounded by a stiff extracellular matrix called the cell wall, put under tension by the internal hydrostatic pressure known as turgor that typically ranges from 0.1 to 1 MPa [8]. Growth is achieved by modulating turgor pressure and cell wall mechanical properties. How can we relate growth to cell mechanics? What is the mechanical status of plant tissues? We here focus on the evidence for forces, as measurements of mechanical properties were reviewed elsewhere [8, 51, 62, 68].

1.2.1 Forces in Tissues and Cells

It has been observed that when peeling a stem, the outer tissue shrinks whereas the internal tissue expands, suggesting that internal cell layers are in compression while the outer layers are under tension [67]. Similarly, cutting a plant tissue leads to a deformation—the cut opens or remains closed according to whether the tissue is in tension or not. Measuring such deformations thus yields information on mechanical stress patterns. For instance, the epidermis of the early sunflower capitulum is under tension in its center and under circumferential compression in the concave region that surrounds the center [23].

Turgor pressure results from osmotic pressure, and so is determined by the difference in osmolyte concentration between the cell and the outer medium. Hence, it is possible to plasmolyze cells by increasing the outer concentration of osmolytes, removing turgor pressure and the tension in the cell wall. Comparing plasmolyzed and turgid cells yields the elastic strain in turgid cell walls and gives information on the stress pattern in cell walls [63].

1.2.2 Forces and Growth

The interplay between turgor pressure and cell wall mechanical properties is at the core of our current understanding of plant cell growth. The growth of the plant cell is due to the yielding of the cell wall under the tension generated by turgor pressure, in addition to the synthesis and export of new wall materials by the cell. From the mechanical point of view, cell wall tension induces deformations that depend on cell wall rheology.

In the simplest rheological model, the cell wall is considered purely elastic: it behaves like a spring. In this case, the deformation, or strain ϵ, depends only on the stress σ applied on the spring, or the cell wall:

$$\epsilon = \sigma / E \tag{1.2}$$

where E the stiffness modulus of the cell wall. The principal limitation of this model is that the spring reverts to its rest shape when the force is released: such a cell wall does not grow. This issue is dealt with within the framework of incremental growth: The spring is loaded and the equilibrium length is taken as the next rest length of the spring. This new rest length initiates the next loading step. Thus growth is modeled as a succession of steps of loading and updating of the rest length.

A more realistic representation of the cell wall is a viscoelastic material. In this case, it behaves like the association of a spring and a damper (see Fig. 1.2):

$$\frac{d\epsilon}{dt} = \sigma/\mu + \frac{d\sigma}{dt}/E \tag{1.3}$$

where μ is the dynamic viscosity. In a few models of growth [16], elasticity is fully neglected and only the damper is accounted for. Note that the incremental approach to growth is equivalent to a viscoelastic model if a timescale is associated with step increments [11]. Here, when the force is released, the material does not revert to its original configuration: this rheology allows cell wall growth.

One of the most elaborate rheological models of the cell wall behavior was introduced in [54], and proposes that the cell wall behaves as a visco-elasto-plastic material (see Fig. 1.2): the cell wall behaves as an elastic material when the stress is lower than a threshold σ_0, but as a viscoelastic material if the stress is larger ($|f|^{(+)} = 0$ if $f < 0$ and else $|f|^{(+)} = f$):

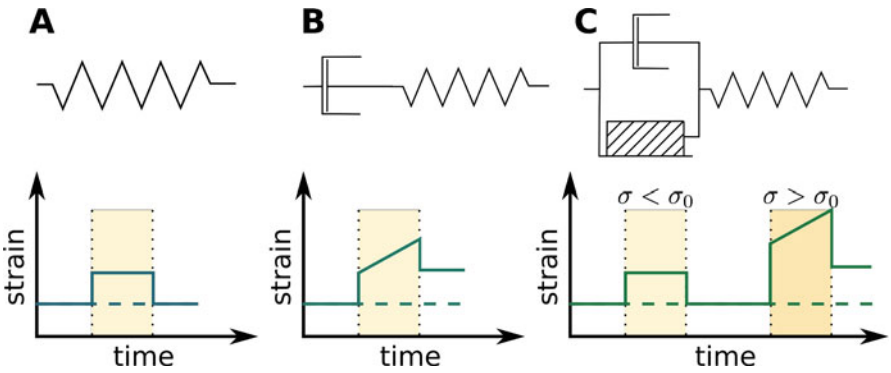

Fig. 1.2 Simple rheological behaviors and plant growth. (**a**) A purely elastic material behaves like a spring: it returns to its original state when applied stress is released. (**b**) A viscoelastic material (of Maxwell type) behaves like the association of a spring and a damper; rate of damper elongation depends on the applied stress. (**c**) The behavior of a visco-elasto-plastic material depends on the value of the force applied: if the stress is smaller than the material-specific threshold σ_0, it behaves elastically, otherwise it behaves viscoelastically. The bottom plots show strain as a function of time, with force applied during the period highlighted in yellow

$$\frac{d\epsilon}{dt} = |\sigma - \sigma_0|^{(+)}/\mu + \frac{d\sigma}{dt}/E \tag{1.4}$$

Cell wall rheological parameters can be measured, using techniques reviewed in [51, 63, 68]. Depending on the question addressed, one can use one or the other of these rheological models. Note that the rheological parameters of the cell wall can be heterogeneous and/or anisotropic. Full models require the generalization of these simple rheological models to 2 or 3 dimensions and the assembly of simple bricks to account for cell and/or tissue geometry and for links between cellular processes and cell wall mechanics.

1.3 Modeling Morphogenesis

1.3.1 Different Types of Models

Models of morphogenesis fall into two main categories: models considering a continuous growing medium and models individualizing each cell.

Continuous models are usually used for large organs, comprising thousands of cells, where cell size is very small compared to organ size. Most studies considered flat organs such as leaves, petals, or sepals (see, e.g., [40, 41]). The surface of the organ is modeled as a 2D surface embedded in 3D space, assuming the thickness is small with respect to other dimensions. 3D models are less common, one example being the morphogenesis of fruits [17]. A widespread assumption for other organs

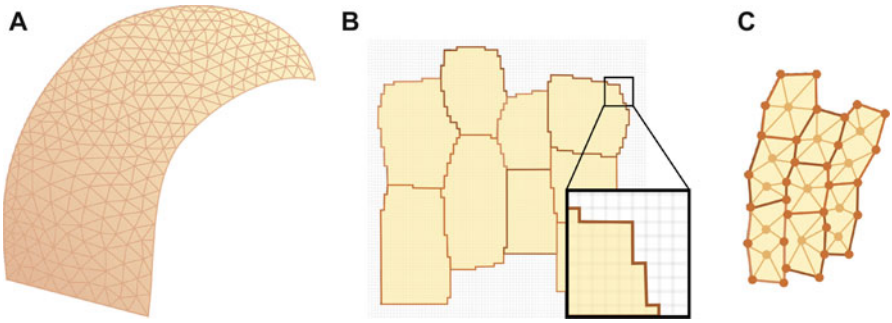

Fig. 1.3 Main types of models: Examples in two dimensions. (**a**) Continuous model: the surface of the organ is continuous, for instance represented by a triangulated mesh. (**b**) Cellular Potts model, where cells are defined on a non-deformable grid. (**c**) Vertex-based model, the cells are defined by vertices (circles) and their edges with the neighboring cells

such as stems is that the epidermis is dominant in the control of growth, because the surface cell wall is thicker and stiffer than internal walls (see, e.g., [9] for a discussion). This enables modeling the dynamics of the surface of the organ [39]. Formally, these models come in the form of partial differential equations, often obtained by accounting for cell wall rheology and mechanical equilibrium. The finite element method is often used to solve them because the triangular meshes are well-suited to domains of arbitrary geometry.

Cells are individualized in other types of models, such as cellular Potts models or vertex-based models (see Fig. 1.3). In the cellular Potts model, cells are defined on a discrete fine grid, and the status of each point of the grid is updated depending on a set of rules, leading to the movement of the edges of the cells. This framework was originally developed for physical systems such as foams. It is widely used in the animal field, and can have a finer subcellular resolution than the vertex-based models discussed hereafter. Nevertheless, the cellular Potts model seems to have been used only once for plants, in the context of auxin concentration dynamics in a growing root [35]. Indeed, this framework is not well-suited for an elastic material and is commonly used to model purely viscous materials such as animal cells.

Vertex models are broadly used to investigate plant development [25, 49]. Cells are often assumed to be polygonal, so that cell shape is defined by the position of vertices and their dynamics in space. Sometimes, edges are allowed to be curved, for instance assumed to be arc of circles [20]. The mechanical elements can be placed at cell edges (e.g. spring-damper systems) accounting for anticlinal cell walls [20, 25], or on the whole surface of the cell accounting for periclinal cell walls [22, 49, 61]. Such models can also incorporate gene regulatory networks, cell–cell communication, or cell division [25].

More recently, 2D models started to combine vertex-based and continuous approaches, with cells individualized and their cell walls represented as continuous structures [14, 30]. This enables to simultaneously model cell scale behavior (cell division, cell shape) and consider the mechanical properties of the periclinal cell

wall at a subcellular resolution. This approach was extended to 3D, enabling for the first time to model morphogenesis a 3D tissue at cellular resolution [12]: by accounting for epidermis and for internal layers, the authors investigated the relative role of cell layers in the outgrowth of organ primordia in the shoot apical meristem.

1.3.2 Implementation of Growth

So far, growth has been modeled using two different approaches. In the first type of description, growth rate, direction, and anisotropy are specified or inferred from a gene regulatory network at each point in time and space of the simulation. The tissue is assumed to be elastic and the equilibrium state is computed from force balance [41]. This amounts to prescribing the rest length of an assembly of springs and computing their equilibrium lengths.

In the second type of description, the cell wall's mechanical properties such as elastic modulus or viscosity (these properties may be anisotropic) are specified or inferred from regulatory networks, and the mechanical equilibrium under loading by turgor pressure defines the current growth rate. In this framework, the tissue deformation depends on the tissue rheology, which can be one of the types previously presented: elastic, viscoelastic, or visco-elasto-plastic.

1.4 Case Studies: The Use of Models to Understand Plant Morphogenesis

We now illustrate the concepts outlined above by discussing representative physical models of morphogenesis.

1.4.1 Morphogenesis of an Isolated Plant Cell

A first step towards understanding organ morphogenesis is to study cell morphogenesis. A classical system of interest is the pollen tube, which is one of the model systems for tip growth: elongation of the tube by expansion of the cell wall localized at the cap of the cylinder. During pollination, the pollen grain lands on the summit of the carpel and germinates. The pollen tube emerges and subsequently grows into the carpel reaching the ovule. How a cell can grow in such directional manner has been extensively investigated.

Considering the cell wall as a hyperelastic (extension of linear elasticity to large deformation) membrane and using an incremental approach to growth, [32] showed that a lower elastic modulus at the tip of the tube was sufficient to produce

self-similar tip growth. A fully viscous model led to similar conclusions [16]. In the visco-elasto-plastic model developed in [24], the authors also needed a softer tip; they found, however, that the cell wall anisotropy was required to retrieve self-similar tip growth, except in very few specific cases. In these three studies, the equations were numerically solved based on the circumferential symmetry of the pollen tube, making this modeling framework difficult to extend to other systems. The study in [28] released this assumption of axisymmetry and used the finite element method for numerical solutions. They implemented an incremental approach to growth, considering the cell wall as an elastic material. The influence of anisotropy and of the steepness of the gradient of stiffness over the edges of the tube were tested, and the interaction between these two quantities allowed self-similar growth depending on the parameters: a steeper cell wall stiffness gradient was associated with a more isotropic cell wall to produce self-similar growth. Finally, [58] introduced a model coupling cell wall chemistry with mechanics, assuming a viscoelastic rheology in which the viscosity depends on the concentration of cross-links in the wall. In addition to self-similar tip growth, they retrieved the oscillations in growth rate and tube diameter observed in fast growing tubes.

1.4.2 Growth Motion of an Elongated Organ

Plants cannot move, but they react to their environment: for instance, the main shoot and the main root bend towards the gravity vector. The molecular mechanisms involved are relatively well described, and the integration of these mechanisms during growth has been investigated using models coupling biomechanical and biochemical processes [31, 70]. The differential localization of auxin transporters from the PIN-FORMED family leads to differential concentrations of auxin, which in turn cause differential growth rates along the transverse axis of roots and hypocotyl, ultimately inducing a bending of the organ. These studies focused on the relationship between auxin, cell differential growth, and bending initiation, but they did not fully investigate how the vertical orientation of the organ is reached. Actually, when only gravity-sensing is taken into account, the shoot oscillates around the axis of gravity whereas real shoots reach this orientation [6]. Sensing the local curvature (proprioception) needs to be included to reproduce the observed dynamics of stem curving, in the case of both gravitropism [6] and phototropism [7].

1.4.3 Shaping a Sheet-Like Organ

Volvox is a green algae in the form of a spherical sheet of cells with an aperture. One major even in the morphogenesis of Volvox is its inversion: the organism turns inside out. The inversion of the sheet of cells is associated with a sequence of deformations where cells firstly circularly invaginate at the equator, accompanied by the posterior

hemisphere which moves into the anterior and inverts as well [38]. Finally, the aperture stretches out over the posterior hemisphere. The deformation of the sheet is associated with cell shape changes, but their role in the sheet inversion remained poorly understood. Modeling the sheet as an elastic material in which the rest state is actively controlled (this is formally similar to the models prescribing growth), the authors showed that the shift in the sheet curvature at the equator combined with the contraction of the posterior hemisphere were sufficient to trigger this major morphogenetic event [38].

In angiosperms, the main sheet-like organs are leaves and petals. The shapes of such organs are very diverse and can change drastically from one species to another. The underlying regulatory networks are very intricate and it is difficult to relate them to final organ shapes. The Snapdragon corolla, in particular, has a very elaborated, asymmetric shape. The wild-type and several morphogenetic mutants have been successfully modeled using a continuous approach based on incremental elastic growth [34]. This model removes mechanical stress at each step when using the equilibrium configuration to define the following rest configuration. This stress is induced by spatial gradients in growth rates (e.g. fast growing regions exert pressure on other regions) and is called residual stress.

Models prescribing mechanical properties instead of growth rates generally account for residual stress [12]. Such residual stress may have significant effect on morphogenesis, for instance in the case of thin organs. Larger growth rates at the edge of these organs induces compressive stress there, which leads to the buckling of the edges of the organ into a wavy shape [2]. Such waviness of edges is observed in leaves and petals of many species, such as in Lily [44, 45]. Based on this, it is likely that a specific regulation of growth rates is required for leaves or petals to remain flat [1, 52].

1.4.4 Feedback Through Mechanical Signals

Mechanics are at the core of morphogenesis: growing cells interact mechanically during morphogenesis, relaxing and generating residual stress. Can this mechanical stress have an impact on cell behavior? It has been shown that plant cells can sense and react to mechanical stress by orientating cortical microtubule networks in the direction of maximal principal stress [36], leading to the synthesis of cellulose microfibrils in this direction and to the mechanical reinforcement of the cell wall along mechanical stress. The consequences on organ morphogenesis were investigated in Arabidopsis thaliana sepals [37]. The authors used an incremental model, with a prescribed elastic modulus, and a mechanical anisotropy imposed by the anisotropy of stress and following the same orientation. They obtained a gradient in growth rates with a slowly growing tip and a fast growing base, leading to a transverse tension in the tip and a mechanical reinforcement there. By comparing simulations with mutants affected in sensing mechanical stress, they showed that this mechanical feedback enabled the modulation of organ shape. Similarly, a vertex

model accounting for cell divisions and viscoelastic periclinal cell walls helped showing that cell divisions follow the direction of maximal stress in the shoot apical meristem [46]. In all these studies, it was assumed that cells sensed mechanical stress.

However, cells could sense either stress or strain. The two are usually correlated, but the direction of maximal strain and maximal stress may differ when the material is mechanically anisotropic. [13] investigated which of the two sensing mechanisms was more plausible in the case of microtubule orientation and cellulose synthesis. They modeled the plant epidermis as an elastic 2D surface embedded in 3D and pressurized from inside by turgor, increasing elastic modulus along the direction of the maximal stress or along maximal strain, and they examined the subsequent elastic strain, considered as a proxy for growth. Simulations where the cellulose orientation followed stress were in accordance with experimental observations, whereas simulations where cellulose oriented depending on strain were less stable and disagreed with observations.

Mechanical cues can also guide differentiation [26]. Models have started addressing how this may pattern growing organs. For instance, the epidermis of leaves is stiffer than internal tissues, so that the effect of turgor is tension in the epidermis and compression in internal tissues. Models assumed that such compression, when above a threshold, leads to the differentiation of ground cells into pro-vascular cells [21, 43]. This mechanism is sufficient to produce venation patterns that are similar to the patterns observed in dicotyledon leaves [21, 43]. However, it also established that biochemical patterning by auxin flow is crucial for venation [10]. It may well be that the combination chemical signals and mechanical signals provides robustness to vascular patterning.

1.4.5 Variability and Morphogenesis

All the models discussed so far are deterministic: They describe the expected average behavior of the system. However, cells in an organ are variable [50]. We here consider three examples of mechanical variability in flat organs.

Modeling the wheat leaf using a cell-based model which takes turgor pressure and water movements into account, [71] showed that turgor pressure was variable between the cells of the wheat leaf, and that this variability correlated with cell identity.

In Arabidopsis sepals, a mutant showing more variability of shape was less heterogenous spatially than wild-type concerning growth rates and mechanical properties [40]. An incremental organ growth model with random mechanical properties showed that increasing the correlation length of elastic modulus allowed to retrieve the observed changes from wild type to mutant [40].

The growth of a leaf depends on its venation pattern, because veins are stiffer than ground tissue. Areas surrounded by veins (areoles) grow at different rates, depending on their geometry and on the thickness of neighboring veins, which