

Sio-long Ao · Haeng Kon Kim
Mahyar A. Amouzegar *Editors*

Transactions on Engineering Technologies

World Congress on Engineering and
Computer Science 2017

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Preface

A large international conference on Advances in Engineering Technologies and Physical Science was held in San Francisco, California, USA, October 25–27, 2017, under the auspices of the World Congress on Engineering and Computer Science (WCECS 2017). The WCECS 2017 is organized by the International Association of Engineers (IAENG). IAENG, originally founded in 1968, is a nonprofit international association for the engineers and the computer scientists. The WCECS Congress serves as an excellent platform for the members of the engineering community to meet and exchange ideas. The Congress in its long history has found a right balance between theoretical and application development, which has attracted a diverse group of researchers, leading its rapid expansion. The conference committees have been formed with over 200 members including research center heads, deans, department heads/chairs, professors, and research scientists from over 30 countries. The full committee list is available at the Congress' website: www.iaeng.org/WCECS2017/committee.html. WCECS conference is truly an international meeting with a high level of participation from many countries. The response to WCECS 2017 conference call for papers was outstanding, with more than five hundred manuscript submissions. All papers went through rigorous peer review process, and the overall acceptance rate was 50.39%.

This volume contains 27 revised and extended research articles, written by prominent researchers, participating in the Congress. Topics include engineering mathematics, electrical engineering, communications systems, computer science, chemical engineering, systems engineering, manufacture engineering, and industrial applications. This book offers the state of the art of tremendous advances in engineering technologies and physical science and applications; it also serves as an exceptional source of reference for researchers and graduate students working with/on engineering technologies and physical science and applications.

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Chapter 1

Classical Young Measures Generated by Oscillating Sequences with Uniform Representation



Andrzej Z. Grzybowski and Piotr Puchała

Abstract The paper is devoted to the theory of classical Young measures. It focuses on the situation where a sequence of rapidly oscillating functions has uniform representation in a sense that is defined in this article. There is stated a proposition characterizing the Young measures generated by such a class of sequences. This characterization enables one to find an explicit formulae for the density functions of these generated measures as well as the computations of the values of the related Young functionals. Examples of possible applications of the new results are presented as well.

Keywords Classical Young measures · Non-convex optimization · Numerical computations · Oscillating sequences · Uniform distribution · Uniform representation

1.1 Introduction

Non-convex optimization problems are at the core of various contemporary engineering applications. They arise e.g. in optimal control, nonlinear evolution equations, variational calculus, micromagnetic phenomena in ferromagnetic materials as well as in microstructures theory in continuum mechanics. It appears however that the optimization problems may not possess classical minimizers especially when elements of the minimizing sequences oscillate rapidly. The following example, attributed to

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Oscar Bolza and Laurence Chisholm Young, illustrates the nature of the problem, see [7].

Example 1 We are minimizing the integral functional \mathcal{J} acting on a Sobolev space $W^{1,4}(0, 1)$ of real functions defined on the unit interval in \mathbb{R} and such that their first derivative belongs to the space $L^4(0, 1)$. The functional \mathcal{J} is of the form

$$\mathcal{J}(u) = \int_0^1 \left[u^2 + \left(\left(\frac{du}{dx} \right)^2 - 1 \right)^2 \right] dx.$$

We impose the boundary conditions $u(0) = 0 = u(1)$.

It can be proved that $\inf \mathcal{J} = 0$.

Consider the function

$$u(x) = \begin{cases} x, & \text{for } x \in [0, \frac{1}{4}) \\ \frac{1}{2} - x, & \text{for } x \in [\frac{1}{4}, \frac{3}{4}) \\ x - 1, & \text{for } x \in [\frac{3}{4}, 1]. \end{cases}$$

Then the sequence $u_n(x) := \frac{1}{n}u(nx)$ (divergent strongly but convergent weakly in $W^{1,4}(0, 1)$) is the minimizing sequence for \mathcal{J} , that is it satisfies the condition $\mathcal{J}(u_n) \rightarrow \inf \mathcal{J}$. However, we have $\inf \mathcal{J} \neq \mathcal{J}(\lim u_n)$: if the limit u_0 of (u_n) were the function satisfying the equality $\inf \mathcal{J} = \mathcal{J}(u_0)$, it would have to satisfy the conditions: $u_0 \equiv 0$ and $\frac{du_0}{dx} = \pm 1$ a.e. (with respect to the Lebesgue measure on $[0, 1]$), which is impossible. This means that \mathcal{J} does not attain its infimum. It is seen, that the number of ‘teeth’ of u_n (with slope ± 1) grows with n , that is the elements of the minimizing sequence oscillate more and more ‘wildly’ around zero.

Such a behavior of the sequences requires a generalization of the notion of a solution for such problems. It often can be achieved by means of Young measures.

Young measures theory has a long history. It starts with the seminal work [12] of L. C. Young who introduced the notion (called by himself “generalized curves”) to provide extended solutions for some non-convex problems in variational calculus. He developed these pioneering ideas in [13].

Nowadays we are provided with vast literature where the Young measures are defined under different assumptions about underlying spaces and analysed from different standpoints. However this paper focuses on the classical Young measures related to sequences of rapidly oscillating functions.

This article is a revised and enlarged version of the talk [5] given by the first author during the World Congress on Engineering and Computer Science 2017 in San Francisco, USA. It is organized as follows. In the next section we introduce some preliminary definitions and results. In Sect. 1.3 we define some classes of fast-oscillating sequences and state new proposition that allows us to find explicit forms of the density functions of related classical Young measures in various situations. Section 1.4 presents some examples that illustrate the possible applications of the main result stated in Sect. 1.3. Finally we make some remarks about possible further extensions and applications.

1.2 Preliminary Definitions and Results

We now introduce basic notions of the Young measure theory from the point of view of nonlinear elasticity. Our presentation follows the approach taken in [8], where the reader is referred to for detailed information along with necessary notions from functional analysis and further bibliography. Another book treating Young measures thoroughly in the context the optimization theory and variational calculus is [11].

Let Ω be a nonempty, open and bounded subset of \mathbf{R}^d with smooth boundary. Denote by $L^\infty(\Omega)$ the Banach space of essentially bounded functions defined on Ω with values in a compact set $K \subset \mathbf{R}^l$. Let (f_n) be a sequence of functions converging to some function f_0 weakly* in L^∞ and denote by φ a continuous real valued function with domain \mathbf{R}^l . By the continuity of φ the sequence $\varphi(f_n)$ is uniformly bounded in L^∞ norm and Banach-Alaoglu theorem yields the existence of the (not relabeled) subsequence such that $\varphi(f_n) \rightarrow g$ weakly* in L^∞ . However, in general g is not $\varphi(f_0)$, moreover, it is not even a function with domain in \mathbf{R}^l . To quote from [8]:

The Young measure associated with (f_n) furnishes the link among (f_n) , f_0 , g and φ .

Recall that a function $H: \mathbf{R}^d \times \mathbf{R}^l \rightarrow \mathbf{R} \cup \{\infty\}$ is called a *Carathéodory function* if it is measurable with respect to the first and continuous with respect to the second variable.

We now state the basic existence theorem for Young measures in its full generality.

Theorem 1 (see Theorem 2.2 in [8]) *Let $\Omega \subset \mathbf{R}^d$ be a measurable set and let $z_n: \Omega \rightarrow \mathbf{R}^l$ be measurable functions such that*

$$\sup_n \int_{\Omega} h(|z_n|) dx < \infty,$$

where $h: [0, \infty) \rightarrow [0, \infty)$ is a continuous, nondecreasing function such that $\lim_{t \rightarrow \infty} h(t) = \infty$. There exist a subsequence, not relabeled, and a family of probability measures $\nu = \{\nu_x\}_{x \in \Omega}$ (the associated Young measure) depending measurably on x with the property that whenever the sequence $(H(x, z_n(x)))$ is weakly convergent in $L^1(\Omega)$ for any Carathéodory function $H(x, \lambda): \Omega \times \mathbf{R}^l \rightarrow \mathbf{R} \cup \{\infty\}$, the weak limit is the function

$$\overline{H}(x) = \int_{\mathbf{R}^l} H(x, \lambda) d\nu_x(\lambda).$$

- Definition 1** (i) the family of probability measures $\nu = \{\nu_x\}_{x \in \Omega}$ of the above Theorem is called the *Young measure associated with the sequence (z_n)* (or *Young measure generated by the sequence (z_n)*);
- (ii) if the Young measure $\nu = \{\nu_x\}_{x \in \Omega}$ does not depend on $x \in \Omega$, then it is called *homogeneous Young measure* and is denoted merely by ν .

Remark 1 The homogeneous Young measures, although being the ‘simplest examples of the species’, are very important both in theory and in applications. For instance, the Young measures calculated in [7] as examples in micromagnetism or elasticity, are homogeneous ones.

One may also look at the Young measure as at object associated with *any* measurable function defined on a nonempty, open, bounded subset Ω of \mathbf{R}^d with values in a compact subset K of \mathbf{R}^l . Such a conclusion can be derived from the Theorem 3.6.1 in [11]. Due to this theorem it can be proved that the Young measure associated with a simple function is homogeneous and is the convex combination of Dirac measures (such measures are called *discrete*). These Dirac measures are concentrated at the values of the simple function under consideration while coefficients of the convex combination are proportional to the Lebesgue measure of the sets on which the respective values are taken on by the function; see [9] for details and more general results concerning simple method of obtaining explicit form of Young measures associated with oscillating functions (similar, although mathematically more complicated situation, is met in elasticity when the deformed body has a laminate structure; see e.g. Sect. 4.6 in [7]).

This result gives rise to the use of computer tools in performing one of the main tasks when dealing with Young measures: calculating the explicit form of the Young measure. Generally this is very difficult or sometimes even impossible to perform. It has however turned out, that using the approach presented in [11] and the notion of a *quasi-Young measure* introduced in [9], the Monte Carlo simulation can be successfully used to partial treatment of the mentioned task (apparently, quasi-Young measures associated with Borel functions are precisely *the* homogeneous Young measures associated with them, see [10] for the proof). The word ‘partial’ refers to the fact that the simulation has worked well for the particularly simple Young measure: the one associated with the continuous piecewise affine real function. In the example presented in [2] proposed computer routine passes the χ^2 Pearson test. Moreover, in the article [3] two more routines for Monte Carlo simulating of the Young measures associated with such functions has been presented and compared. The analysis of those algorithms has led to the much more general *theoretical* result: the characterisation of a Young measure associated with *any* Borel function. The main result stated there provides direct link between the Young measure concepts and the probability theory. Namely, the following theorem holds:

Theorem 2 (see [6]) *Let $f: \mathbf{R}^d \supset \Omega \rightarrow K \subset \mathbf{R}^l$ be a Borel function with Young measure ν . Then ν is the probability distribution of the random variable $Y = f(U)$, where U has a uniform distribution on Ω .*

The above theorem opens the possibility not only for broadening the class of functions whose Young measures can be calculated via Monte Carlo simulation but also for calculating the values of the classical (i.e. generated by sequences of rapidly oscillating functions) Young measures. The Monte Carlo method in the second opportunity has been done in [4]. The main topic of this article is to present yet another way.

Now, let us recall the notion of classical Young measure associated with a sequence of oscillating functions $\{f_k\}$, see e.g. [11].

Definition 2 The classical Young measure generated by the sequence $\{f_k\}$ is a family of probability measures $\nu = \{\nu_x\}_{x \in \Omega}$ satisfying the condition:

For any Carathéodory function H

$$\int_{\Omega} H(x, f_k(x)) dx \xrightarrow{k \rightarrow \infty} \int_{\Omega} \int_K H(x, y) d\nu_x(y) dx \tag{1.1}$$

The application that assigns to any Carathéodory function H the integral given on the right-hand side of the above equation is called *Young functional*. Its values on H will be denoted here as $YF(h)$, while the integrals on the left-hand side of (1.1) will be denoted as $C(f_k, H)$.

Basically, the above definition presents the original understanding of the Young measure, as introduced in his work [13]. These measures are of our main concern in this paper.

1.3 Main Results

This section contains main results of the article. We first recall the notion of the sequence whose elements are violently oscillating functions and then consider classical Young measures generated by such sequences. Finally we formulate proposition characterizing those Young measures.

1.3.1 Rapidly Oscillating Sequences with Uniform Representation

Let function $f: [a, b) \rightarrow K \subset \mathbf{R}$ be a Borel function defined on the interval $[a, b)$, $b > a$, and let $f^e: \mathbf{R} \rightarrow K$ be the periodic extension of f (with the period equal to $T = b - a$).

Let Ω be a given interval. A sequence $\{f_k\}$ of functions $f_k: \Omega \rightarrow K$, $k = 1, 2, \dots$ defined by the formula

$$f_k(x) = f^e(kx), \quad x \in \Omega \tag{1.2}$$

will be called a *Rapidly Oscillating Sequence with Uniform representation* f , and denoted as $ROSU(f)$. In such a case we will also say that f generates rapidly oscillating sequence $\{f_k\}$. Note, that the interval Ω —the domain of elements of $ROSU(f)$ —does not have to be the same as $[a, b)$, i.e. the domain of f .

Example 2 In this example we present illustrative plots of some elements of $ROSU(f)$, where

$$f(x) = 2 - 2 \sin(x), \quad x \in [0, 3\pi/2) \tag{1.3}$$

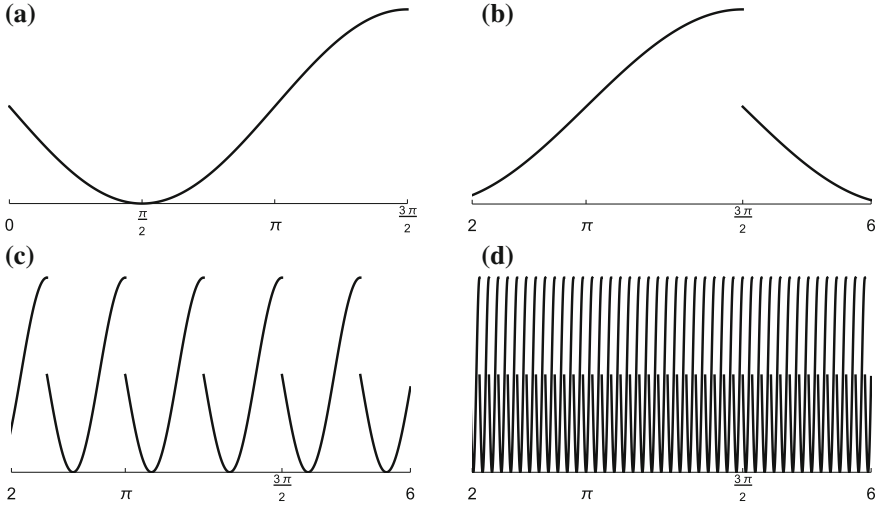


Fig. 1.1 A function f given by Eq. (1.3) and functions f_1 , f_5 and f_{50} belonging to $\text{ROSU}(f)$ with the domain $\Omega = [2, 6)$. Plots of the functions f , f_1 , f_5 and f_{50} are labeled as **a**, **b**, **c** and **d**, respectively

Its purpose is not only to illustrate the behaviour of ROSU (which is quite obvious, in fact) but also to illustrate the concept of classical Young measure associated with the sequence $\{f_k\}$. The plots of the function f given by 1.3 as well as of exemplary elements of $\text{ROSU}(f)$ with the domain $\Omega = [2, 6)$ are presented in Fig. 1.1. Namely it shows f , f_1 , f_5 and f_{50} .

It can be easily seen that the graphs of f_k are getting denser when k tends to infinity. Unfortunately, a conventional weak* cluster point of $\{f_k\}$ loses most of the information about the fast oscillations in $\{f_k\}$ because, in some sense, it takes into account only the mean values of $\{f_k\}$ —as integrals do. That is why we need a new concept of the limit and here the theory of Young measures helps us. If ν_x is the Young measure associated with $\{f_k\}$ then, roughly speaking, for any measurable set $A \subset K$ the intuitive meaning of $\nu_x(A)$ is the probability that for an *infinitesimally* small neighbourhood S of $x \in \Omega$ and sufficiently large k 's we can “find” $f_k(s)$ in A , when s changes within S . The “density” of the values in K can be observed in the plot (d) (for f_{50}) where the “more probable values” create darker straps in the figure.

1.3.2 Classical Young Measures Generated by the $\text{ROSU}(f)$

It results directly from the definition of $\text{ROSU}(f)$ that its behaviour, as k tends to infinity, is exactly the same in every neighbourhood of any $x \in \Omega$. In other words, its asymptotic behaviour in an arbitrarily small interval $I \subset \Omega$ does not depend on where the interval is placed within the domain. Consequently, it is obvious that the

classical Young measure generated by the $\text{ROSU}(f)$ is the homogeneous Young measure (i.e. it does not depend on $x \in \Omega$).

Now, let U_S denote a random variable uniformly distributed on S . Let us consider two functions $g_1 : \Omega_1 \rightarrow K$ and $g_2 : \Omega_2 \rightarrow K$. We will say that the two functions *identically transform a uniform distribution* if the distributions of the random variables $g_1(U_{\Omega_1})$ and $g_2(U_{\Omega_2})$, are the same. This fact will be denoted by $g_1 \approx g_2$. Obviously the “ \approx ” is the equivalence relation.

Note that if the $\text{ROSU}(f)$ is defined on the same interval as the generating function f , then *any* of its elements transform a uniform distribution identically as the function f , i.e. $f_k \approx f$ for any $k = 1, 2, \dots$. Now let us consider the case where the $\text{ROSU}(f)$ is defined on interval Ω that is different than the domain $[a, b)$ of the generating function f . In such a case it can be seen that $f_k(U_\Omega) \xrightarrow{D} f(U_{[a,b)})$, where \xrightarrow{D} denotes the *convergence in distribution*, see [1]. In other words the distribution of $f(U_{[a,b)})$ is a vague limit of the sequence of distributions of $f_k(U_\Omega)$. Indeed, for k 's large enough so the length of interval Ω is greater than $(b - a)/k$ we have for *any* measurable subset $A \subset K$:

$$|P(f_k(U_\Omega) \in A) - P(f(U_{[a,b)}) \in A)| < 1/k$$

Finally, by Theorem 2 we know, that the distribution of $Y = f(U_{[a,b)})$ is the Young measure associated with the function f . Thus we can formulate the following result concerning classical Young measures.

Proposition 1 *Classical Young measure generated by $\text{ROSU}(f)$ is the homogeneous Young measure. This measure is identical with the distribution of the random variable $Y = f(U_{[a,b)})$.*

1.4 Illustrative Examples

On the basis of the above general description of the classical Young measure generated by $\text{ROSU}(f)$ we can obtain a number of rules which allow one to find an explicit form of the classical Young measure in various specific cases. For example, let us consider the following situation.

Let $[a, b)$ be the interval—domain of the function f . Let us consider an open partition of $[a, b)$ into a number of open intervals I_1, I_2, \dots, I_n such that the intervals are pairwise disjoint and $\bigcup_i^n \bar{I}_i = [a, b)$, where \bar{A} denotes the closure of the set A .

Let function f be continuously differentiable on each interval of the partition and let $f'(x) \neq 0$ for all $x \in \bigcup_{i=1}^n I_i$.

Recall that for any set A the symbol $\mathbf{1}_A$ stands for the characteristic function of this set, i.e. $\mathbf{1}_A(x) = 1$ if $x \in A$ and $\mathbf{1}_A(x) = 0$ otherwise.

Using the well-known probabilistic result concerning the distributions of such functions of random variables we can obtain the following corollary of the Proposition 1.

Corollary 1 *The classical Young measure generated by any ROSU(f) is a homogeneous one and its density g with respect to the Lebesgue measure on K is of the following form*

$$g(y) = \frac{1}{b-a} \sum_{l=1}^n |h_l'(y)| \mathbf{1}_{D_l}(y) \quad (1.4)$$

where h_l is the inverse of f on the interval I_l , while $D_l = f(I_l)$ is the domain of h_l .

To show how Proposition 1 works in practice, let us consider a specific function f and an exemplary Carathéodory function H for which both sides of the Eq. (1.1) can be computed precisely.

Example 3 Let us consider a function $f(x) = \sin x$ defined on the interval $[0, 2\pi)$. Now, let the ROSU(f) be defined on the interval $[2, 4)$.

Let us compute the integrals $C(f_k, H)$ that appears on the left-hand side of the Eq. 1.1 for the exemplary Carathéodory function $H(x, y) = xy^2$. We get

$$\begin{aligned} C(f_k, H) &= \int_2^4 H(x, f_k(x)) dx = \int_2^4 x \sin^2(kx) dx \\ &= \frac{1}{8k^2} [\cos(4k) - \cos(8k) + 4k(6k + \sin(4k) - 2 \sin(8k))] \end{aligned}$$

The left hand side of the Eq. 1.1 is a limit of the above expression when k tends to infinity, so it equals 3.

In order to compute the value of the Young functional on H , i.e. the integral given by right-hand side of the Eq. 1.1 we need to know the classical Young measure generated by the ROSU(f). For this purpose we make use of the Corollary 1 and receive the following formula:

$$g(y) = \frac{1}{\pi \sqrt{1-y^2}} \mathbf{1}_{(-1,1)}(y)$$

Thus the value of the Young functional in the considered case is the following (recall that the Young measure ν_x is homogeneous in this case, so the subscript ‘ x ’ is omitted):

$$\begin{aligned} YF(H) &= \int_{\Omega} \int_K H(x, y) d\nu(y) dx = \int_{(2,4)} \left(\int_{(-1,1)} H(x, y) g(y) dy \right) dx \\ &= \int_2^4 \int_{-1}^1 \frac{xy^2}{\pi \sqrt{1-y^2}} dy dx = 3 \end{aligned}$$

As we can see, the “whole information” about the rapid oscillations in this case is contained in the classical Young measure and—due to the Proposition 1—can be

revealed with the help of the Corollary 1. In this example the considered generating function f has continuously differentiable extension f^e and the integrations needed to compute the $C(f_k, H)$ were easy to perform. But although this example is intentionally simple, it perfectly shows the benefits resulting from our proposition. It is quite clear that even in this case, where the oscillation have such smooth nature, the computation of the limit of $\{C(f_k, H)\}$ for more complex Carathéodory functions could be a much more difficult task than the calculation of the value of the Young functional $YF(H)$. Moreover, the problem is getting even more difficult if the extension f^e of the generating function f is piecewise continuous with countably many discontinuity points. The next example deals with such a case.

Example 4 Let us consider the generating function f given by Eq. 1.3 and $ROSU(f)$ introduced in Example 1. Although we use again sinus function as the “basis” for the definition of f , it appears that due to the discontinuity of its extension f^e the general symbolic form for the integrals $C(f_k, H)$ cannot be computed, as long as k is unspecified. They can be only computed for given values of k (and not for large values) or approximated numerically, but even this can be very challenging task for such a functions. Moreover it is certainly insufficient for calculation of the limiting value, which is our aim. With the help of Wolfram Mathematica 10.4 software we computed numerically the integrals in the considered case assuming the Carathéodory functions $H(x, y) = x^2y^3$. The exemplary computed values are as follows (recall that the domain of $ROSU(f)$ is the interval $[2, 6)$): $C(f_5, H) = 941.71, C(f_{10}, H) = 891.05, C(f_{50}, H) = 942.32, C(f_{100}, H) = 956.47, C(f_{500}, H) = 953.92, C(f_{600}, H) = 955.75$.

As we can see it is not easy to guess what is the limit value of the sequence $\{C(f_k, H)\}$.

Now to compute this limit let us make use of the Young concept. For this purpose we need the density function g of the classical Young measure generated by the $ROSU(f)$. With the help of Corollary 1 in this case we obtain:

$$g(y) = \frac{4}{3\pi\sqrt{4 - (y - 2)^2}} \mathbf{1}_{(0,2)}(y) + \frac{2}{3\pi\sqrt{4 - (y - 2)^2}} \mathbf{1}_{[2,4)}(y)$$

Thus the value of the Young functional on H is the following:

$$\begin{aligned}
\text{YF}(H) &= \int_{\Omega} \int_K H(x, y) d\nu(y) dx \\
&= \int_2^6 \int_0^2 \frac{4x^2 y^3}{3\pi \sqrt{4 - (y - 2)^2}} dy dx \\
&\quad + \int_2^6 \int_2^4 \frac{2x^2 y^3}{3\pi \sqrt{4 - (y - 2)^2}} dy dx \\
&= \frac{832(45\pi - 44)}{27\pi}
\end{aligned}$$

The above limit value could hardly be guessed on the basis of the approximately computed values of the elements of $\{C(f_k, H)\}$. The decimal form of the limit, which is 955.09, differs from the computed numerically value of $C(f_{600}, H) = 955.92$. It is worth mentioning at this point that the numerical integration of $C(f_k, H)$ for $k > 600$ failed to converge due to highly oscillatory integrand.

1.5 Final Remarks

The probability theory provides us with a number of different versions of the theorems concerning the shapes of distributions of functions of random variables/vectors. Consequently various other rules for computing explicit formulae for the density functions of classical Young measures generated by $\text{ROSU}(f)$ can also be developed on the basis of the new result stated in Proposition 1. We are also sure that the same approach enables development of analogous results related to rapidly oscillating sequences with uniform representation which are defined on open and bounded subsets of \mathbf{R}^d . It is promising direction of future research.

The possibility of derivation of explicit formulae for the density functions of classical Young measures is not the only benefit resulting from our Proposition 1. In many interesting cases explicit formulae for these densities cannot be found. For instance, if one wants to make use of Corollary 1 they have to obtain the inverses of f on particular subintervals of its domain, but it is not always possible. However in all such cases thanks to Proposition 1 one may use directly Monte Carlo simulations in order to compute values of the Young functionals. That fact significantly broaden the range of possible applications of our main result.

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Chapter 2

Energy Requirements Estimation Models for Iron and Steel Industry Applied to Electric Steelworks



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Abstract The price of electric energy depends on additional factors since the introduction of renewable energy sources, which has changed the basics of electricity production and the determination of its price. Iron and steel industries strongly require forecasting procedures for the energy amount of their production cycles: today production planning is performed without taking into account that the difference in electricity price between night and day can overcome 500%. The aim of this work is to create a model allowing to estimate energy requirements for steel industry; the model correctness is assessed, for both energy and power analysis, by comparison with real data. A planning tool is employed to provide data to a computer platform able to assess, on the basis of required energy, the best market on which power can be purchased ensuring money saving for the steelworks.

Keywords Computer simulation · Consumption forecasting · Decision support system · Electric power · Energy market · Industrial process optimization · Steelworks · Production planning · Production process · Software tool

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2.1 Introduction

Nowadays, electric grid users are very heterogeneous: from domestic user to second homes, from city lighting to industrial plants of remarkable consumption. Such a differentiated panorama implies the necessity to produce electric energy in the moment in which the same is required; therefore the installed power must be sufficient to satisfy a demand which is not constant with time. In steelworks industry [1, 2], not long time ago, the Companies focus was mainly turned to production process itself, to its control and automation. The success of this focusing is evident, and the quality standards offered by the market, which nowadays are taken for granted, are extremely high, so as applications, which are more and more advanced. In order not to risk the falling of the Occidental industries competitiveness compared to the big Asiatic Companies, it is necessary to become competitive not only on a technological and quality horizon, but also, and above all, to be able to excel in the customer satisfaction field, being able to suit in a repeatable manner, customers requirements in terms of delivery time, building a stable process of Planning, Insertion and Management of the orders. Such system requires firstly a production order and a definition of the status quo; after that, an optimized production planning needs to be implemented to satisfy the orders. It must be taken into account that during production problems may occur that can require an immediate production re-programming. The introduction of a virtual planning, in all the possible problems that can require a production re-programming, allows to assess in a virtual time the plant status; the system is able to process and show the new organization of the production line. At the same time, the system is able to interact in a continuative manner with the energy market basing on plant necessities. In this paper, the Authors propose a planning method for a complex steelworks plant. The proposed method performance is assessed by comparison with real data obtained by measurements on the plant. The analysis involved fully productive days, days with up and down power ramps, non productive days and anomalous days. To provide thorough terms of comparison, the MAPE (mean absolute percentage error), the Least Squares and the ANOVA (analysis of variance) methods were employed.

2.2 The Complexity of the Electric System

The complexity of the inter-relation between demand and offer, imposes an equally complex management of the electric system [3, 4], which can be divided into: (1) Power generation plants: these are characterized by a mix of technologies suitable for satisfying the time-varying behavior of the demand, which appears not only within 24 hours, but also within weeks and in general during the whole year. In order to assure power supply to users, the installed power needs to be 10–15% higher than peak demand power. In fact, other than maximum yearly demand, installed power needs to satisfy the power for programmed or extraordinary events, e.g. maintenance or

renovation, failures or natural causes such as the incomplete filling of hydro-electric basins due to rain scarcity. For this reason, a certain number of power plants are kept in operation H24 (e.g. nuclear plants, flowing water plants and modern coal plants, which supply for the base-load), while the plants characterized by a short startup time (e.g. gas turbines, combined cycle plants, reciprocating engines and water basin hydroelectric plants) are employed to satisfy the peak demand.

(2) Transmission grid: this connects power generation plants to the distribution grid and represents the place in which electric energy demand and offer meet. Along the whole chain going from generation to final use, energy losses inevitably occur by Joule effect. Generally electric power is transmitted at high voltage to reduce losses, which increase with increasing distance between generation and use locations.

A transmission grid is composed by:

- three lines, usually overhead, which transport power connecting two electric stations, or an energy immission/withdrawal point;
- electric stations, used both to distribute the power between the lines of a network and for transferring electricity between different voltage networks;
- load rephasing systems;
- switches;
- regulation and control systems.

Electric power transmission grids are structured in a knit way, in order to make available alternative routes in case of breakdown or to share the load on the network.

(3) distribution grid: this brings electricity to the final consumer. The national transmission grid is connected to the distribution grid, which supplies the electricity to the final user with a voltage dependent on the type of use. It is divided into:

- High voltage (HV) grid, ensuring the primary distribution of electric power. This grid includes lines with voltage values between 30 and 150 kV;
- primary cabins, transforming electric power from high voltage to medium voltage;
- Medium voltage (MV) lines, from 1 kV up to 30 kV;
- secondary cabins, transforming electric power from medium to low voltage;
- Low voltage (LV) lines, characterized by voltage values lower than 1 kV;
- Low voltage grid, feeding all domestic users.

The energy produced by the electric system is subdivided into six classes, to account for internal uses and losses:

- Gross production: it represents the sum of the energy produced by all the plants;
- Net production: this represents the gross production minus the generation plants requirements;
- Energy for pumping: this represents the energy required in water basin pumping systems to pump the water from the lower to the higher basin;
- Production intended for consumption: this is net production minus the pumping energy;

- Electric energy required by the grid: this represents the energy production intended for consumption minus the exported energy plus the imported energy;
- Electrical consumption: this represents the energy required by the grid minus the transmission losses.

2.3 The Large Energy Consumers: Steelworks

The production size of electric steelworks is between 1 and 2 million tons per year; the furnaces casting capacity is limited, which imposes a high frequency of casting in modern systems to maintain a high cycle production. Electric-powered steelworks, smaller than the integral cycle ones, allow the construction of plants with an acceptable relationships between investment and production capacity.

Electric ovens are small ovens with a maximum production capacity of 12 tons approximately. Their development and use has in many cases been held back by the cost of energy, which significantly affects the cost of steelmaking.

The melting furnaces with electric heating can be divided into two large categories:

- electric arc furnaces (EAF);
- Induction furnaces.

Induction furnaces with magnetic cores are made of a metallic bath contained in an annular crucible of small cross-section forming the secondary of a transformer whose primary is wound on a frame-shaped iron core, located in a vertical plane. Alternated current is delivered in the primary, so that a low voltage but high intensity current is induced in the metallic bath, warming it by Joule effect. Their application in the iron and steel industry is very limited, while it is more extensive in the field of special metallurgy such as copper and nickel.

Induction furnaces without magnetic core have had a remarkable development for the production of high quality steel, or foundry for the production of cast iron.

In ovens without core, the primary consists of a copper spiral with a tubular section traveled internally by a stream of cooling water and wrapped around a refractory crucible containing the metal bath.

The arc furnace operates through an electrical discharge that melts the metal; to reduce the consumption of the electrode, steelworks have begun to introduce pre-heated scrap. This type of furnace is formed by a cylindrical crucible with a rounded bottom and whose walls form the vat. The vault covers the vat and, rotating on a peripheral vertical axis, opens the oven allowing to quickly load it from the top through the charging baskets.

The power involved may range from 500 to over 100 000 kVA; the potentiality of the oven is measured by the diameter of the basin. Consumption is in the order of 500–700 kWh per ton of product. Once produced in semi-finished steel, the final desired shape must be impressed. This part of the steelmaking process is common to the two oven types; there are two ways to proceed: casting into ingots and following rolling, or continuous casting.

The competitiveness of such plants is based on the ability to achieve very important economies of scale in terms of production volumes; the optimal production threshold for large steelworks ranges between 5 and 10 million tons per year.

2.4 Steel Manufacturing Plant

The first phase of the Electric Arc Furnace is the loading of the scrap, which usually comes from the demolition of steel structures or process residuals. The scrap is transported above the electric oven through a basket which, by overturning, allows the scrap to be poured inside the vat; here the scrap begins the melting process through the electric arcs that appear between the electrodes and the scrap itself. Once liquid state is reached, pressurized oxygen is blown in the reaction volume, enabling to withdraw undesired chemical elements, such as nitrogen and phosphorus. In this phase, slags are formed, collecting on the top of the metallic bath. When the steel has reached the prescribed value of alloying elements and the ideal temperature for the subsequent treatments, it is poured into the ladle; this is then taken to the Ladle Furnace, in which molten steel is kept in temperature by means of electrodes and is further added with alloying elements. In Fig. 2.1 is represented the Electric Arc Furnace, completed by the various flows of materials and energy and the main auxiliary equipment.

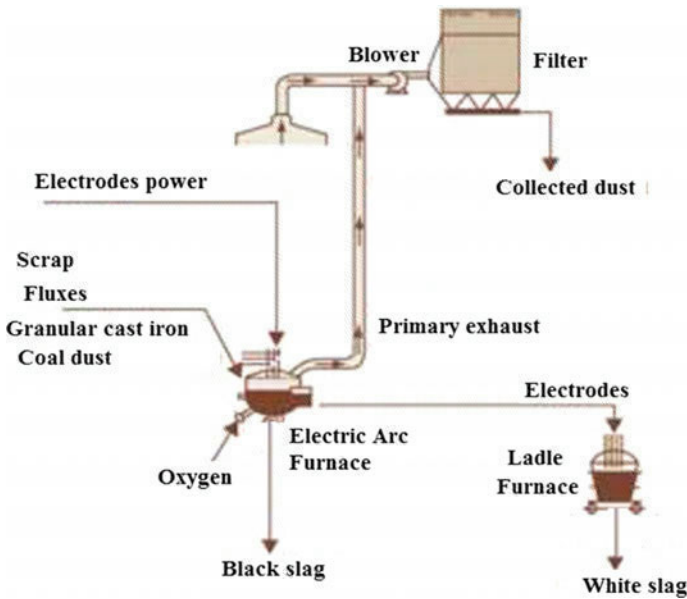


Fig. 2.1 Process of the electric arc furnace

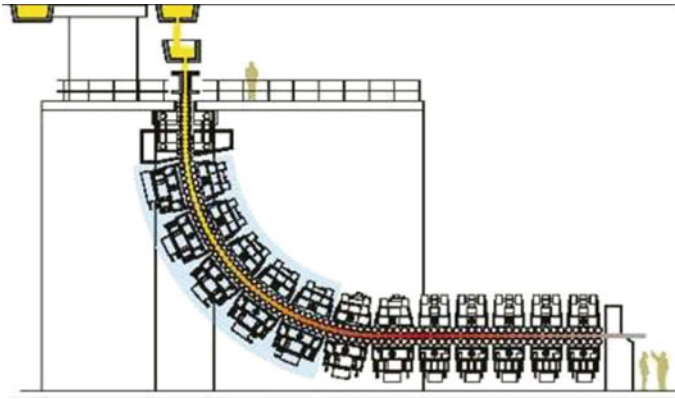


Fig. 2.2 Process of the electric arc furnace

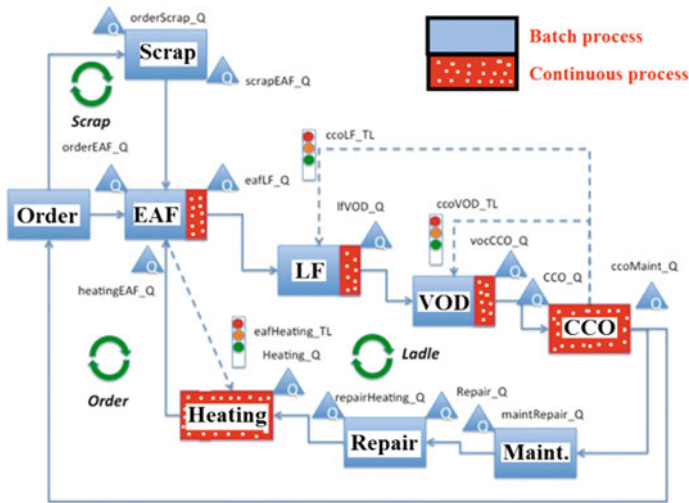


Fig. 2.3 Process scheme of the analyzed steelworks

According to the plant type, steel can then pass directly to a Continuous Casting Machine, assuming different shapes, or be placed in molds, solidified in ingots and then milled. Continuous Casting Machine is an oscillating mold, in which liquid steel is poured assuming the shape impressed by the casting channel. Usually, the mold is constituted by water-cooled copper walls.

The longer is the path made by steel, the longer will be the cooling phase, therefore modern machines are developed along a curved line, as visible in Fig. 2.2.

Figure 2.3 represents the scheme of the operations of the steelworks analyzed in this paper. The processes represented in figure are the following:

- *Order*: arriving orders;
- *Scrap*: scrap warehouse;
- *EAF*: Electric Arc Furnace;
- *LF*: Ladle Furnace;
- *VOD*: degassing treatment;
- *CCO*: Continuous casting;
- *Maint and Repair*: processes of repair and maintenance of the ladles;
- *Heating*: ladles heating process.

Rectangular blocks correspond to the processes, ie the treatments that the initial product undergoes to get the finished product. Triangles represent the queues in input and output of the processes.

Operations in full color (CCO and Heating) are continuous processes, those in dotted color are batch, those with both the colors (EAF, LF, VOD) present both a discrete and a continuous component. Continuous arrows indicate the processes temporal dependencies and order. Traffic lights and dotted arrows, with direction opposite to continue arrows, indicate the functional dependency between processes: the process from which arrow starts commands the process under the traffic light (for example, the EAF regulates the arrival of a ladle from the Heating process). The three green circular arrows identify three types of cycle:

- scrap cycle (Scrap) following the Scrap, EAF, LF, VOD, CCO and Order path;
- order cycle (Order) that follows the route Order, EAF, LF, VOD, CCO;
- ladles cycle (Ladle) that follows the path Heating, EAF, LF, VOD, CCO, Maint. and Repair.

The following Fig. 2.4 provides an overview of the plant studied in this work, highlighting for the mentioned components the hourly capacity of each individual machine.

In Fig. 2.4 is visible the flow scheme of the whole process, including the material flow rates processed by each component. The examined plant provides two production lines; in particular:

- EAF: Electrical Arc Furnace that can work 7 days a week; after that it requires maintenance (these operations take between 8 and 12 hours)
 - EAF A: processing every 75 min with 110t/h capacity;
 - EAF B: processing every 60 min with 150t/h capacity;
- LF: Ladle Furnaces with the same capacity of the EAF of its line; they consume 20 MW;
- CASTER: it is the station where casting starts. The plant has three casters:
 - NNS (Near Net Caster): capacity 100t/h, it mainly supplies billets to Large Section Mill (LSM);
 - B CASTER: capacity between 100 and 160t/h, it supplies billets to Medium Section Mill (MSM);

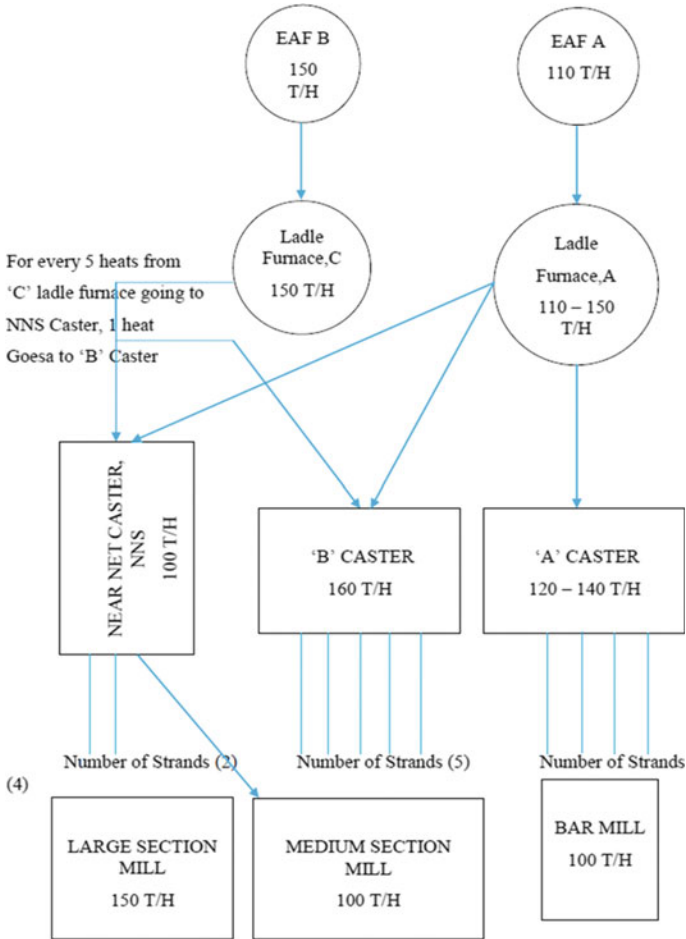


Fig. 2.4 Process flow diagram

- A CASTER: capacity 100t/h, it supplies billets to Bar Section Mill (BSM).
- MILLER: three rolling mills.

2.5 Consumption Data

The steelworks consumptions have been monitored and collected in a data-base for the period between 1/1/2010 and 30/9/2015, excluding the year 2012 for which it was not possible to obtain an exhaustive documentation.

Each day has been divided into 3 shifts (from 9 p.m. of the day before to 4.00 a.m. of the current day, from 5.00 a.m. to 12.00 a.m. of the current day, from 1.00 p.m. to 8.00 p.m. of the current day). The average of hourly required power has been calculated, such as the average of all day. Based on this, can be identified:

- Days of production: if the average of the absorbed power is 21 MW at least;
- Days of ramp down production if: (i) The second half of the first shift, the second shift and the first half of the third are productive; (ii) The day hasn't been classified as productive; (iii) The first half of the third shift, the second half of itself and the first half of the first shift (the one related to the day after) have to be characterized by a ramp down;
- We have a ramp down if:
 1. The second half of the previous shift is productive;
 2. The first half of the next shift is no-productive;
 3. The average of the half under exam is higher than the average of the next half;
 4. The first value of the half under exam is higher than the value associated to the last hour of the same half;
 5. The next half is no-productive.
- Days of ramp up production if
 1. The second half of the first shift, the first half of the second or the second half of the same must be characterized by a rising ramp;
 2. The second half of the second shift have to be productive just like all the third shift and the first half of the first shift (the one connected to the next day);
 3. The day must not have already been classified as fully productive.
- We have a ramp up if:
 1. the second half of the previous shift is non-productive;
 2. the first half of the next shift is productive;
 3. the average of the half in question is less than average of the later half;
 4. the first value of the half under examination, corresponds to the first hour of the same, is less than the value associated with the last hour of the same half;
 5. the half later is productive.
- Days are classified as no-production if:
 1. the first half and the second half of the first shift are not productive;
 2. the second and the third shift are not productive;
 3. the day hasn't been categorized as productive with one of the two ramps.
- Abnormal days: any day does not fit into any of the classes above described.

Each of the 1732 days analyzed has been assigned to the category with the following result:

- 336 whole days production;
- 523 no-productive days;
- 81 productive days characterized by ramp down;
- 88 productive days characterized by ramp;
- 704 abnormal days.

2.6 Construction of Production Profiles Based on Planning

In this section will be described the steps to create a simplified ideal production profile basing on which it is possible to organize and manage the purchase of electric energy, avoiding to buy useless energy in those days in which the plant is stopped or only partially operative.

An optimized production plan, based on times that each machine uses for fulfilling its task, was implemented. In particular, in the following are described the processing times for a complete production process.

EAF: 60 min; Transport between EAF and LF: 5 min; LF: 40 min; Transport between LF and CCO/VD: 5 min; Degassing (VOD): 20 min; Continuous casting (CCO): 40 min. The last two processes were considered as one for simplicity, creating a unique process CCO/VOD lasting 60 min.

The last two processes were considered as one for simplicity, creating a unique process CCO/VOD lasting 60 min.

For each of the three equipments, a chart was built (Fig. 2.5), showing the absorbed MW (y axis) in function of time (x axis); the equipment consumptions are estimated in 50 MW for EAF, 20 MW for LF and 1 MW per both degassing and CCO.

Planning begins from zero-day and the first process of each equipment doesn't consider the time interval needed to reach the processing temperature.

To optimize working time, in both the lines of the plant the second casting starts when the first semi-finished piece is exiting from the ladle furnace (LF) (gap time of 5 min) in order to avoid delays which would turn into costs for missed production.

The treatment in ladle furnaces LF-A and LF-B lasts for 40 min, after which billets are transported, in a 5 min time, to degassing machine. Degassing treatment lasts 20 min, downstream of which there is the continuous casting plant which terminates its work after 40 min.

Reporting on a single diagram the power required by all the three components (EAF, LF and CCO/VOD) in order to complete a production phase, in the ideal case of considering the temperature ramps instantaneous (zero startup time), to transform a whole steel batch into finished product, and overlapping of three of these optimized power diagrams, opportunely shifted in time one each other, the diagram in Fig. 2.5 is obtained.

Summing in function of time the power values and considering a base-load power for auxiliaries equal to 5 MW, the diagram of the total absorbed power is obtained, as visible in Fig. 2.6.

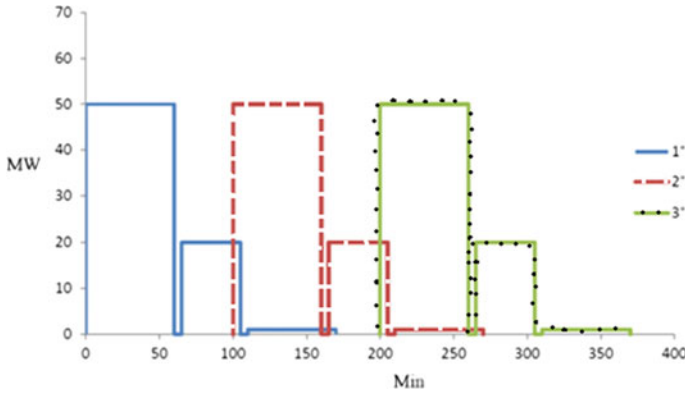


Fig. 2.5 Three cycles in series

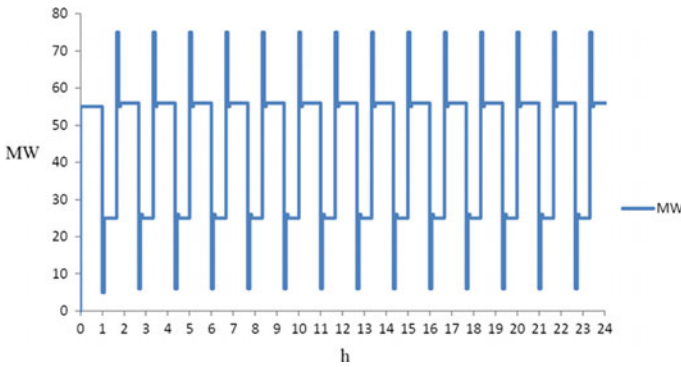


Fig. 2.6 Total absorbed power

From this diagram it is possible to estimate the energy amount required for each hour of plant operation, as visible in Fig. 2.7.

2.6.1 MAPE

The MAPE (Mean Absolute Percentage Error) is defined as: $(\text{Expected Value} - \text{Real Value}) / \text{Real Value}$. Such value was calculated for all the days typologies above described, both for energy and for power values, obtaining the following results:

- production days: 27,2% (energy) and 32% (power);
- no-production days: 76,7% (energy) and 84% (power);
- production days with ramp up: 12,7% (energy) and 13% (power);
- production days with ramp down: 80,4% (energy) and 81% (power);
- anomalous days: 38% (energy) and 76% (power).