Theory of Periodic Conjugate Heat Transfer

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With 41 Figures and 11 Tables

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For Tatiana, my wife

Preface

The book was conceived to give the exhaustive answer to a question how thermophysical and geometrical parameters of a body affect heat transfer characteristics under conditions of thermohydrodynamic fluctuations. An applied objective of the book consisted in the development of a universal method of prediction of the average heat transfer coefficient for periodic conjugated processes of heat transfer.

Real "stationary" processes of heat transfer, as a rule, can be considered stationary only on average. Actually, except for a of purely laminar flows, periodic, quasi-periodic and various random fluctuations of parameters (velocities, pressure, temperatures, momentum and energy fluxes, vapor content, interphase boundaries, etc.) about their average values always exist in any fluid flow. Owing to the conjugate nature of the interface "fluid flow–streamlined body," both fluctuation and average values of temperatures and heat fluxes on which surface of heat transfer takes place generally depend on thermophysical and geometrical characteristics of the heat transferring wall. In this connection, a principle question arises about the possible influence of the material and thickness of a wall on the heat transfer coefficient, which is actually the key parameter of convective heat transfer. The facts of such an influence were earlier noticed in experimental investigations of heat transfer at nucleate boiling, dropwise condensation, as well as in some other cases. In these studies, heat transfer coefficients determined as a ratio of the average heat flux on the surface and the average temperature difference between the wall and the fluid could differ noticeably for various materials of the wall (and also for its different thicknesses). These experimental information resulted in the necessity to introduce various correction factors for to "stationary" heat transfer coefficients determined based on the theory of convective heat transfer. As it is believed, such corrections (frequently purely empirical), with any of them being valid for each particular case, brought additional uncertainty in the concept of the average heat transfer coefficient.

Clarity to this question was for the first time brought by my highly respected scientific supervisor Prof. D.A. Labuntsov (1929–1992), who developed a concept of a *true heat transfer coefficient* in 1976. According to this concept, actual values of the heat transfer coefficient (for each point of the heat transferring surface and at each moment of time) are determined solely by hydrodynamic characteristics of the fluid flow and consequently *do not depend* on parameters of the body. Fluctuations of parameters occurring in the fluid flow will result in respective *fluctuations of the true heat transfer coefficient*, also *independent* of the material and thickness of a wall. Then, from a solution of the heat conduction equation with a boundary condition of the third kind, it is possible to find a temperature field in the body (and, hence, on the heat transfer surface) and, as a result, to determine the required *experimental heat transfer coefficient* as a ratio of an average heat flux by an average temperature difference. This value (determined in traditional heat transfer experiments and used in applied calculations) according to its definition should depend on the conjugation parameters.

A study of interrelations of the heat transfer coefficients averaged based on different procedures (*true* and *experimental*) was the main subject of the book of Labuntsov and Zudin (1984) "Processes of heat transfer with periodic intensity," Energoatomizdat, Moscow (in Russian). One of the fundamental results obtained in this book was that the *average experimental value* of the heat transfer coefficient is always less than the *average true value* of this parameter. The present book has arisen as a natural continuation and development of the concept of *true heat transfer coefficient*. The link with the book of Labuntsov and Zudin (1984) is reflected in the first three chapters. The other chapters generalize new results obtained and published by myself since the year of 1991.

Chapter 1 presents a qualitative description of the method proposed as a tool for investigations of periodic conjugate convective–conductive problems "fluid flow–streamlined body." Several particular cases of physical problems including heat transfer processes with periodic fluctuations are outlined briefly in this chapter.

In Chap. 2, an analysis is carried out to a boundary problem for the two-dimensional unstationary heat conduction equation with a periodic boundary condition of the third kind. To characterize quantitatively the thermal effects of a solid body on the average heat transfer, a concept of a *factor of conjugation* is introduced. It is shown that the quantitative effect of the conjugation in the problem can be rather significant.

Chapter 3 outlines a general solution of a boundary problem for the equation of heat conduction with a periodic boundary condition of the third kind. Analytical solutions presented here comprise cases of three characteristic laws of the variation of the true heat transfer coefficient, namely, harmonic, inverse harmonic, and stepwise.

In Chap. 4, a universal algorithm of a general approximate solution of the problem is developed. On its basis, solutions are obtained of several problems for various laws of periodic fluctuations of the true heat transfer coefficient.

Chapter 5 considers conjugate periodic heat transfer for "complex" cases of an external heat supply (or removal): Heat transfer at a contact either with environment or with a second body. A generalized solution for the factor of conjugation for bodies of the "standard form" is obtained. A problem of the conjugate heat transfer for a case of bilateral periodic heat transfer is also investigated in this chapter.

In Chap. 6, an analysis is given for the cases of asymmetric and nonperiodic fluctuations of the true heat transfer coefficient.

Chapter 7 represents some applied problems of the periodic conjugate heat transfer theory such as jet impingement onto a surface, dropwise condensation and nucleate boiling.

The author is deeply grateful to Prof. Wilfried Roetzel (Helmut-Schmidt-Universität, Universität der Bundeswehr Hamburg), and also Prof. Karl Stephan and Prof. Bernhard Weigand (Universität Stuttgart) for the fruitful discussions and their valuable advices promoting an improvement of the overall description of the material outlined in this book.

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Moscow, July, 2007 *Y. B. Zudin*

Contents

Abbreviations

ATHTC	Averaged true heat transfer coefficient
BC	Boundary condition
EHTC	Experimental heat transfer coefficient
FC	Factor of conjugation
HTC	Heat transfer coefficient
PTE	Parameter of the thermal effect
TBC	Thermal boundary conditions
THTC	True heat transfer coefficient

Symbols

A_k, A_k^*	Complex conjugate eigenvalues
B_k, B_k^*	Complex conjugate eigenfunctions
b	Amplitude of oscillations of the true heat transfer coefficient
$C_\mathrm{f}/2$	Friction factor
c	Specific heat ($\mathrm{J\,kg^{-1}\,K^{-1}}$)
d_0	Nozzle diameter (m)
F_k	Real parts of eigenfunctions
h	True heat transfer coefficient (THTC) ($\mathrm{Wm^{-2}\,K^{-1}}$)
$\langle h \rangle$	Averaged true heat transfer coefficient (ATHTC) ($\mathrm{Wm^{-2}\,K^{-1}}$)
$\langle \bar{h} \rangle$	Dimensionless averaged true heat transfer coefficient or Biot number
h_m	Experimental heat transfer coefficient (EHTC) ($\mathrm{Wm^{-2}\,K^{-1}}$)
h_0	Steady-state heat transfer coefficient ($\mathrm{Wm^{-2}\,K^{-1}}$)
$\bar{h}_0$	Dimensionless stationary heat transfer coefficient
h_fg	Specific enthalpy of evaporation ($\mathrm{J\,kg^{-1}}$)
I_n	Imaginary parts of eigenfunctions
Ja	Jacob number
k	Thermal conductivity ($\mathrm{Wm^{-1}\,K^{-1}}$)
L	Distance between nucleate boiling sites (m)
m	Inverse Fourier number
n_F	Number of boiling sites ($\mathrm{m^{-2}}$)
K	Ratio of thermal potentials of contacting media
p	Pressure (Pa)
Pr	Prandtl number
q	Heat flux density ($\mathrm{W\,m^{-2}}$)
$\langle q \rangle$	Averaged heat flux density ($\mathrm{W\,m^{-2}}$)
$\hat{q}$	Oscillating heat flux density ($\mathrm{W\,m^{-2}}$)
q_V	Volumetric heat source ($\mathrm{W\,m^{-3}}$)
R_n	Real parts of eigenvalues
R_*	Critical radius of vapor nucleus (m)

St	Stanton number
t	Dimensionless time
T_{s}	Saturation temperature (K)
u	Velocity ($\mathrm{m\,c^{-1}}$)
u_0	Free stream velocity ($\mathrm{m\,c^{-1}}$)
u_*	Friction velocity ($\mathrm{m\,c^{-1}}$)
U	Overall heat transfer coefficient ($\mathrm{Wm^{-2}\,K^{-1}}$)
$\langle U \rangle$	Averaged true overall heat transfer coefficient ($\mathrm{Wm^{-2}\,K^{-1}}$)
U_{m}	Experimental overall heat transfer coefficient ($\mathrm{Wm^{-2}\,K^{-1}}$)
$\langle \bar{U} \rangle$	Dimensionless averaged true overall heat transfer coefficient
E	Generalized factor of conjugation
X	Spanwise coordinate (m)
x	Dimensionless spanwise coordinate
Z	Coordinate along the surface of heat transfer (m)
Z_0	Spatial periods of oscillation (m)
z	Dimensionless coordinate along the heat transfer surface

Greek Letter Symbols

α	Thermal diffusivity ($\mathrm{m^2\,c^{-1}}$)
Γ	Shear stress ($\mathrm{N\,m^{-2}}$)
δ	Wall thickness (flat plate) (m)
$\bar{\delta}$	Dimensionless wall thickness (flat plate)
δ_{f}	Thickness of liquid film (m)
ε	Factor of conjugation (FC)
ϑ	Temperature (K)
$\langle \vartheta \rangle$	Averaged temperature (K)
$\hat{\vartheta}$	Oscillating temperature (K)
ϑ_0	Free stream temperature (K)
$\vartheta^{\bullet}$	Gradient of oscillating temperature ($\mathrm{K\,m^{-1}}$)
ϑ_{Σ}	Total temperature difference in the three-part system (K)
θ	Dimensionless oscillations temperature
$\theta^{\bullet}$	Dimensionless gradient of the oscillation temperature (or dimensionless heat flux density)
ξ	Generalized coordinate of a progressive wave
ξ_{ϑ}	Phase shift between oscillation of true heat transfer coefficient and temperature
ξ_{q}	Phase shift between oscillation of true heat transfer coefficient and heat flux
μ	Dynamic viscosity ($\mathrm{kgm^{-1}\,s^{-1}}$)
ν	Kinematic viscosity ($\mathrm{m^2}\,s^{-1}$)
ρ	Density ($\mathrm{kg\,m^{-3}}$)

σ	Surface tension ($\mathrm{N\,m^{-1}}$)
τ	Time (s)
τ_0	Time period of oscillation (s)
Φ_k	Imaginary parts of eigenfunctions
χ	Parameter of thermal effect (PTE)
ψ	Periodic part of the heat transfer coefficient
ω	Frequency ($\mathrm{s^{-1}}$)

Subscripts

+	Active period of heat transfer
−	Passive period of heat transfer
f	Fluid
g	Gas
0	External surface of a body (at $X = 0$)
δ	Heat transfer surface (at $X = \delta$)
min	Minimal value
max	Maximal value
w	Another (second) body

Definition of Nondimensional Numbers and Groups

$\langle\bar{h}\rangle = \frac{\langle h\rangle Z_0}{k}$	Dimensionless averaged true heat transfer coefficient or Biot Number
$\bar{h}_0 = \frac{h_0 Z_0}{k}$	Dimensionless stationary heat transfer coefficient
$E = \frac{U_\mathrm{m}}{\langle U\rangle}$	Generalized factor of conjugation
$Ja = \frac{\rho_\mathrm{f} c_\mathrm{pf}\vartheta}{\rho_\mathrm{g} h_\mathrm{fg}}$	Jacob number
$K = \sqrt{\frac{kc\rho}{k_\mathrm{f} c_\mathrm{f}\rho_\mathrm{f}}}$	Ratio of thermal potentials of the contacting media
$m = \frac{Z_0^2}{\alpha\tau_0}$	Inverse Fourier number
$Pr = \frac{\nu_\mathrm{f}}{\alpha_\mathrm{f}}$	Prandtl number
$St = \frac{q}{\rho_\mathrm{f} c_\mathrm{f} u_0 \vartheta_0}$	Stanton number
$\langle\bar{U}\rangle = \frac{\langle U\rangle Z_0}{k}$	Dimensionless averaged true overall heat transfer coefficient
$\bar{U}_\mathrm{m} = \frac{U_\mathrm{m} Z_0}{k}$	Dimensionless experimental overall heat transfer coefficient
$\bar{\delta} = \frac{\delta}{Z_0}$	Dimensionless wall thickness (flat plate)
$\varepsilon = \frac{h_\mathrm{m}}{\langle h\rangle}$	Factor of conjugation

1

Introduction

1.1 Heat Transfer Processes Containing Periodic Oscillations

1.1.1 Oscillation Internal Structure of Convective Heat Transfer Processes

Real stationary processes of heat transfer, as a rule, can be considered stationary only on the average. Actually (except for the purely laminar cases), flows are always subjected to various periodic, quasiperiodic and other casual oscillations of velocities, pressure, temperatures, momentum and energy fluxes, vapor content and interphase boundaries about their average values. Such oscillations can be smooth and periodic (wave flow of a liquid film or vapor, a flow of a fluctuating coolant over a body), sharp and periodic (hydrodynamics and heat transfer at slug flow of a two-phase media in a vertical pipe; nucleate and film boiling process), on can have complex stochastic character (turbulent flows). Oscillations of parameters have in some cases spatial nature, in others they are temporal, and generally one can say that the oscillations have mixed spatiotemporal character.

The theoretical base for studying instantly oscillating and at the same time stationary on the average heat transfer processes are the unsteady differential equations of momentum and energy transfer, which in case of two-phase systems can be notated for each of the phases separately and be supplemented by the conditions of the physical interface on the boundaries between phases (conditions of conjugation). An exhaustive solution of the problem could be a comprehensive analysis with the purpose of a full description of any particular fluid flow and heat transfer pattern with all its detailed characteristics, including various fields of oscillations of its parameters.

However, at the time being such an approach can not be realized in practice. The problem of modeling turbulent flows [1] can serve as a vivid example.

As a rule at its theoretical analysis, Reynolds-averaged Navier–Stokes equations are considered, which describe time-averaged quantities of fluctuating parameters, or in other words turbulent fluxes of the momentum and energy. To provide a closed description of the process, these correlations by means of various semiempirical hypotheses are interrelated with time-averaged fields of velocities and enthalpies. Such schematization results in the statement of a stationary problem with spatially variable coefficients of viscosity and thermal conductivity. Therefore, as boundary conditions here, it is possible to set only respective stationary conditions on the heat transfer surface of such a type as, for example, "constant temperature," "constant heat flux."

It is necessary to specially note, that the replacement of the full "instant" model description with the time-averaged one inevitably results in a loss of information on the oscillations of fluid flow and heat transfer parameters (velocities, temperatures, heat fluxes, pressure, friction) on a boundary surface. Thus the theoretical basis for an analysis of the interrelation between the temperature oscillations in the flowing ambient medium and in the body is omitted from the consideration. And generally saying, the problem of an account for possible influence of thermophysical and geometrical parameters of a body on the heat transfer at such a approach becomes physically senseless. For this reason, such a "laminarized" form of the turbulent flow description is basically not capable of predicting and explaining the wall effects on the heat transfer characteristics, even if these effects are observed in practice. The problem becomes especially complicated at imposing external oscillations on the periodic turbulent structure that takes place, in particular, in flows over aircraft and spacecraft. Unresolved problems of closing the Navier–Stokes equations in combination with difficulties of numerical modeling make a problem of detailed prediction of a temperature field in the flowing fluid very complicated. In some cases, differences between the predicted and measured local *heat transfer coefficient* (HTC) exceeds 100%.

In this connection the direction in the simulation of turbulent flows based on the use of the primary transient equations [2] represents significant interest. The present book represents results of numerical modeling of the turbulent flows in channels subjected to external fields of oscillations (due to vortical generators etc.). It is shown that in this case an essentially anisotropic and three-dimensional flow pattern emerges strongly different from that described by the early theories of turbulence [1]. In the near-wall zone, secondary flows in the form of rotating "vortical streaks" are induced that interact with the main flow. As a result, oscillations of the thermal boundary layer thickness set on, leading to periodic enhancement or deterioration of heat transfer. Strong anisotropy of the fluid flow pattern results in the necessity of a radical revision of the existing theoretical methods of modeling the turbulent flows. So, for example, the turbulent Prandtl number being in early theories of turbulence [1] a constant of the order of unity (or, at the best, an indefinite scalar quantity), becomes a tensor.

It is necessary to emphasize that all the mentioned difficulties are related to the nonconjugated problem when the role of a wall is reduced only to maintenance of a *boundary condition* (BC) on the surface between the flowing fluid and the solid wall.

1.1.2 Problem of Correct Averaging the Heat Transfer Coefficients

The basic applied task of the book is the investigation into the effects of a body (its thermophysical properties, linear dimensions and geometrical configuration) on the traditional HTC, measured in experiments and used in engineering calculations. Processes of heat transfer are considered stationary on average and fluctuating instantly. A new method of investigation of the conjugate problem "fluid flow–body" is presented. The method is based on a replacement of the complex mechanism of oscillations of parameters in the flowing coolant by a simplified model employing a varying "true heat transfer coefficient" specified on a heat transfer surface.

The essence of the developed method can be explained rather simply. Let us assume that we have perfect devices measuring the instant local values of temperature and heat fluxes at any point of the fluid and heated solid body. Then the hypothetical experiment will allow finding the fields of temperatures and heat fluxes and their oscillations in space and in time, as well as their average values and all other characteristics. In particular, it is possible to present the values of temperatures (exactly saying temperature heads or loads, i.e., the temperatures counted from a preset reference level) and heat fluxes on a heat transfer surface in the following form:

$$\vartheta = \langle \vartheta \rangle + \hat{\vartheta}, \tag{1.1}$$

$$q = \langle q \rangle + \hat{q}, \tag{1.2}$$

i.e., to write them as the sum of the averaged values and their temporal oscillations. For the general case of spatiotemporal oscillations of characteristics of the process, the operation of averaging is understood here as a determination of an average with respect to time τ and along the heat transferring surface (with respect to the coordinate Z). The *true heat transfer coefficient* (THTC) is determined on the basis of (1.1) and (1.2) according to Newton's law of heat transfer [3, 4]:

$$h = \frac{q}{\vartheta}. \tag{1.3}$$

This parameter can always be presented as a sum of an averaged part and a fluctuating additive:

$$h = \langle h \rangle + \hat{h}. \tag{1.4}$$

It follows from here that the correct averaging of the HTC is as follows

$$\langle h \rangle = \left\langle \frac{q}{\vartheta} \right\rangle . \tag{1.5}$$

Therefore we shall call parameter $\langle h \rangle$ an *averaged true heat transfer coefficient* (ATHTC). The problem consists in the fact that the parameter $\langle h \rangle$ cannot be directly used for applied calculations, since it contains initially the unknown information of oscillations $\hat{\vartheta}, \hat{q}$. This fact becomes evident if (1.5) is rewritten with the help of (1.1) and (1.2):

$$\langle h \rangle = \left\langle \frac{\langle q \rangle + \hat{q}}{\langle \vartheta \rangle + \hat{\vartheta}} \right\rangle . \tag{1.6}$$

The purpose of the heat transfer experiment is the measurement of averaged values of an averaged temperature $\langle \vartheta \rangle$ and a heat flux $\langle q \rangle$ on the surfaces of a body and determination of the traditional HTC

$$h_{\mathrm{m}} = \frac{\langle q \rangle}{\langle \vartheta \rangle} . \tag{1.7}$$

The parameter h_{m} is fundamental for carrying out engineering calculations, designing heat transfer equipment, composing thermal balances, etc. However it is necessary to point out that transition from the initial Newton's law of heat transfer (1.3) to the restricted (1.7) results in the loss of the information of the oscillations of the temperature $\hat{\vartheta}$ and the heat fluxes $\hat{q}$ on the wall.

Thus, it is logical to assume that the influence of the material and the wall thickness of the body taking part in the heat transfer process on HTC h_{m} uncovered in experiments is caused by noninvariance of the value of h_{m} with respect to the Newton's law of heat transfer. For this reason we shall refer further to the parameter h_{m} as to an *experimental heat transfer coefficient* (EHTC).

Thus, we have two alternative procedures of averaging the HTC: true (1.5) and experimental (1.7). The physical reason of the distinction between $\langle h \rangle$ and the h_{m} can be clarified with the help of the following considerations:

- Local values $\langle \vartheta \rangle$ and $\langle q \rangle$ on a surface where heat transfer takes place are formed as a result of the thermal contact of the flowing fluid and the body.
- Under conditions of oscillations of the characteristics of the coolant, temperature oscillations will penetrate inside the body.
- Owing to the conjugate nature of the heat transfer in the considered system, both fluctuating $\hat{\vartheta}, \hat{q}$, and averaged $\langle \vartheta \rangle , \langle q \rangle$ parameters on the heat transfer surface depend on the thermophysical and geometrical characteristics of the body.
- The ATHTC $\langle h \rangle$ directly follows from Newton's law of heat transfer (1.3) (which is valid also for the unsteady processes) and consequently it is determined by hydrodynamic conditions in the fluid flowing over the body.