Lecture Notes in Electrical Engineering 90

Sio Iong Ao
Len Gelman
*Editors*

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# Preface

A large international conference in Electrical Engineering and Applied Computing was held in London, U.K., 30 June–2 July, 2010, under the World Congress on Engineering (WCE 2010). The WCE 2010 was organized by the International Association of Engineers (IAENG); the Congress details are available at: http://www.iaeng.org/WCE2010. IAENG is a non-profit international association for engineers and computer scientists, which was founded originally in 1968. The World Congress on Engineering serves as good platforms for the engineering community to meet with each other and exchange ideas. The conferences have also struck a balance between theoretical and application development. The conference committees have been formed with over two hundred members who are mainly research center heads, faculty deans, department heads, professors, and research scientists from over 30 countries. The conferences are truly international meetings with a high level of participation from many countries. The response to the Congress has been excellent. There have been more than one thousand manuscript submissions for the WCE 2010. All submitted papers have gone through the peer review process, and the overall acceptance rate is 57%.

This volume contains fifty-five revised and extended research articles written by prominent researchers participating in the conference. Topics covered include Control Engineering, Network Management, Wireless Networks, Biotechnology, Signal Processing, Computational Intelligence, Computational Statistics, Internet Computing, High Performance Computing, and industrial applications. The book offers the state of the art of tremendous advances in electrical engineering and applied computing and also serves as an excellent reference work for researchers and graduate students working on electrical engineering and applied computing.

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# Contents

# Chapter 1
# Mathematical Modelling for Coal Fired Supercritical Power Plants and Model Parameter Identification Using Genetic Algorithms

**Omar Mohamed, Jihong Wang, Shen Guo, Jianlin Wei, Bushra Al-Duri, Junfu Lv and Qirui Gao**

**Abstract** The paper presents the progress of our study of the whole process mathematical model for a supercritical coal-fired power plant. The modelling procedure is rooted from thermodynamic and engineering principles with reference to the previously published literatures. Model unknown parameters are identified using Genetic Algorithms (GAs) with 600MW supercritical power plant on-site measurement data. The identified parameters are verified with different sets of measured plant data. Although some assumptions are made in the modelling process to simplify the model structure at a certain level, the supercritical

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coal-fired power plant model reported in the paper can represent the main features of the real plant once-through unit operation and the simulation results show that the main variation trends of the process have good agreement with the measured dynamic responses from the power plants.

**Nomenclature**

| | |
|---|---|
| $ff$ | Fitness function for genetic algorithms |
| $ffr$ | Pulverized fuel flow rate (kg/s) |
| $h$ | Enthalpy per unit mass (MJ/kg) |
| $K$ | Constant parameter |
| $k$ | Mass flow rate gain |
| $m$ | Mass (kg) |
| $\dot{m}$ | Mass flow rate (kg/s) |
| $P$ | Pressure of a heat exchanger (MPa) |
| $\dot{Q}$ | Heat transfer rate (MJ/s) |
| $R$ | Response |
| $T$ | Temperature (°C) |
| $t$ | Time (s) |
| $\tau$ | Time constant (s) |
| $U$ | Internal energy (MJ) |
| $V$ | Volume of fluid ($m^3$) |
| $\dot{W}$ | Work rate or power (MW) |
| $x$ | Generator reactance (p.u) |
| $y$ | Output vector |
| $\rho$ | Density ($kg/m^3$) |
| $\chi$ | Valve opening |
| $\delta$ | Rotor angle (rad) |
| $\theta$ | Mechanical angle (rad) |
| $\omega$ | Speed (p.u) |
| $\Gamma$ | Torque (p.u) |

**Subscripts**

| | |
|---|---|
| $a$ | Accelerating |
| $air$ | Air |
| $e$ | Electrical |
| $d$ | Direct axis |
| $ec$ | Economizer |
| $hp$ | High pressure turbine |
| $hx$ | Heat exchanger |
| $i$ | Inlet |
| $ip$ | Intermediate pressure turbine |
| $me$ | Mechanical |
| $ms$ | Main steam |
| $m$ | Measured |

| | |
|---|---|
| *o* | Outlet |
| *out* | Output of the turbine |
| *q* | Quadrature axis |
| *rh* | Reheater |
| *sh* | Superheater |
| *si* | Simulated |
| *ww* | Waterwall |

**Abbreviations**

| | |
|---|---|
| BMCR | Boiler maximum continuous rate |
| ECON | Economizer |
| GA | Genetic algorithm |
| HP | High pressure |
| HX | Heat exchanger |
| IP | Intermediate pressure |
| MS | Main steam |
| RH | Reheater |
| SC | Supercritical |
| SH | Superheater |
| WW | Waterwall |

## 1.1 Introduction

The world is now facing the challenge of the issues from global warming and environment protection. On the other hand, the demand of electricity is growing rapidly due to economic growth and increases in population, especially in the developing countries, for example, China and India. With the consideration of environment and sustainable development in energy, renewable energy such as wind, solar, and tidal wave should be only resources to be explored in theory. But the growth in demand is also a heavy factor in energy equations so the renewable energy alone is unlikely able to generate sufficient electricity to fill the gap in the near future. Power generation using fossil fuels is inevitable, especially, coal fired power generation is found to be an unavoidable choice due to its huge capacity and flexibility in load following. As a well know fact, the conventional coal fired power plants have a huge environmental impact and lower energy conversion efficiencies. Any new coal fired power plants must be cleaner with more advanced and improved technologies.

Apart from Carbon Capture and Storage, supercritical power plants might be the most suitable choice with consideration of the factors in environmental enhancement, higher energy efficiency and economic growth. However, there has been an issue to be addressed in its dynamic responses and performance in relation with conventional subcritical plants due to the difference in the process structure and energy storage drum [1]. The characteristics of supercritical plants require the considerable attention and investigation. Supercritical boilers have to be once-through type boilers because there is not distinction between water and steam phases in supercritical process so there is no drum to separate water steam mixture. Due to the absence of the drum, the once-through boilers have less stored energy and faster responses than the drum-type boilers. There are several advantages of supercritical power plants [2, 3] over traditional subcritical plants include:

- Higher cycle efficiency (Up to 46%) and lower fuel consumption.
- Reduced $CO_2$ emissions per unit power generation.
- Be fully integratable with $CO_2$ capture technology.
- Fast load demand following (in relatively small load demand changes).

However, some concerns are also raised in terms of its dynamic responses with regards to the demand for dynamic response speed. This is mainly caused by its once-through structure, that is, there is no drum to store energy as a buffer to response rapid changes in load demand.

The paper is to develop a mathematical model for the whole plant process to study dynamic responses aiming at answering the questions in dynamic response speed. From the literature survey, several models have been reported with emphasis on different aspects of the boiler characteristics. Studying the dynamic response and control system of once-through supercritical (SC) units can be traced back to 1958 when work was done on a time-based simulation for Eddystone I unit of Philadelphia Electric Company and the work was extended for simulation of Bull run SC generation unit later in 1966 [4].

Yutaka Suzuki et al. modelled a once through SC boiler in order to improve the control system of an existing supercritical oil-fired plant. The model was based on nonlinear partial differential equations, and the model was validated through simulation studies [5]. Wataro Shinohara et al. presented a simplified state space model for SC once through boiler-turbine system and designed a nonlinear controller [6]. Pressure node model description was introduced by Toshio Inoue et al. for power system frequency simulation studies [7]. Intelligent techniques contributions have yielded an excellent performance for modeling. Neural network has been introduced to model the SC power plant with sufficiently accurate results if they are trained with suitable data provided by operating unit [8]. However, neural network performances are unsatisfactory to simulate some emergency conditions of the plant because NN method depends entirely on the data used for the learning process, not on physical laws. Simulation of SC boilers may be achieved either theoretically based on physical laws or empirically based on experimental work. In this paper, the proposed mathematical model is based on thermodynamic principles and the model parameters are identified by using the data obtained from a 600MW SC power plant [9]. The simulation results show that the model is trustable to simulate the whole once-through mode of operation at a certain level of accuracy.

## 1.2 Mathematical Model of the Plant

### *1.2.1 Plant Description*

The unit of a once-through supercritical 600MW power plant is selected for the modelling study. The schematic view of the boiler is shown in Fig. 1.1. Water from the feedwater heater is heated in the economizer before entering the superheating stages through the waterwall. The superheater consists of three sections which are low temperature superheater, platen superheater, and final stage superheater. The main outlet steam temperature is about 571°C at the steady state and a pressure is 25.5 MPa. There are 2 reheating sections in the boiler for reheating the steam exhausted from the high pressure turbine. The inlet temperature of the reheater is 309°C and the outlet temperature is nearly 571°C and average pressure is 4.16 MPa. The reheated steam is used to energize the intermediate pressure turbine. The mechanical power is generated through multi-stage turbines to provide an adequate expansion of the steam through the turbine and subsequently high thermal efficiency of the plant.

### *1.2.2 Assumptions Made for Modelling*

Assumptions are made to simplify the process which should be acceptable by plant engineers and sufficient to transfer the model from its complex physical model to lead to simple mathematical model for the research purpose. Some of these assumptions are usually adopted for modelling supercritical or subcritical boilers [10]. Modelling in the work reported in the paper, the following general assumptions are made:

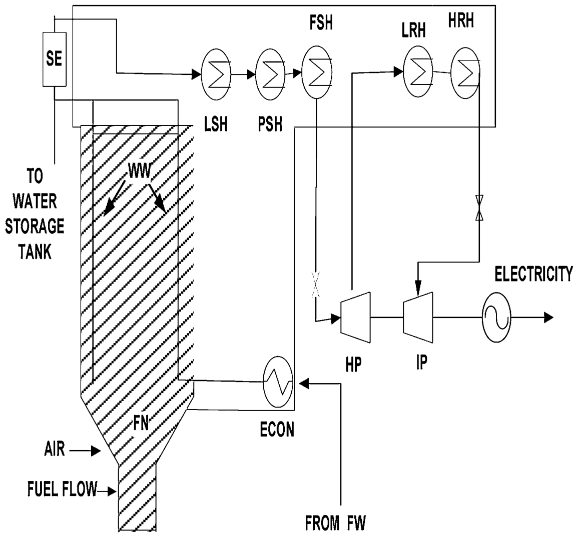


**Fig. 1.1** Schematic view of the plant

- Fluid properties are uniform at any cross section, and the fluid flow in the boiler tubes is one-phase flow.
- In the heat exchanger, the pipes for each heat exchanger are lumped together to form one pipe.
- Only one control volume is considered in the waterwall.
- The dynamic behaviour of the air and gas pressure is neglected.

## *1.2.3 The Boiler Model*

### 1.2.3.1 Heat Exchanger Model

The various heat exchangers in the boiler are modelled by the principles of mass and energy balances. The sub-cooled water in the economizer is transferred directly to a supercritical steam through the waterwall without passing the evaporation status. The equations are converted in terms of the derivatives (or variation rates) pressure and temperature of the heat exchanger. The mass balance equation of the heat exchanger (control volume) is:

$$\frac{dm}{dt} = \dot{m}_i - \dot{m}_o \tag{1.1}$$

For the constant effective volume, Eq. 1.1 will be:

$$V\frac{d\rho}{dt} = \dot{m}_i - \dot{m}_o$$

The density is a differentiable function of two variables which can be the temperature and pressure inside the control volume, thus we have:

$$V\left(\left.\frac{\partial \rho}{\partial P}\right|_T \cdot \frac{dP}{dt} + \left.\frac{\partial \rho}{\partial T}\right|_P \cdot \frac{dT}{dt}\right) = \dot{m}_i - \dot{m}_o$$

The energy balance equation:

$$\frac{dU_{hx}}{dt} = \dot{Q}_{hx} + \dot{m}_i h_i - \dot{m}_o h_o$$

Also,

$$\begin{aligned}\frac{dU_{hx}}{dt} = {} & V\left[h\left(\left.\frac{\partial \rho}{\partial P}\right|_T \cdot \frac{dP}{dt} + \left.\frac{\partial \rho}{\partial T}\right|_P \cdot \frac{dT}{dt}\right) + \rho\left(\left.\frac{\partial \mathrm{h}}{\partial P}\right|_T \cdot \frac{dP}{dt} + \left.\frac{\partial h}{\partial T}\right|_P \cdot \frac{dT}{dt}\right)\right] \\ & - V\frac{dP}{dt}V\left[h\left(\left.\frac{\partial \rho}{\partial P}\right|_T \cdot \frac{dP}{dt} + \left.\frac{\partial \rho}{\partial T}\right|_P \cdot \frac{dT}{dt}\right) + \rho\left(\left.\frac{\partial h}{\partial P}\right|_T \cdot \frac{dP}{dt} + \left.\frac{\partial h}{\partial T}\right|_P \cdot \frac{dT}{dt}\right)\right] \\ & - V\frac{dP}{dt}\dot{Q}_{hx} + \dot{m}_i h_i - \dot{m}_o h_o\end{aligned} \tag{1.2}$$

Combining (1.1) and (1.2) to get the pressure and temperature state derivatives,

$$\dot{P} = \frac{\dot{Q}_{hx} + \dot{m}_i H_i - \dot{m}_o H_o}{\tau} \tag{1.3}$$

$$\dot{T} = C(\dot{m}_i - \dot{m}_o) - D\dot{P} \tag{1.4}$$

Where:

$$H_i = \left( h_i - h - \frac{\rho \frac{\partial h}{\partial T}\big|_P}{\frac{\partial \rho}{\partial T}\big|_P} \right) \tag{1.5}$$

$$H_o = \left( h_o - h - \frac{\rho \frac{\partial h}{\partial T}\big|_P}{\frac{\partial \rho}{\partial T}\big|_T} \right) \tag{1.6}$$

$$\tau = V \left( \rho \frac{\partial h}{\partial P}\bigg|_T - \frac{\rho \frac{\partial \rho}{\partial P}\big|_T \cdot \frac{\partial h}{\partial T}\big|_P}{\frac{\partial \rho}{\partial T}\big|_P} - 1 \right) \tag{1.7}$$

$$C = \frac{1}{V \frac{\partial \rho}{\partial T}\big|_P} \tag{1.8}$$

$$D = \frac{\frac{\partial \rho}{\partial P}\big|_T}{\frac{\partial \rho}{\partial T}\big|_P} \tag{1.9}$$

The temperature of the superheater is controlled by the attemperator. Therefore, the input mass flow rate to the superheater is the addition of the SC steam and the water spray from the attemperator. The amount of attemperator water spray is regulated by opening the spray valve which responds to a signal from the PI controller. This prevents the high temperature fluctuation and ensures maximum efficiency over a wide range of operation.

#### 1.2.3.2 Fluid Flow

The fluid flow in boiler tubes for one-phase flow is :

$$\dot{m} = k \cdot \sqrt{\Delta P} \tag{1.10}$$

Equation 1.10 is the simplest mathematical expression for fluid flow in boiler tubes. The flow out from the reheater and main steam respectively are:

$$\dot{m}_{rh} = K_1' \frac{P_{rh}}{\sqrt{T_{rh}}} \chi_{rh} \tag{1.11}$$

$$\dot{m}_{ms} = K_2' \frac{P_{ms}}{\sqrt{T_{ms}}} \chi_{ms} \tag{1.12}$$

The detailed derivation of (1.11) and (1.12) can be found in [11].

## 1.2.4 Turbine/Generator Model

### 1.2.4.1 Turbine Model

The turbine is modeled through energy balance equations and then is combined with the boiler model.

The work done by high pressure and intermediate pressure turbines are:

$$\dot{W}_{hp} = \dot{m}_{ms} \cdot (h_{ms} - h_{out}) \tag{1.13}$$

$$\dot{W}_{ip} = \dot{m}_{rh} \cdot (h_{rh} - h_{out}) \tag{1.14}$$

The mechanical power of the plant:

$$P_{me} = \dot{W}_{hp} + \dot{W}_{ip} \tag{1.15}$$

Up to Eq. 1.14, the boiler-turbine unit is model in a set of combined equations and can be used for simulation if we assume that the generator is responding instantaneously. However, the dynamics of the turbines' speeds and torques must be affected by the generator dynamics and injecting the mechanical power only into the generator model will not provide this interaction between the variables. To have a strong coupling between the variables in the models of the turbine-generator, torque equilibrium equations for the turbine model are added to the turbine model:

$$\dot{\omega}_{hp} = \frac{1}{M_{hp}}\left[\Gamma_{hp} - D_{hp}\omega_{hp} - K_{HI}(\theta_{hp} - \theta_{ip})\right] \tag{1.16}$$

$$\dot{\theta}_{hp} = \omega_b(\omega_{hp} - 1) = (\omega_{hp} - 1) \tag{1.17}$$

$$\dot{\omega}_{ip} = \frac{1}{M_{ip}}\left[\Gamma_{ip} - D_{ip}\omega_{ip} + K_{HI}(\theta_{hp} - \theta_{ip}) - K_{IG}(\theta_{hp} - \theta_g)\right] \tag{1.18}$$

$$\dot{\theta}_{ip} = \omega_b(\omega_{ip} - 1) = (\omega_{ip} - 1) \tag{1.19}$$

Note that, for two-pole machine: $\theta_g = \delta$

### 1.2.4.2 Generator Model

The generator models are reported in a number of literatures; a third order non-linear model is adopted in our work [12]:

$$\dot{\delta} = \Delta\omega \tag{1.20}$$

$$J\Delta\dot{\omega} = \Gamma_a = \Gamma_m - \Gamma_e - D\Delta\omega \tag{1.21}$$

$$\dot{e}'_q = \frac{1}{T'_{do}}\left(E_{FD} - e'_q - \left(x_d - x'_d\right) i_d\right) \tag{1.22}$$

$$\Gamma_e(\text{p.u}) \approx P_e(\text{p.u}) \approx \frac{V}{x'_d} e'_q \sin\delta + \frac{V^2}{2}\left(\frac{1}{x_q} - \frac{1}{x'_d}\right)\sin 2\delta \tag{1.23}$$

## 1.3 Model Parameter Identification

### *1.3.1 Identification Procedures*

The parameters of the model which are defined by the formulae from (1.3) to (1.7) and the other parameters of mass flow rates' gains, heat transfer constants, turbine, and generator parameters are all identified by Genetic Algorithms in a sequential manner. Even though some of these parameters are inherently not constant, these parameters are fitted directly to the actual plant response to save time and effort. Various data sets of boiler responses have been chosen for identification and verification. First, the parameters of pressure derivatives equations are indentified. Then, the identification is extended to include the temperature equations, the turbine model parameters and finally generator model parameters.

The measured responses which are chosen for identification and verification are:

- Reheater pressure.
- Main SC steam pressure.
- Main SC steam temperature.
- Mass flow rate of SC steam from boiler main outlet to HP turbine.
- Mass flow rate of reheated steam from reheater outlet to the IP turbine.
- Turbine speed.
- Infinite bus frequency.
- Generated power of the plant.

In recent years, Genetic Algorithms optimization tool has been widely used for nonlinear system identification and optimization due to its many advantages over conventional mathematical optimization techniques. It has been proved that the GAs tool is a robust optimization method for parameters identification of sub-critical boiler models [13]. Initially, the GAs produces random values for all the parameters to be identified and called the initial population. Then, it calculates the corresponding fitness function to recopy the best coded parameter in the next generation. The GAs termination criteria depend on the value of the fitness function. If the termination criterion is not met, the GA continues to perform the three main operations which are reproduction, crossover, and mutation. The fitness function for the proposed task is:

$$ff = \sum_{n=1}^{N} (R_m - R_{si})^2 \tag{1.24}$$

The fitness function is the sum of the square of the difference between measured and simulated responses for each of the variables mentioned in this section. $N$ is the number of points of the recorded measured data, The load-up and load-down data have been used for identification. The changes are from 30% to 100% of load and down to 55% to verify the model derived. The model is verified from a ramp load up data and steady state data to cover a large range of once-through operation. The model has been also verified by a third set of data. The GAs parameters setting for identification are listed below:

Generation: 100
Population type: double vector
Creation function: uniform
Population size: 50–100
Mutation rate: 0.1
Mutation function: Gaussian
Migration direction: forward
Selection: stochastic uniform

Figure 1.2 shows some of the load-up identification results. It has been observed that the measured and simulated responses are very well matched for the power generated and they are also reasonably matched for the temperature. Some parameters of the boiler model are listed in Table 1.1 and for heat transfer rates are listed in Table 1.2.

### *1.3.2 Model Parameter Verification*

The validation of the proposed model has been performed using a number of data sets which are the load down and steady state data. Figure 1.3 shows some of the simulated verification results (load-down and steady state simulation). From the results presented, it is obvious that the model response and the actual plant response are well agreed to each other.

## 1.4 Concluding Remarks

A mathematical model for coal fired power generation with the supercritical boiler has been presented in the paper. The model is based on thermodynamic laws and engineering principles. The model parameters are identified using on-site operating

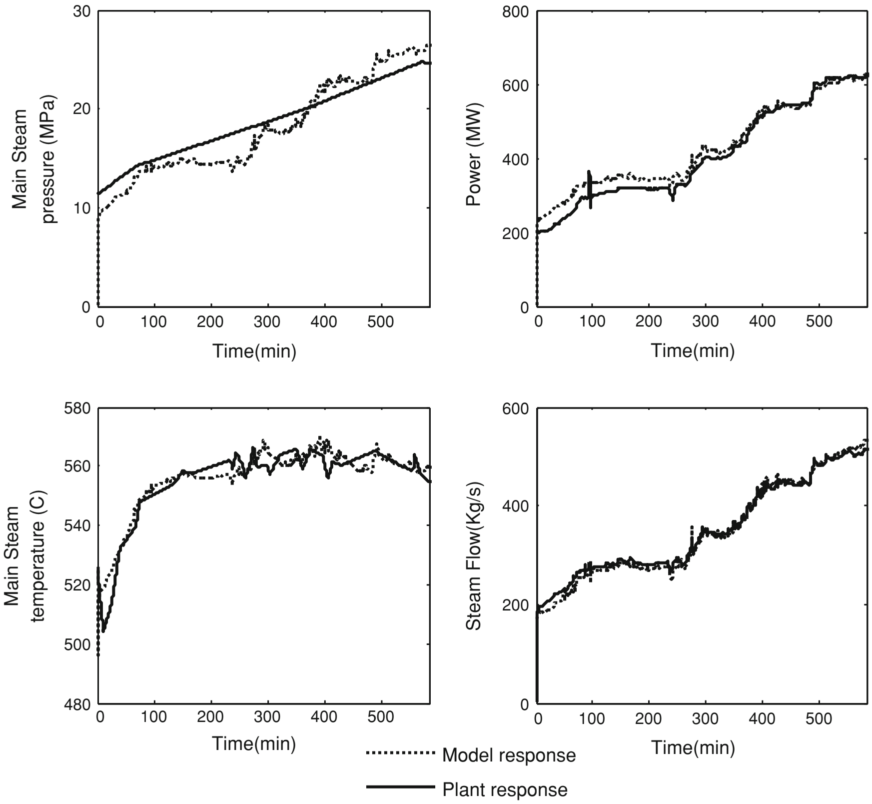


**Fig. 1.2** Identification results

**Table 1.1** Heat exchanger parameter

| HX | $H_i$ | $H_o$ | $C$ | $D$ |
|---|---|---|---|---|
| ECON | 10.2 | 13.6 | 2.1e-6 | −3.93 |
| WW | 12.2 | 13.3 | −1.2e-6 | −0.1299 |
| SH | 20.5 | 45.9 | 1e-6 | −3.73 |
| RH | 19.8 | 22.0 | −1e-6 | −17.9 |

**Table 1.2** Heat transfer rate

| $\tau_1$(s) | $K_{ec}$ | $K_{ww}$ | $K_{sh}$ | $K_{rh}$ |
|---|---|---|---|---|
| 9.3 | 5.7785 | 7.78 | 23.776 | 21.43 |

data recorded. The model is then verified by using different data sets and the simulation results show a good agreement between the measured and simulated data. For future work, the model will be combined with a nonlinear mathematical

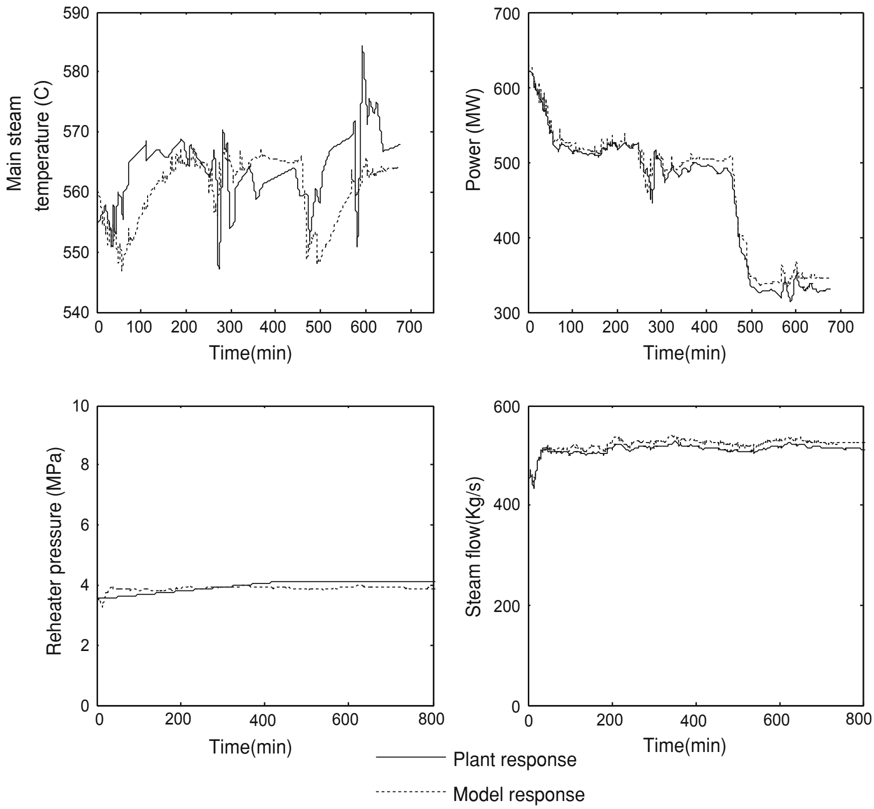


**Fig. 1.3** Verification results

model of coal mill to obtain a complete process mathematical model from coal preparation to electricity generation. It is expected that the mill local control system should have great contributions in enhancing the overall control of the plant.

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## References

1. Kundur P (1981) A survey of utility experiences with power plant response during partial load rejection and system disturbances. IEEE Trans Power Apparatus Syst PAS-100(5): 2471–2475
2. Laubli F, Fenton FH (1971) The flexibility of the supercritical boiler as a partner in power system design and operation: part I. IEEE Trans Power Apparatus Syst PAS-90(4): 1719–1724

3. Laubli F, Fenton FH (1971) The flexibility of the supercritical boiler as a partner in power system design and operation: part II. IEEE Trans Power Apparatus Syst PAS-90(4): 1725–1733
4. Littman B, Chen TS (1966) Simulation of bull-run supercritical generation unit. IEEE Trans Power Apparatus Syst 85:711–722
5. Suzuki Y, Sik P, Uchida Y (1979) Simulation of once-through supercritical boiler. Simulation 33:181–193
6. Shinohara W, Kotischek DE (1995) A simplified model based supercritical power plant controller. In: Proceeding of the 35th IEEE Conference on Decision and Control, vol 4, pp 4486–4491
7. Inoue T, Taniguchi H, Ikeguchi Y (2000) A Model of Fossil Fueled Plant with Once-through Boiler for Power System Frequency Simulation Studies. IEEE Trans Power Syst 15(4): 1322–1328
8. Lee KY, Hoe JS, Hoffman JA, Sung HK, Won HJ (2007) Neural network based modeling of large scale power plant. IEEE Power Engineering Society General Meeting No (24–28):1–8
9. Mohamed O, Wang J, Guo S, Al-Duri B, Wei J (2010) Modelling study of supercritical power plant and parameter identification using genetic algorithms. In: Proceedings of the World Congress on Engineering II, pp 973–978
10. Adams J, Clark DR, Luis JR, Spanbaur JP (1965) Mathematical modelling of once-through boiler dynamics. IEEE Trans Power Apparatus Syst 84(4):146–156
11. Salisbury JK (1950) Steam turbines & their cycles. Wiley, New York
12. Yu Y-N (1983) Electric power system dynamics. Academic Press, New York
13. Ghaffari A, Chaibakhsh A (2007) A simulated model for a once through boiler by parameter adjustment based on genetic algorithms. Simul Model Pract Theory 15:1029–1051

# Chapter 2
# Sequential State Computation Using Discrete Modeling

**Dumitru Topan and Lucian Mandache**

**Abstract** In this paper we present a sequential computation method of the state vector, for pre-established time intervals or punctually. Based on discrete circuit models with direct or iterative companion diagrams, the proposed method is intended to a wide range of analog dynamic circuits: linear or nonlinear circuits with or without excess elements or magnetically coupled inductors. Feasibility, accessibility and advantages of applying this method are demonstrated by the enclosed example.

## 2.1 Introduction

The discretization of the circuit elements, followed by corresponding companion diagrams, leads to discrete circuit models associated to the analyzed analog circuits [1–3]. Using the Euler, trapezoidal or Gear approximations [4, 5], simple discretized models are generated, whose implementation leads to an auxiliary active resistive network. In this manner, the numerical computation of desired dynamic quantities becomes easier and faster. Considering the time constants of the circuit, the discretization time step can be adjusted for reaching the solution optimally, in terms of precision and computation time.

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The discrete modeling of nonlinear circuits assumes an iterative process too, that requires updating the parameters of the companion diagram at each iteration and each integration time step [5, 6]. If nonzero initial conditions exist, they are computed usually through a steady state analysis performed prior to the transient analysis.

The discrete modeling can be associated to the state variables approach [6, 7], as well as the modified nodal approach [5, 8], the analysis strategy being chosen in accordance with the circuit topology, the number of the energy storage circuit elements (capacitors and inductors) and the global size of the circuit.

The known computation algorithms based on the discrete modeling allow the sequential computation, step by step, along the whole analysis time, of the state vector or output vector directly [5, 9, 10]. In this paper, one proposes a method that allows computing the state vector punctually, at the moments considered significant for the dynamic evolution of the circuit. Thus, the sequential computation for pre-established time subdomains is allowed.

## 2.2 Modeling Through Companion Diagrams

The time domain analysis is performed for the time interval $[t_0, t_f]$, bounded by the initial moment $t_0$ and the final moment $t_f$. It can be discretized with the constant time step $h$, chosen sufficiently small in order to allow using the Euler, trapezoidal or Gear numerical integration algorithms [1–5]. One can choose $t_0 = 0$ and $t_f = wh$, where $w$ is a positive integer.

The analog circuit analysis using discrete models requires replacing each circuit element through a proper model according to its constitutive equations. In this way, if the Euler approximation is used, the discretization equations and the corresponding discrete circuit models associated to the energy storage circuit elements are shown in Table 2.1, for the time interval $[nh,\ (n+1)h]\,,\ h<w$.

The tree capacitor voltages $\mathbf{u}_C$ and the cotree inductor currents $\mathbf{i}_L$ [7, 8] are chosen as state quantities, assembled in the state vector $\mathbf{x}$. The currents $\mathbf{I}_C$ of the tree capacitors and the voltages across the cotree inductors $\mathbf{U}_L$ are complementary variables, assembled in the vector $\mathbf{X}$.

At the moment $t = nh$, the above named vectors are partitioned as:

$$\mathbf{x}^n = \begin{bmatrix} \mathbf{u}_C^n \\ \mathbf{i}_L^n \end{bmatrix}, \quad \mathbf{X}^n = \begin{bmatrix} \mathbf{I}_C^n \\ \mathbf{U}_L^n \end{bmatrix} \tag{2.1}$$

with obvious significances of the vectors $\mathbf{u}_C^n,\ \mathbf{i}_L^n,\ \mathbf{I}_C^n,\ \mathbf{U}_L^n$.

For the magnetically coupled inductors, the discretized equations and the companion diagram are shown in Table 2.1, where the following notations were used:

$$\begin{aligned} R_{11}^{n+1} &= \frac{L_{11}}{h}, \quad R_{12}^{n+1} = \frac{L_{12}}{h}, \quad e_1^{n+1} = \frac{L_{11}}{h} i_1^n + \frac{L_{12}}{h} i_2^n, \\ R_{22}^{n+1} &= \frac{L_{22}}{h}, \quad R_{21}^{n+1} = \frac{L_{21}}{h}, \quad e_2^{n+1} = \frac{L_{22}}{h} i_2^n + \frac{L_{21}}{h} i_1^n. \end{aligned} \tag{2.2}$$

**Table 2.1** Discrete modeling of the energy storage elements

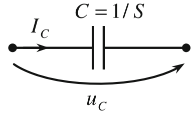
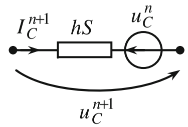
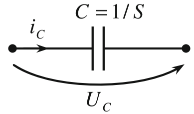
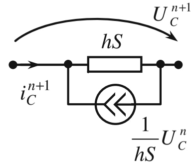
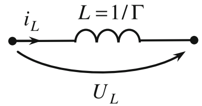
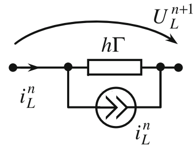
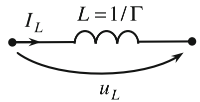
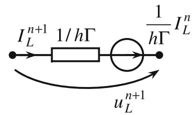
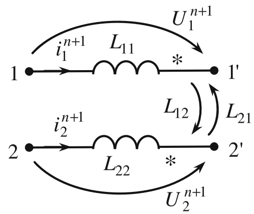
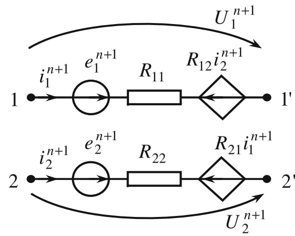
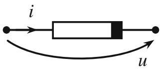
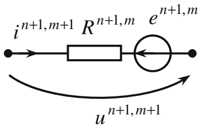
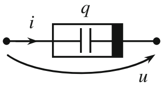
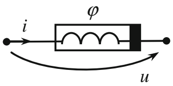
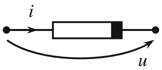
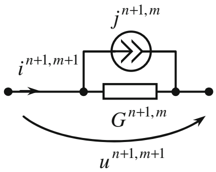
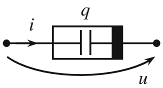
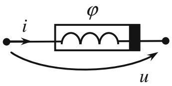
| Element | Symbol | Discretized expressions | Companion diagram |
|---|---|---|---|
| Tree capacitor |   | $u_C^{n+1} = u_C^n + hSI_C^{n+1}$ |   |
| Excess capacitor |   | $i_C^{n+1} = \frac{1}{hS}\left(U_C^{n+1} - U_C^n\right)$ |   |
| Cotree inductor |   | $i_L^{n+1} = i_L^n + h\Gamma U_L^{n+1}$ |   |
| Excess inductor |   | $u_L^{n+1} = \frac{1}{h\Gamma}\left(I_L^{n+1} - I_L^n\right)$ |   |
| Magnetically coupled inductor pair |   | $U_1^{n+1} = R_{11}i_1^{n+1} - R_{11}i_1^n$ $+R_{12}i_2^{n+1} - R_{12}i_2^n$ <br> $U_2^{n+1} = R_{21}i_1^{n+1} - R_{21}i_1^n$ $+R_{22}i_2^{n+1} - R_{22}i_2^n$ |   |

**Table 2.2** Iterative discrete modeling

| Element | Iterative dynamic parameter | Companion diagram | Notations in the companion diagram |
|---|---|---|---|
|   $u = \hat{u}(i)$ | $R^{n+1,m} = \left(\frac{\partial u}{\partial i}\right)_{i=i^{n+1,m}}$ |   | $R^{n+1,m} = R^{n+1,m}$<br>$e^{n+1,m} = u^{n+1,m} - R^{n+1,m} \cdot i^{n+1,m}$ |
|   $u = \hat{u}(q)$ | $C^{n+1,m} = \left(\frac{\partial q}{\partial u}\right)_{u=u^{n+1,m}}$ | | $R^{n+1,m} = hS^{n+1,m}$<br>$e^{n+1,m} = u^{n+1,m} - hS^{n+1,m} \cdot i^{n+1,m}$ |
|   $\varphi = \hat{\varphi}(i)$ | $L^{n+1,m} = \left(\frac{\partial \varphi}{\partial i}\right)_{i=i^{n+1,m}}$ | | $R^{n+1,m} = \frac{1}{h} L^{n+1,m}$<br>$e^{n+1,m} = u^{n+1,m} - \frac{1}{h} L^{n+1,m} \cdot i^{n+1,m}$ |
|   $i = \hat{i}(u)$ | $G^{n+1,m} = \left(\frac{\partial i}{\partial u}\right)_{u=u^{n+1,m}}$ |   | $G^{n+1,m} = G^{n+1,m}$<br>$j^{n+1,m} = i^{n+1,m} - G^{n+1,m} \cdot u^{n+1,m}$ |
|   $q = \hat{q}(u)$ | $S^{n+1,m} = \left(\frac{\partial u}{\partial q}\right)_{q=q^{n+1,m}}$ | | $G^{n+1,m} = \frac{1}{h} C^{n+1,m}$<br>$j^{n+1,m} = i^{n+1,m} - \frac{1}{h} C^{n+1,m} \cdot u^{n+1,m}$ |
|   $i = \hat{i}(\varphi)$ | $\Gamma^{n+1,m} = \left(\frac{\partial i}{\partial \varphi}\right)_{\varphi=\varphi^{n+1,m}}$ | | $G^{n+1,m} = h\Gamma^{n+1,m}$<br>$j^{n+1,m} = i^{n+1,m} - h\Gamma^{n+1,m} \cdot u^{n+1,m}$ |

For nonlinear circuits, the state variable computation at the moment $t = (n+1)h$ requires an iterative process that converges towards the exact solution [4, 5]. A second upper index corresponds to the iteration order (see Table 2.2). Similar results to those of Tables 2.1 and 2.2 can be obtained using the trapezoidal [5, 11] or Gear integration rule [4, 5].

## 2.3 Sequential and Punctual State Computation

The treatment with discretized models assumes substituting the circuit elements with companion diagrams, which consist in a resistive model diagram. It allows the sequential computation of the circuit solution.

### 2.3.1 Circuits Without Excess Elements

If the given circuit does not contain capacitor loops nor inductor cutsets [7, 8], the discretization expressions associated to the energy storage elements (Table 2.1, lines 1 and 3), using the notations (2.1), one obtains

$$\mathbf{x}^{n+1} = \mathbf{x}^n + h \begin{bmatrix} \mathbf{S} & 0 \\ 0 & \Gamma \end{bmatrix} \mathbf{X}^{n+1}, \tag{2.3}$$

where $\mathbf{S}$ is the diagonal matrix of capacitor elastances and $\Gamma$ is the matrix of inductor reciprocal inductances.

Starting from the companion resistive diagram, the complementary variables are obtained as output quantities [5, 10, 11] of the circuit

$$\mathbf{X}^{n+1} = \mathbf{E}\,\mathbf{x}^n + \mathbf{F}\,\mathbf{u}^{n+1}, \tag{2.4}$$

where $\mathbf{E}$ and $\mathbf{F}$ are transmittance matrices, and $\mathbf{u}^{n+1}$ is the vector of input quantities [7, 8] at the moment $t = (n+1)h$.

From (2.3) and (2.4) one obtains an equation that allows computing the state vector sequentially, starting from its initial value $\mathbf{x}^0 = \mathbf{x}(0)$ until the final value $\mathbf{x}^w = \mathbf{x}(wh)$:

$$\mathbf{x}^{n+1} = \mathbf{M}\,\mathbf{x}^n + \mathbf{N}\,\mathbf{u}^{n+1}, \tag{2.5}$$

where

$$\mathbf{M} = 1 + h \begin{bmatrix} \mathbf{S} & 0 \\ 0 & \Gamma \end{bmatrix} \mathbf{E}, \tag{2.6}$$

**1** being the identity matrix, and

$$\mathbf{N} = h \begin{bmatrix} \mathbf{S} & 0 \\ 0 & \Gamma \end{bmatrix} \mathbf{F}. \tag{2.7}$$

Starting from Eq. 2.5, through mathematical induction, the useful formula is obtained as

$$\mathbf{x}^n = \mathbf{M}^n \mathbf{x}^0 + \sum_{i=1}^{n} \mathbf{M}^{n-k} \mathbf{N}\,\mathbf{u}^k, \tag{2.8}$$

where the upper indexes of the matrix $\mathbf{M}$ are integer power exponents. The formula (2.8) allows the punctual computation of the state vector at any moment $t = nh$, if the initial conditions of the circuit and the excitation quantities are known.

If a particular solution $\mathbf{x}_p(t)$ of the state equation exists, it significantly simplifies the computation of the general solution $\mathbf{x}(t)$. Using the Euler numerical integration method, one obtains [5]:

$$\mathbf{x}^{n+1} = \mathbf{M}\left(\mathbf{x}^n - \mathbf{x}_p^n\right) + \mathbf{x}_p^{n+1}. \tag{2.9}$$

The sequentially computation of the state vector implies the priory construction of the matrix E, according to Eqs. 2.6 and 2.9. This action requires analyzing an auxiliary circuit obtained by setting all independent sources to zero in the given circuit.

Starting from Eq. 2.9, the expression

$$\mathbf{x}^n = \mathbf{M}^n\left(\mathbf{x}^0 - \mathbf{x}_p^0\right) + \mathbf{x}_p^n \tag{2.10}$$

allows the punctual computation of the state vector.

### *2.3.2 Circuits with Excess Elements*

The excess capacitor voltages [8, 11], assembled in the vector $\mathbf{U}_C$, as well as the excess inductor currents [5, 7, 8], assembled in the vector $\mathbf{I}_L$, can be expressed in terms of the state variables and excitation quantities, at the moment $t = nh$:

$$\begin{bmatrix} \mathbf{U}_C^n \\ \mathbf{I}_L^n \end{bmatrix} = \begin{bmatrix} \mathbf{K}_1 & 0 \\ 0 & \mathbf{K}_2 \end{bmatrix} \mathbf{x}^n + \begin{bmatrix} \mathbf{K}_1' & 0 \\ 0 & \mathbf{K}_2' \end{bmatrix} \mathbf{u}^n, \tag{2.11}$$

where the matrices $\mathbf{K}_1$, $\mathbf{K}_1'$ and $\mathbf{K}_2$, $\mathbf{K}_2'$ contain voltage and current ratios respectively.

Using the Table 2.1, the companion diagram associated to the analyzed circuit can be obtained, whence the complementary quantities are given by:

$$\mathbf{X}^{n+1} = \mathbf{E}\,\mathbf{x}^n + \mathbf{E}_1 \begin{bmatrix} \mathbf{U}_C^n \\ \mathbf{I}_L^n \end{bmatrix} + \mathbf{F}\,\mathbf{u}^n, \tag{2.12}$$

the matrices $\mathbf{E}$, $\mathbf{E}_1$ and $\mathbf{F}$ containing transmittance coefficients.

Considering Eqs. 2.11 and 2.12, the recurrence expression is obtained from (2.5), allowing the sequential computation of the state vector:

$$\mathbf{x}^{n+1} = \mathbf{M}\,\mathbf{x}^n + \mathbf{N}\,\mathbf{u}^{n+1} + \mathbf{N}_1\,\mathbf{u}^n, \tag{2.13}$$

where

$$\begin{aligned} &\mathbf{M} = 1 + h\begin{bmatrix} \mathbf{S} & 0 \\ 0 & \Gamma \end{bmatrix}(\mathbf{E} + \mathbf{E}_1\mathbf{K}), \\ &\mathbf{N} = h\begin{bmatrix} \mathbf{S} & 0 \\ 0 & \Gamma \end{bmatrix}\mathbf{F}, \quad \mathbf{N}_1 = h\begin{bmatrix} \mathbf{S} & 0 \\ 0 & \Gamma \end{bmatrix}\mathbf{E}_1\mathbf{K}', \\ &\mathbf{K} = \begin{bmatrix} \mathbf{K}_1 & 0 \\ 0 & \mathbf{K}_2 \end{bmatrix}, \quad \mathbf{K}' = \begin{bmatrix} \mathbf{K}_1' & 0 \\ 0 & \mathbf{K}_2' \end{bmatrix}. \end{aligned} \tag{2.14}$$