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Daniel Lenz
Florian Sobieczky
Wolfgang Woess
Editors

Random Walks, Boundaries and Spectra

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Daniel Lenz
Florian Sobieczky
Wolfgang Woess
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Editors
Daniel Lenz
Mathematisches Institut
Universität Jena
Ernst-Abbe-Platz 2
D-07737 Jena
Germany
daniel.lenz@uni-jena.de

Florian Sobieczky
Institut für Mathematik C
TU Graz
Steyrergasse 30
A-8010 Graz
Austria
sobieczky@tugraz.at

Wolfgang Woess
Institut für Mathematik C
TU Graz
Steyrergasse 30
A-8010 Graz
Austria
woess@tugraz.at

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Random Walks on random subgraphs of transitive graphs

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Preface

This book contains the joint proceedings of the workshop on **Boundaries** that took place in Graz, from June 29–July 3, and the **Alp-Workshop** that was held immediately afterwards in Sankt Kathrein am Offenegg, on the weekend July 4–5, 2009.

The two events were dedicated to related subjects.

The aim of the **Boundaries** workshop was to bring together mathematicians working on groups, graphs, manifolds, etc., in the context of probability (random walks, Brownian motion), harmonic analysis, potential theory, ergodic theory, geometric group theory and related topics. The title indicates a central topic but was not to be considered the exclusive theme.

The scientific committee of the meeting consisted of Tatiana Nagnibeda-Smirnova (Geneva), Christophe Pittet (Marseille), Hamish Short (Marseille), and Wolfgang Woess (Graz).

The local organisation rested on the shoulders of Ecaterina Sava and Wolfgang Woess at Graz University of Technology in the capital of Styria, southeastern province of Austria.

Three special guests were particularly featured in view of their "milestone birthdays" taking place in 2009:

- Donald I. Cartwright (Sydney; 60th birthday)
- Vadim A. Kaimanovich (Bremen; 50th birthday)
- Massimo Picardello (Rome; 60th birthday)

Each of these three has given substantial contributions to the mathematical subject of the workshop, and to each of them, a half-day session was dedicated, featuring in particular their own (respective) invited talks. In the present volume, we display their lists of publications (state of September, 2010).

The **Alp-Workshop 2009** was devoted to "Spectral and probabilistic properties of random walks on random graphs". The aim was a discussion between experts from spectral theory, ergodic theory and probability theory about the special topics of random walk theory in which the methods from group theory and harmonic analysis fail: Discrete structures with much irregularity, such as Percolation, Random Graphs, or Branching Processes were the main focus. Instead of a detailed discussion of each talk we refer to the attached programme. During the first afternoon-session, there were six twenty-minutes talks by young researchers of whom several have contributed to the proceedings.

The Alp-Workshop was organised by Florian Sobieczky with the budget of project P18703 ("Random Subgraphs of Transitive Graphs") of the Austrian Science Foundation (FWF). Furthermore, the main part of the publication cost of these proceedings was carried by the budget of this research project.

The "Almenland" in the mountains east of Graz provided a picturesque environment for the interdisciplinary discussion about random walks. Its remoteness allowed inviting more people with the given budget while keeping a high standard of the venue.

The editing of the proceedings contributed by the Alp-Workshop's participants was undertaken by Daniel Lenz and Florian Sobieczky. The contributions from the Boundaries-Workshop were edited by Wolgang Woess. All articles underwent anonymous refereeing by experts from the respective field.

We would like to thank everyone who was directly or indirectly involved in helping to organise these meetings.

This volume is dedicated to

Donald I. Cartwright

Massimo A. Picardello

Vadim A. Kaimanovich

October 2010,

Daniel Lenz
Florian Sobieczky
Wolfgang Woess

Programme of the Workshop on "Boundaries"

June 29th (Mon.)

09:00–09:10	Opening
09:10–10:10	**Francois Ledrappier**, University of Notre Dame *Linear drift for the Brownian motion on covers*
10:10–10:40	Coffee & Registration
10:40–11:10	**Martin Dunwoody**, University of Southampton *An inaccessible graph*
11:20–1150	**Panos Papazoglou**, University of Athens *Topology of boundaries and splittings*
12:00–12:20	**Barbara Bobikau**, University of Wroclaw *Spectral properties of a class of random walks on locally finite groups*
12:20–14:30	Lunch
14:30–15:20	**Massimo Picardello**, Tor Vergata University in Rome *Harmonic functions on homogeneous trees and buildings*
15:30–16:00	**Sara Brofferio**, University of Paris-Sud 11 *Poisson boundary of matrix groups with rational coefficients*
16:10–16:40	Coffee
16:40–17:30	**Yves Guivarc'h**, University of Rennes *Random walk in a random medium on Z, and random walks on homogeneous spaces*
17:40–18:00	**Daniele D'Angeli**, University of Geneva *The boundary action of the Basilica group*

June 30th (Tue.)

09:30–09:50	**Tim Riley**, Cornell University *How wild can a group with a quadratic Dehn function be?*
10:00–10:30	Coffee
10:40–11:11	**Anton Thalmaier**, University of Luxembourg *The Poisson boundary of certain Cartan-Hadamard manifolds of unbounded curvature*
11:20–11:50	**Alexander Gnedin**, Utrecht University *Boundaries of the generalised Pascal triangles and larger graded graphs*
12:00–12:20	**Jeremy Macdonald**, McGill University *Compressed words and automorphisms in fully residually free groups*
12:20–14:30	Lunch
14:30–15:20	**Tim Steger**, University of Sassari *Background on fake planes*
15:30–16:00	**Jean Lécureux**, Claude Bernard University Lyon 1 *Combinatorial boundaries of buildings*

16:10–16:40	Coffee
16:40–17:30	**Donald Cartwright**, University of Sidney *The* 50 *fake projective planes*
17:40–18:00	**Bernhard Krön**, University of Vienna *Vertex cuts, ends and group splittings*

July 1st (Wed.)

09:00–09:50	**Anna Erschler**, University of Paris-Sud 11 *Boundaries of amenable groups*
10:00–10:50	**Poster Session** & Coffee Poster: Elisabetta Candellero, Lorenz Gilch, Motoko Kotani, Jeremy Macdonald, Sebastian Müller, Svetla Vassileva
10:50–11:20	**Matthias Keller**, Universität Jena *Heat transfer to the boundary on discrete graphs*
11:30–12:00	**Erin Pearse**, University of Iowa & University of Oklahoma *Resistance analysis of infinite networks*
Afternoon	Excursion

July 2nd (Thu.)

09:00–09:50	**James Parkinson**, University of Sydney *Random walks on p-adic groups and affine buildings*
10:00–10:30	Coffee
10:40–11:10	**Agelos Georgakopoulos**, Graz University of Technology *Uniqueness of currents in an electrical network of finite total resistance*
11:20–11:50	**Jörg Schmeling**, Lund University *Large dimension of limit sets of Kleinian groups and transience of critical random walks*
12:00–12:20	**Riddhi Shah**, Jawaharlal Nehru University *Distal actions on locally compact groups*
12:20–14:30	Lunch
14:30–15:20	**Vadim Kaimanovich**, University of Ottawa *Random graphs, stochastic homogenization and equivalence relations*
15:30–16:00	**Alexander Bendikov**, University of Wroclaw *On a class of random walks on groups with infinite number of generators*
16:00–16:40	Coffee
16:40–17:30	**Volodymyr Nekrashevych**, Texas A& M University *Hyperbolic duality*
17:40–18:00	**Fréderic Mathéus**, LMAM University of South-Brittany *Poisson boundary of free-by-cyclic groups*

July 3rd, (Fri.)

09:00–09:50	**Klaus Schmidt**, University of Vienna *Sandpiles and the harmonic model*
10:00–10:40	Coffee
10:40–11:10	**Tatiana Smirnova-Nagnibeda**, University of Geneva *Sandpiles and self-similar groups*
11:20–11:50	**Markus Neuhauser**, RWTH Aachen *Further examples to a question of Atiyah*
11:50–13:30	Lunch
13:30–14:00	**Michael Björklund**, Hebrew University *Sharp sumset inequalities for Bohr sets*
14:10–15:00	**Anatoly Vershik**, St.Petersburg State University *Adjoint dynamics to a question of Atiyah*

Programme of the Alp-Workshop 2009

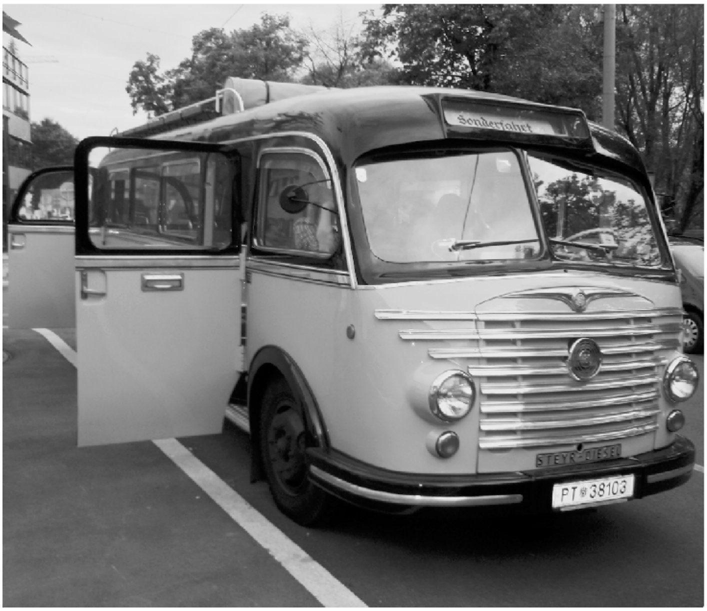
July 4th (Sat.)

09:15–09:30	Welcome
09:30–10:15	**Christoph Pittet**, University of Aix-Marseille 1 *Return probabilities and spectral distribution of Laplace operators*
10:20–11:05	**Peter Müller**, Ludwigs Maximilians University Munich *Ergodic properties of randomly coloured aperiodic point sets*
11:05–11:20	Coffee
11:20–12:05	**Tatyana Turova**, Lund University *Asymptotic size of the largest cluster in inhomogeneous random graphs: sub-critical and critical phases*
12:10–12:55	**Vadim Kaimanovich**, Jacobs University Bremen *Stochastic homogenization of graphs: case studies*
12:55–14:00	Lunch
14:00–16:30	**Short Talks-Session** & Coffee Wolfgang Spitzer, Bernt Metzger, Radoslaw Wojciechowski, Matthias Keller, Sebastian Müller, Erin Pearse
Evening	**Hike and Dinner** at Mountain Cabin

July 5th (Sun.)

10:00–10:45	**Daniel Lenz**, Universität Jena *Amenability of Horocyclic Products of uniformly growing trees*
10:45–11:00	Coffee
11:00–11:45	**Tatiana Smirnova-Nagnibeda**, Geneva University *Amenability and percolation*

11:50–12:35	**Jörg Schmeling**, Lund University *Random trees generated by a dynamical system and the structure of typical orbits*
12:35–14:00	Lunch
14:00–14:45	**Franz Lehner**, Graz University of Technology *On the Eigenspaces of Lamplighter Random Walks and Percolation Clusters on Graphs*
14:50–15:55	**Poster-Session** & Coffee Erin Pearse, Lorenz Gilch, Ecaterina Sava, Wilfried Huss, Seon Hee Lim, Michael Matter, Uta Freiberg, Elisabetta Candellero
16:00–16:45	**Peter Mörters**, University of Bath *Simultaneous multifractal analysis of branching and visibility measure on a Galton-Watson tree*
17:00–17:45	**Ivan Veselić**, TU Chemnitz *Percolation clusters on Caley graphs and their spectra*
18:00–18:45	**Tyll Krüger, Rainer Siegmund-Schultze**, TU Berlin *Epidemic processes on networks and generalisations*



A **Steyr 480a** “Postbus” waiting for its passengers to board before taking them to St. Kathrein am Offenegg, the venue of the Alp-Workshop 2009.

Donald I. Cartwright

Research Publications

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Massimo A. Picardello

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Vadim A. Kaimanovich

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An Inaccessible Graph

M.J. Dunwoody

Abstract. An inaccessible, vertex transitive, locally finite graph is described. This graph is not quasi-isometric to a Cayley graph.

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Keywords. Ends of graphs, quasi-isometry.

1. Introduction

Let X be a locally finite connected graph. A *ray* is a sequence of distinct vertices $v_0, v_1, \dots$ such that v_i is adjacent to v_{i+1} for each $i = 1, 2, \dots$. Obviously for a ray to exist, the graph X has to be infinite. For any two vertices $u, v \in VX$ let $d(u, v)$ be the length of a shortest path joining u, v.

We say that two rays R, R' belong to the same *end* ω, if for no finite subset F of VX or EX do R_1 and R_2 eventually lie in distinct components of $X \setminus F$. We define $\mathcal{E}(X)$ to be the set of ends of X.

We say that ω is *thin* if it does not contain infinitely many vertex disjoint rays. As in [16] the end ω is said to be *thick* if it is not thin.

In their nice paper [16] Thomassen and Woess define an accessible graph. A graph X is *accessible* if there is some natural number k such that for any two ends ω_1 and ω_2 of X, there is a set F of at most k vertices in X such that F separates ω_1 and ω_2, i.e., removing F from X disconnects the graph in such a way that rays R_1, R_2 of ω_1, ω_2 respectively eventually lie in distinct components of $X \setminus F$.

A finitely generated group G is said to have more than one end ($e(G) > 1$) if its Cayley graph $X(G, S)$ with respect to a finite generating set S has more than one end. This property is independent of the generating set S chosen. Stallings [14] showed that if $e(G) > 1$ then G *splits* over a finite subgroup, i.e., either $G = A *_C B$ where C is finite, $C \neq A, C \neq B$ or G is an HNN extension $G = A*_C = \langle A, t | t^{-1}ct = \theta(c)\rangle$, where C is finite, $C \leq A$ and $\theta : C \to A$ is an injective homomorphism. A group is *accessible* if the process of successively factorizing factors that split in a decomposition of G eventually terminates with factors that are finite or one ended.

Thomassen and Woess show that the Cayley graph of a finitely generated group G is accessible if and only if G is accessible. In [5, 6] I have given examples of inaccessible groups, and so not every locally finite connected graph is accessible.

Let ω be an end of X. As in [16], p. 259 define $k(\omega)$ to be the smallest integer k such that ω can be separated from any other end by at most k vertices. If this number does not exist, put $k(\omega) = \infty$.

Thomassen and Woess show that X is accessible if and only if $k(\omega) < \infty$ for every end ω. We say that an end ω is *special* if $k(\omega) = \infty$.

In this paper we construct a locally finite, connected, inaccessible, vertex transitive graph X. The property of being inaccessible is invariant under quasi-isometry. If X, Y are graphs, then a quasi-isometry $\theta : X \to Y$ induces a bijection $\mathcal{E}(\theta) : \mathcal{E}(X) \to \mathcal{E}(Y)$ which takes thick ends to thick ends, and special ends to special ends. One can put a topology on $\mathcal{E}(X)$ in a natural way. The map $\mathcal{E}(\theta)$ is then a homeomorphism.

Woess asked in [17, 15] if every vertex transitive, locally finite graph is quasi-isometric to a Cayley graph. It was shown in [11, 12] that the Diestel-Leader graph $DL(m, n), m \neq n$ (see [3] or [17]) is not quasi-isometric to a Cayley graph, answering the question of Woess. It is shown here that the graph X is another example. I originally thought that X was hyperbolic, and the fact that X was not quasi-isometric to a Cayley graph then followed because a hyperbolic group is finitely presented, and would therefore have an accessible Cayley graph by [4]. However there are arbitrarily large cycles in X for which the distance apart of two vertices in the cycle is the same as that in X. This cannot happen in a hyperbolic graph. It seems likely that a hyperbolic graph must be accessible.

The vertex transitive graph X we construct is based on a construction in [7]. In that paper, Mary Jones and I construct a finitely generated group G for which $G \cong A *_C G$ where C is infinite cyclic. The vertex set of the graph X is the set of left cosets of D in G, where D has index 2 in C. One could take the vertex set of X to be the left cosets of A or C as they are commensurable with D. In fact it is easier to work with a G-graph Y quasi-isometric to X, in which there are two orbits of vertices for the action of G on Y.

In general, if a group G is the commensurizer of a subgroup H, and G is generated by $H \cup S$, then one can construct a vertex transitive, connected graph, in which the vertices are the cosets of H, and there are edges (H, sH) for each $s \in S$. If G actually normalizes H, then this graph is a Cayley graph for G/H. Conversely if X is a connected, vertex transitive, locally finite graph and H is the stabilizer of a vertex v, then G is the commensurizer of H and G is generated by $H \cup S$, where S is any subset of G with the property that for each u adjacent to v there is an $s \in S$ such that $sv = u$.

The graph Y has an orbit of cut points, i.e., vertices whose removal disconnects the graph. It is well known that cut points in a graph give rise to a tree decomposition. This is described – for example – in [10], in which the theory of structure trees is extended to graphs that can be disconnected by removing finitely many vertices rather than finitely many edges. The cut point tree T for Y has two orbits of vertices under G. One orbit corresponds to the set of 2-blocks, where each 2-block is a maximal 2-connected subgraph, and the other orbit corresponds to the cut points. It is then shown that after a subdivision and two folding operations, each of which is a quasi-isometry, and removing spikes (a spike is an edge with a vertex of degree one) each 2-block becomes a graph isomorphic to Y. Thus the graph Y has a self-similarity property that comes from the fact that $G \cong A *_C G$ where C is infinite cyclic. One would not expect this to happen in a Cayley graph, as it is not possible that for a finitely generated group G to be isomorphic to $A *_C G$ where C is finite. This follows from a result of Linnell [13], which indicates that in a process of successively factorizing factors that split in a decomposition of an inaccessilbe group G, the size of the finite groups over which the factors split must increase.

Thus after carrying out the subdivision and folding operations, the graph $Y = Y_1$ becomes a graph Y_2 which has a single orbit of disconnecting edges. Removing (the interior of) all these edges will give a single orbit of points each with stabilizer a conjugate of A, and a second orbit, consisting of 2-blocks each of which is isomorphic to Y, with stabilizer conjugate to the subgroup of G which is the second factor in the decomposition $G \cong A *_C G$. If we repeat this process $n-1$ times, then we a obtain a graph Y_n which has $n-1$ orbits of disconnecting edges. Removing these edges produces $n-1$ orbits of vertices each of which has finite stabilizer, isomorphic to A, and a single orbit of 2-blocks each of which is isomorphic to Y. Let B_n be one of these blocks. The graph Y has an orbit of subgraphs each of which is a trivalent tree. Let Z be a particular trivalent subtree of Y. Although the folding operations do involve folding Z, the result of the operations is another trivalent tree. We will see that any two rays in Z represent a particular special end ω of Y. There will also be uncountably many special ends that do not correspond to a translate of Z. A ray representing a special end must eventually lie in a translate of B_n, since otherwise it will represent a thin end. However the initial number x_n of points in the ray outside a translate of B_n may tend to infinity with n. There will be uncountably many such special ends. If the ray eventually ends up in a translate of Z, then x_n is bounded, since each translate of Z lies in a translate of B_n. Since each translate of B_n contains a translate of Z, the orbit of ω is dense in the space of special ends.

We will show that in a Cayley graph, if there is a countable set of special ends which is dense in the subspace of all special ends, then there must be a special end corresponding to a 1-ended subgraph. There is no special end of Y corresponding to a 1-ended subgraph, and so the graph Y cannot be quasi-isometric to a Cayley graph.

As it is important in our construction, we repeat the description of G below. In another paper [8], Mary Jones and I went on to construct a finitely generated group G_1 for which $G_1 \cong G_1 *_{C_1} G_1$ with C_1 infinite cyclic. It might be expected that the coset graph X_1 of C_1 in G_1 has similar properties to X. This will not be the case. Although X_1 is inaccessible and locally finite, it is quasi-isometric to a Cayley graph. This is because C_1 contains a central subgroup Z as a subgroup of finite index. Then X_1 is quasi-isometric to the Cayley graph of G_1/Z.