Gert-Martin Greuel
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Sebastià Xambó-Descamps *Editors*

Singularities, Algebraic Geometry, Commutative Algebra, and Related Topics

Festschrift for Antonio Campillo on the Occasion of his 65th Birthday

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Preface

The decision to prepare a volume of contributions in honor of Antonio Campillo was taken at the University of Valladolid (Spain) during 19–23 June 2017 on the occasion of the 4th Joint Congress of the Spanish and Mexican Mathematical Societies.

The Congress paid homage to Campillo by including two plenary lectures to present his contributions to mathematics and his service and leadership in a variety of institutions, two special sessions on Singularities and Algebraic Geometry, and a special scientific day to celebrate more closely his mathematical work.

The announcement of this book project was sent to the list of all participants at the Congress enlarged with the names of other colleagues with whom Campillo has had significant interactions in his professional career.

This volume collects the refereed papers that were submitted following that announcement. After the opening article, which is meant to be a first approach to Campillo's biography, we have grouped the papers in five categories: Singularities (ten papers), Algebraic Geometry (five), Commutative Algebra (five), Algebraic Codes (three), and Other Topics (three). The variety of topics correlates well with the very broad research interests of Antonio Campillo.

The grouping is of course imperfect, as some papers would easily fit in more than one category, but we hope that the classification respects nevertheless the main area in which each paper is ascribed. Within each category, papers that explicitly connect with published work of Campillo are placed first, and these, as well as the others, are ordered alphabetically by the family name of the first author.

We are most grateful to all the authors (two per paper on average) for their positive response, their patience with the long refereeing process, and above all, for their scientific contribution to the volume. Thanks also to the generosity and keen judgments of the many anonymous referees.

Going back to the beginning, we are thankful to the Mexican and Spanish mathematical societies for having embraced the idea of paying tribute to Antonio Campillo on the occasion of their 4th joint meeting, and to the University of Valladolid, in particular to its Mathematical Institute (IMUVA), for providing excellent organization for all activities in that week.

Antonio Campillo has been our colleague and friend for many years, in a variety of circumstances, and it is a pleasant duty to express our acknowledgment to him for his productive career, his dedication to causes that promote the welfare of institutions and professional communities, his vision regarding academic progress and research organization, his engagement in increasing public awareness of mathematics, and of course for all his achievements in mathematics.

We wish you, Antonio, a most happy 65th birthday and that your career may continue to be, for many years to come, as productive and inspirational as it has been hitherto.

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Acronyms

AMS	American Mathematical Society
ANECA	Agencia Nacional de Evaluación de la Calidad y Acreditación
ANEP	Agencia Nacional de Evaluación y Prospective
BBVA	Banco de Bilbao Vizcaya Argentaria
BMS	Belgium Mathematical Society, SMB
CDM	Conferencia de Decanos de Matemáticas
CEMAT	Comité Español de Matemáticas
CIMPA	Centre International de Mathématiques Pures et Appliquées
EMS	European Mathematical Society
FESPM	Federación Española de Sociedades de Profesores de Matemáticas
FPI	Formación de Personal Investigador
HNE	Hamburger-Noether expansion
ICIAM	International Congress on Industrial and Applied Mathematics
IMU	International Mathematical Union
MCyT	Ministerio de Ciencia y Tecnología
MO	Mathematical Olympiad
MPE	Mathematics of Planet Earth
MR	Mathematical Reviews
MSP	Minero Siderúrgica de Ponferrada
RSEF	Real Sociedad Española de Física
RSEQ	Real Sociedad Española de Química
RSME	Real Sociedad Matemática Española
SCM	Societat Catalana de Matemàtiques
SEIO	Sociedad de Estadística e Investigación Operativa
SEMA	Sociedad Española de Matemática Aplicada
SEN	Sociedad Española de Neurociencias
SIMAI	Società Italiana di Matematica Applicata e Industriale
SMB	Société Mathématique de Belgique
SML	Société Mathématique du Luxembourg
SMM	Sociedad Matemática Mexicana
UCM	Universidad Complutense de Madrid

UMI	Unione Matematica Italiana
US	Universidad de Sevilla
UVA	Universidad de Valladolid

Antonio Campillo

A Portrayal of His Life and Work

Francisco Monserrat and Sebastià Xambó-Descamps

Abstract The main purpose is to present a biographical portrayal of Antonio Campillo López. In addition to the most relevant aspects of his life and academic career, we offer a panoramic view of his scientific work in fields such as singularity theory, commutative algebra, algebraic geometry, or coding theory. We also stress that his endeavors have been closely linked to an important work of research training, from which a thriving school has emerged. Finally, his vision and influence will be considered through his numerous institutional and scientific policy responsibilities at all levels.

1 Prelude

The University of Valladolid (UVA) hosted the 4th Joint Meeting of the Real Sociedad Matemática Española (RSME) and the Sociedad Matemática Mexicana (SMM) from June 19 to 22, 2017. The meeting, and a special scientific day on June 23, had the character of a tribute to Antonio Campillo. This tribute was acknowledged from the table of the opening ceremony[1] and further stressed by the fact that the opening plenary lecture had been planned to offer an outline of his

[1] Constituted by Daniel Miguel San José, rector of the UVA; Pablo Raphael de la Madrid, Director of the Cultural Institute of the Embassy of Mexico in Spain; Gelasio Salazar Anaya, President of the SMM; Francisco Marcellán Español, President of the RSME; and by Edgar Martínez Moro, President of the Organizing Committee of the meeting.

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life and achievements.[2] To a good extend, this paper is an elaboration of that lecture aimed at providing a wider and deeper coverage of several aspects, most particularly of his mathematical research.

The tribute befitted the occasion on many counts, as we will point out in due time, but especially because Antonio Campillo was the promoter, in the two terms as President of the RSME (2009–2012 and 2012–2016), of the first four joint meetings of the two societies: Oaxaca (2009), Torremolinos (2012), Zacatecas (2014), and Valladolid (2017).

Let us also mention, for the effect it has had on the the gestation of this work, and also on the scientific relations between the two mathematical communities, that one of the most outstanding projects Antonio Campillo set out in his first mandate was the celebration of the (first) centenary of the RSME (2011). It entailed a vast mobilization in which many people were involved in a variety of initiatives, and one of them was the launching of the *RSME-Universia Gallery of Mathematics, Science and Technology* (colloquially ArbolMat).[3] The mission of this initiative is "to publish scientific profiles of personalities from Latin America, including Spain and Portugal, distinguished by their eminent research in Mathematics or in the use of Mathematics (in science, technology, economics, ...), for their high generativity and influence, and for their ability to inspire younger generations." Fifteen profiles were published in November 2011, 24 in November 2014, and 29 in January 2017. Among the latter there was Antonio Campillo's profile,[4] whose inclusion was decided after finishing in November 2015 his second term as President of the RSME. From the Mexican side, the profiles that have been produced so far are those of José Antonio de la Peña, Xavier Gómez Mont, Santiago López de Medrano, and Raimundo Bautista Ramos, and at the time of this writing those of Francisco González Acuña and Alberto Verjovsky are in the pipeline.

The picture for the ArbolMat portal (Fig. 1) was taken on the occasion of the joint meeting (another really outstanding organizational feat) of the Unione Matematica Italinana (UMI), RSME, Catalan Mathematical Society (SCM), Sociedad Española de Matemática Aplicada (SEMA), and Società Italiana per la Matematica Applicata e Industriale (SIMAI), which was held at the Universidad del País Vasco/Euskal Herriko Unibertsitatea (Bilbao) from 30 June to 4 July 2014. We believe that it captures and transmits, at least to those that know him, characteristic features of his scientific, academic, institutional and organizational personality.

Antonio Campillo was born on 26 November, 1953, in the industrious Ponferrada, the capital of El Bierzo, a region in the North-Western of the León province, which today belongs to the Autonomous Community of Castilla-León, Spain. Founded in the eleventh century, today Ponferrada is the largest city crossed by the

[2] "Antonio Campillo, bosquejos sobre su vida y obra", by S. Xambó (https://mat-web.upc.edu/people/sebastia.xambo/Bios/AC/2017-Xambo--s-AC.pdf)

[3] http://www.arbolmat.com/. Since February 2018, however, ArbolMat belongs solely to RSME and its name is *RSME Gallery of Mathematics, Science and Technology*.

[4] http://www.arbolmat.com/antonio-campillo/

Fig. 1 A. Campillo's picture at the ArbolMat portal. With permission from the author, S. Xambó

rapid Sil River, and the last major town on the Camino de Santiago before reaching Santiago de Compostela. To facilitate the crossing of the river to Santiago pilgrims, the initial wooden bridge was commissioned to be fortified with iron by Osmundo, Bishop of Astorga (1082–1098), and hence the name *Pons Ferrata*, or *Iron Bridge*. León itself is the old *Legio VI* of the Romans, for whom the whole area turned out to be the largest open mining site across the Empire, particularly the gold mines of Las Médulas (now a Unesco World Heritage Site), and an excellent producer of many other goods, notably food and wine.

The modern Ponferrada also owes much to mining, but this time of high quality coal (anthracite) and iron mines that began to be exploited after World War I by the mining and steel company MSP (Minero Siderùrgica de Ponferrada). The MSP built a railway that provided services beyond its use for ore transportation, and a thermal power plant (Central Térmica) that was powered by coal extracted from the mines and popularly known as the "Light Factory" (La Fábrica de Luz) because it supplied electrical power to the city. At its height in over half a century ago, the MSP provided employments and services, including housing for workers, and a school, but two decades later it underwent a deep crisis that led to it being absorbed by the Asturian steel industry. Early in this century, La Fábrica de Luz was rehabilitated, preserving the old structure, into the Ponferrada Energy Museum (Fig. 2).

These are only a few glimpses of a rich cultural heritage that is much cherished by the inhabitants of those lands and cities and a real joy for the outsiders that visit them. Antonio Campillo grew up in this resourceful environment, where he spend his childhood and youth. He attended the MSP school in the period 1960–1963. Being the brightest student, the school endowed him with a grant to attend the public high school Gil y Carrasco, in the City Hall square, where he completed the whole secondary education cycle in the period 1964–1971. At age of 17 he entered the University of Oviedo, where he spent the term 1971–1972 as a student of the first course of the science faculties and engineering schools, which at that time was mandatory for all science and engineering careers. His early decision to pursue Mathematics lead him to the University of Valladolid, which is about 200 km South-East of Ponferrada, to pursue the remaining four terms to get the "Licenciado

Fig. 2 Ponferrada's "La Fábrica de Luz. Energy Museum", an award-winning rehabilitation of the one-century old thermal power plant. Published with the kind permission of the author, José Hevia, through the good offices of Míriam Fernández Cuevas

en Matemáticas" degree, which in todays terms would be equivalent to a Master's degree in Mathematics.

Let us end this prelude with a short description of the sections in which we have divided what lies ahead:

- Sect. 2: A timeline of Antonio Campillo's main academic landmarks.
- Sect. 3: A relation of his PhD students with a brief summary of their theses.
- Sect. 4: A short report on his main collaborators.
- Sect. 5: A selection of his most representative works, with comments, often with quotations from other sources. When appropriate, we emphasize questions and observations that are likely seeds for future research opportunities. At the end we provide a summary of his main contributions in a headlines format.
- Sect. 6: An overview of his services in several fronts, and an assessment of his influence in important aspects of the mathematical and scientific communities.
- 7: References. The full list of Campillo's publications as of this writing (January 2018). References to works of others are given in footnotes inserted at the spot in the text where they are required.

2 Mathematical Education and Academic Positions

Antonio Campillo obtained a 5-year bachelor's degree in Mathematics[5] from the University of Valladolid (UVA) in 1976. In the last three terms of these studies, he won a salary-grant and he finished the degree as the first of his promotion and was the first nation-wide among the granted students in Mathematics. In the next two terms he was Assistant Professor in the same university and was awarded an FPI[6] grant that culminated with a PhD thesis that was defended in 1978.

The thesis advisor was José M. Aroca, a former student (1970) of Pedro Abellanas and Heisuke Hironaka, and the thesis title was *Singularidades de curvas algebroides planas en característica positiva*.[7] It was a remarkable memoir, for its extension and contents, and a no less remarkable feat that the author could complete it in a period that by all standards was very short.

It was published, with an important additional chapter (we will return to this later on), as a Springer Lecture Notes in Mathematics [1]. The main theme is the extension, and theoretical development, of the notion of equisingularity (introduced by Oscar Zariski in the 1960s) for plane curves over an algebraically closed field of any characteristic. Instead of the classical Puiseux expansions, the technical tools used are the parameterizations given by Hamburger-Noether expansions.

The PhD period coincided with those of Ignacio Luengo and Julio Castellanos. With both he has maintained a substantial collaboration along the years. In the period 1978–1982, he had a position of Associate Professor *ad interim* in the UVA. It is the time in which the Sevilla school of Algebraic Geometry and Singularities begins its own journey, inspired by José Luis Vicente Córdoba, another former student of Pedro Abellanas (1973), and that Antonio Campillo visits on several ocasions. In this way he formed a solid friendship with that school, particularly with Luis Narváez, Emilio Briales, Francisco Castro, Ramón Piedra, Javier Herrera, and of course Vicente Córdoba. It was also in this connection that he also met, as early as 1980, Lê Dũng Tráng, Andrei Todorov, Manfred Hermann, Gert-Martin Greuel, Bernard Teissier, and Jean Giraud.

The year 1981 of that period is especially intense, as he and Ignacio Luengo made postdoctoral stays at Columbia University and Harvard University. These allowed him to meet Oscar Zariski and several of his students, including Shreeran Abhyankar (PhD 1956), Heisuke Hironaka (1960), David Mumford (1961), and Joseph Lipman (1965). The exchanges with these researchers, and the motivation provided by their works, boosted his interest in the study of singularities in geometry and soon reached the maturity of being able to propose approaches and algebraic models for their study in any characteristic. Fundamentally, he focused on "classifying singularities from associated algebraic structures, mainly arcs and semigroups, and

[5] Licenciatura en Matemáticas. It was equivalent, roughly speaking, to a master's degree in Mathematics in todays terms.

[6] Formación de Personal Investigador, i.e. Training of Research Personel.

[7] *Singularities of algebroid plane curves in positive characteristic*.

on discovering how to read the geometry and topology directly in those structures" (AC). The thesis first, and the maturity reached in his postdoctoral stays afterwards, are clear signs of him having reached the proficiency and determination to carry on a creative and productive research career.

The academic consolidation came in the period 1982–1984, in which he had an appointment as Associate Professor of Geometry in the UVA (1982–1983), Full Professor of Geometry and Topology in the University of Sevilla (US) (1983–1984), and Full Professor of Algebra in the UVA since 1984. In the year 1983 he served as Director of the Algebra Department of the US, and after his settlement at the UVA, he was Director of the Department of Algebra, Geometry and Topology (1996–1999) and Dean of the Science Faculty (2000–2004). In 1991 he founded the Singacom research group and has been its coordinator ever since.

3 PhD Students

In a private communication about 2 weeks before the lecture referred to in the Foonote 2, Antonio Campillo described how he views himself in relation to scientific cooperation: "My idea has always been to feel inside a school of researchers, that is, to share with one another knowledge and ideas, including taking them to the classroom." This principle is clearly reflected in the ways he has carried out the orientation of doctoral students, to whom we dedicate this section, and in the collaboration with a rather long list of researchers, to whom we devote the next section.

- Félix Delgado de la Mata: *Invariantes numéricos de curvas algebroides con varias ramas. Una descripción explícita*. Universidad de Valladolid (UVA), 1986 (282 p).

Aimed at providing an explicit combinatorial description of several numerical invariants that characterize the equisingularity type of a plane reduced algebroid curve defined over an algebraically closed field, it essentially generalizes and extends the theory that was available in the irreducible case (Puiseux exponents, semigroup of values, indexes of maximal contact, dual graph, local topology, Whitney conditions, and so on) to the case of two or more branches, providing explicit descriptions of each kind of invariant in terms of the other kinds. In this case the equisingularity is defined in terms of the well-understood equisingularity of the branches and the intersection multiplicites of pairs of branches, and the value semigroup is a subsemigroup S of $\mathbb{N}^r$, where r is the number of branches. One of the many novel ideas introduced in this memoir is the concept of *gap* (laguna) relative to S, which is one of the keys for the main result of the thesis, namely the explicit description of the semigroup of values. The number of gaps is finite. This allows to compute the length of $\bar{R}/R$, where R denotes the local ring of the algebroid curve and $\bar{R}$ its integral closure in the total quotient ring of R, and so to characterize the Gorenstein condition of R in terms of the symmetries of the semigroup.

- Carolina Ana Núñez Jiménez: *Anillos saturados de dimensión uno: Clasificación, significado geométrico y aplicaciones*. UVA, 1986 (194 p).

In the Arcata Symposium "Singularities 1981", Antonio Campillo introduced a notion "absolute saturation" (he called it "presaturation"), see [2], and compared it with other analogous notions that had been introduced by Zariski, Teissier or Lipman. "This interesting paper illustrates the advantage of the use of Hamburger-Noether expansions instead of the Puiseux expansions in some problems over fields of arbitrary characteristic." (Doru Ştefănescu, MR713060). Ana Núñez' thesis is focused on the reduced and equicharacteristic one-dimensional complete local rings that are saturated according to Campillo's definition, providing a description of them, of their geometric interpretations, and their classification by means of a complete system of invariants. An outstanding contribution is the ring-theoretical formulation of Abhyankar's so-called inversion formulas for Puiseux exponents.

- Julen Susperregui Lesaca: *Invariantes determinantales de módulos sobre ciertos anillos no conmutativos*. UVA, 1987 (238 p).

Defines and studies the Fitting ideals of a finitely generated $\mathscr{D}$-module using the notion of determinant (introduced by Kossivi Adjamagbo) of a square matrix of elements of $\mathscr{D}$. For a summary, see the author's paper *Déterminant sur des anneaux filtrés* published by the Comptes Rendus de l'Académie de Sciences, Série I (Mathématiques), **293**/9 (1981), 447–449. A similar treatment was given for modules over skew-commutative graded algebras using a suitable definition of determinant for this case. In both cases he determined the geometrical invariants using the Fitting ideals.

- Carlos Marijuán López: *Una teoría birracional para grafos acíclicos*. UVA, 1988 (265 p).

A combinatorial study, in terms of oriented acyclic graphs, of processes, particularly the blowing-up along a subvariety, that concur in the algorithmic desingularization and birrational theory of algebraic curves. One of the main results is that each oriented acyclic graph G is birrationally equivalent to a unique oriented forest, and that this oriented forest classifies G up to birrational equivalence. He also obtained birrational contractions of an oriented forest with a minimal number of vertexes.

- Julio García de la Fuente: *Geometría de los sistemas lineales de series de potencias en dos variables*. UVA, 1989 (118 p).

The topic of this thesis is the birrational reduction of the singularities of singular foliations on a smooth algebraic surface that have a meromorphic first integral, which means that the foliations are given as the tangents to the curves of a pencil $ag + bh = 0$, where $(a : b) \in \mathbb{P}^1$ and g and h are regular functions. The reduction turns out to be perfectly related to the elimination of the pencil base points, a result that has been used by Abhyankar, Artal and Luengo, among others, in their works on dicritical points.

- CARLOS GALINDO PASTOR: *Desarrollos de Hamburger-Noether y equivalencia discreta de valoraciones*. UVA, 1991 (203 p).

The Hamburger-Noether expansion (HNE) of a plane curve germ is especially suitable for the study of its singularity in the case of positive characteristic (see [1]). In the thesis, the concept of HNE is extended to any plane valuation in any characteristic. This allows to refine Spivakovsky's classification of these valuations. In addition, it is proved that the HNE gives rise to parametric equations of the valuations with the consequent advantage from the computational point of view. The concept of the Poincaré series of a valuation is also introduced and an explicit calculation is given in the divisorial case. Finally, the concept of intersection number of germs of foliations and plane valuations is introduced, proving that the family of these values determines the birrational reduction of a foliation.

- ANA JOSÉ REGUERA LÓPEZ: *Proximidad, cúmulos e ideales completos sobre singularidades racionales de superficie*. UVA, 1993 (162 p). Codirigida con MONIQUE LEJUENE-JALABERT.

The geometric theory of complete ideals for rational singularities of an algebraic surface is established by means of a generalization, suitable for this context, of the Enriques-Zariski notion of proximity for infinitely near points associated to their resolution by blowing-up of points. A full dictionary is provided between the geometric theory and the combinatorial theory associated of its dual graph. This dictionary allows to understand the geometry of significant arcs traced on the rational surface singularities.

- JOSÉ IGNACIO FARRÁN MARTÍN: *Construcción y decodificación de códigos álgebro-geométricos a partir de curvas planas: algoritmos y aplicaciones*. UVA, 1997 (142 p).

In the search of an efficient set-up for the algorithmic construction, coding and decoding of Goppa codes based on a smooth projective curve $\tilde{C}$ defined over a finite field, the main idea of this thesis is to cast all the relevant ingredients in terms of a *plane* model C by using a symbolic version of the Hamburger-Noether expansions to obtain $\tilde{C}$ (desingularization of C), the so-called embedded resolution, a basis for the space $L(G)$ used to construct a Goppa code, and the Weirstrass semigroup of a rational branch of C. The most satisfactory results are for the case $G = mP$ (point Goppa codes) and, more specifically, when P is the only branch at infinity of C.

- EDUARDO TOSTÓN VALDÉS: *Estudio y factorización de ideales completos en anillos locales*. UCM, 2002 (138 p).

The main contribution of this memoir is the discovery of a class of complete ideals in which the results of Zariski in dimension two can be generalized to arbitrary dimension. For this class of ideals, which is a wide subclass of toric complete ideals, it follows naturally that the semigroup they form is free, and that results of Zariski-Samuel (*Commutative Algebra*, volume II) and of Lipman (regular case) can be fully extended to this more general situation.

- Francisco Monserrat Delpalillo: *El cono de curvas asociado a una superficie racional. Poliedricidad*. Universidad Jaume I, 2003 (203 p). Codirigida con Carlos Galindo Pastor.

To any projective surface X one can associate a series of convex cones (cone of curves, semi-ample cone, and characteristic cone) that provide information on the geometry of the surface. In this thesis the cone of curves associated to a rational and regular projective surface is studied. More specifically, conditions are established that involve the polyhedricity of that cone. These conditions are of two types: those that depend on the existence of certain effective divisors, and others that depend solely on obtaining the surface from a relatively minimal surface (which may be the projective plane or a Hirzebruch surface). The polyhedricity of the cone of curves has important geometric implications, such as the fact that the number of projective morphisms with connected fibers from X to another variety (contractions) is finite, and, consequently, that the number of (-1)-curves of X (that is, non-singular rational curves with self-intersection -1) is finite.

- Diego Ruano Benito: *Estructura métrica de los códigos lineales. Códigos tóricos generalizados*. UVA, 2007 (100 p). Codirigida con José Ignacio Farrán Martín.

A contribution to the theory of toric codes, that is, algebro-geometric codes on toric varieties obtained by evaluating rational functions at the torus points, and to some generalizations of toric codes. In particular he obtains, using intersection theory, lower bounds for the minimal distance. It also contributes to the study of linear codes in general by introducing structures of toric type that allow to understand and handle them in a way similar to the toric codes.

- Ann Lemahieu: *Poincaré series and zeta functions*. Katholieke Universiteit Leuven, 2007 (113 p). Codirigida con Willem Veys.

One main result is to prove, in terms of the theory developed by Campillo, Gonzalez-Sprinberg and Lejeune-Jalabert [3], the monodromy conjecture for the Igusa function (Denef-Loeser theory) of surface singularities that are not degenerate with respect to a toric cluster in dimension 3, one of the few cases for which it has been possible to prove that conjecture. Another main result is to establish the theory, perform the corresponding calculations, and describe explicitly the geometrical meaning of the Poincaré series of toric varieties with respect to their natural filtractions.

- Rosa María de Frutos Marín: *Perspectivas aritméticas para la Conjetura de Casas-Alvero*. UVA, 2013 (145 p).

Detailed study on the multiple facets and their interrelations of the Casas-Alvero conjecture (2001), according to which a complex univariate polynomial of degree n has only one root if it shares a root with each of its first $n-1$ derivatives. In particular it shows that the conjecture is valid if the polynomial has at most three monomials and, in general, it introduces a numerical discriminant, defined for each

degree n, which is an integer, and whose non-vanishing is equivalent to the validity of the conjecture. On the other hand, since the validity of the conjecture follows from the validity of its reduction modulo some prime number p, it also considers six different reductions modulo p and proves that the validity of either of them for any given p is equivalent to the validity of the remaining five reductions for the same p. This result provides substantially simpler proofs of most known results.

- IRENE MÁRQUEZ CORBELLA: *Combinatorial Commutative Algebra Approach to Complete Decoding*. UVA, 2013 (222 p). Codirigida con EDGAR MARTÍNEZ-MORO.

It explores links between the algebraic structure of a linear code and the complete decoding process and develops an algebraic analysis of the decoding process by means of different mathematical structures associated with certain code families. Alternative algorithms and new techniques are also proposed that allow to relax the conditions of the decoding problem (in general NP-complete) and thereby reducing the resources of space and time necessary to handle it. The information concerning finite rings is represented in an original way by bit strings and their manipulation is based on toric geometry.

4 Collaborators

Antonio Campillo has created and led a dynamic research school composed not only by doctoral students and nearby colleagues, but also by pre and post doctoral students from numerous countries. All this activity has been carried out in step with his ability to attract, generate and manage appropriate resources. So it is not surprising that up till now he has had over 50 scientific collaborators.

Among the PhD students, we find the following list (the order of the names is by the year of the thesis defense, as in the previous section, and the order of the cited works is from oldest to newest):

- Félix DELGADO DE LA MATA: [4–31]
- Carlos MARIJUÁN LÓPEZ: [32–34]
- Julio GARCÍA DE LA FUENTE: [35, 36]
- Carlos GALINDO PASTOR: [35, 37–39]
- Ana José REGUERA LÓPEZ: [35, 40–42]
- José Ignacio FARRÁN MARTÍN: [43–47]
- Francisco MONSERRAT DELPALILLO: [48]
- Ann LEMAHIEU: [49]

Among the remaining collaborators, we consign a selection of those that seem to us to have more significance for the scientific endeavors of Campillo.

- François LOESER (PhD 1983, B. Teissier), Ignacio LUENGO (PhD 1979, J. M. Aroca), Claude SABBAH (PhD 1976, L. D. Tráng): [50]. See Fig. 3, left.

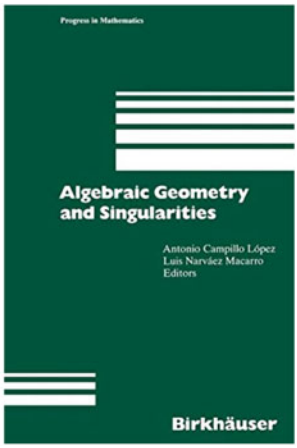

Fig. 3 Editorial work, I: *Courbes* (with permission of the organizers and editors), [50]; *Algebraic geometry and singularities*, [51]

- Monique LEJEUNE-JALABERT (PhD 1973, H. Hironaka): [3, 52–54]
- Pilar PISÓN (PhD 1991, J. L. Vicente): [33, 34, 55–60]
- Gerardo GONZALEZ-SPRINBERG (PhD 1982, J.-L. Verdier): [3, 48, 52–54, 61]
- Philippe THIERRY GIMENEZ (PhD 1993, M. Morales): [62, 63]
- Jorge Alberto GUCCIONE, Juan José GUCCIONE (PhD 1991, O. E. Villamayor), María Julia REDONDO (PhD 1991, O. E. Villamayor), Andrea SOLOTAR (PhD 1988, O. E. Villamayor), and Orlando E. VILLAMAYOR: [64]
- Luis NARVÁEZ (PhD 1984, Lê Dũng Tráng and José Luis Vicente Córdoba) [51]. See Fig. 3, right.
- Sabir GUSEIN-ZADE (PhD 1975, S. P. Novikov and V. I. Arnold): [5–31]
- Jorge OLIVARES (PhD 1994, X. Gómez-Mont): [65–70]
- Santiago ENCINAS (PhD 1996, O. E. Villamayor Uriburu): [71–73]
- Gert-Martin GREUEL (PhD 1973, E. V. Brieskorn), Christoph LOSSEN (PhD 1998, G.-M. Greuel): [74, 75]
- Patrick FITZPATRICK (PhD 1980, L. G. Kovács), Edgar MARTÍNEZ-MORO (PhD 2001, F. J. Galán Simón), Ruud PELLIKAAN (PhD 1985, D. Siersma): [76]
- Gabriel CARDONA (PhD 2002, J. Quer), Alejandro Melle-Hernández (PhD 1996, I. Luengo), Willem VEYS (PhD 1991, J. Denef), W. A. Zúñiga-Galindo (PhD 1996: K.-O. Stöhr) [77]
- Franz-Viktor KUHLMANN (PhD 1990, P. Roquette), Bernard TEISSIER (PhD 1973, H. Hironaka): [78]

5 Fruits and Seeds

The mathematical expertise domains of Antonio Campillo are *commutative algebra*, *singularities* and *algebraic geometry*, as represented by the books [1, 79] (Fig. 4). Among the main lines on which he has oriented his research, we will consider the following: equisingularity and deformations, particularly in positive characteristic; toric geometry; clusters, proximity, complete ideals and cones; algebraic geometry codes; algebraic foliations and polarity;

We also point out some questions and problems that seem to us promising research opportunities. At the end of the section, we include a list of headlines summarizing the major reseach achievements of Campillo so far.

5.1 Books

The memoir [1] essentially coincides with the thesis (1978) extended with a chapter on the singularities of space curves. "These notes intend to give a systematic development of the theory of equisingularity of irreducible algebroid curves over an algebraically closed field of arbitrary characteristic, using as main tool the Hamburger-Noether expansion instead of the Puiseux expansion which is usually employed in characteristic zero." (from the Introduction).

The review in the AMS Mathematical Reviews (MR) is signed by B. Teissier (1982), already a young giant in the theory of singularities by then, and it is worth quoting a few sentences from it: "This very clear book, defining all the concepts it uses, is an excellent introduction to the algebraic aspect of the classification of

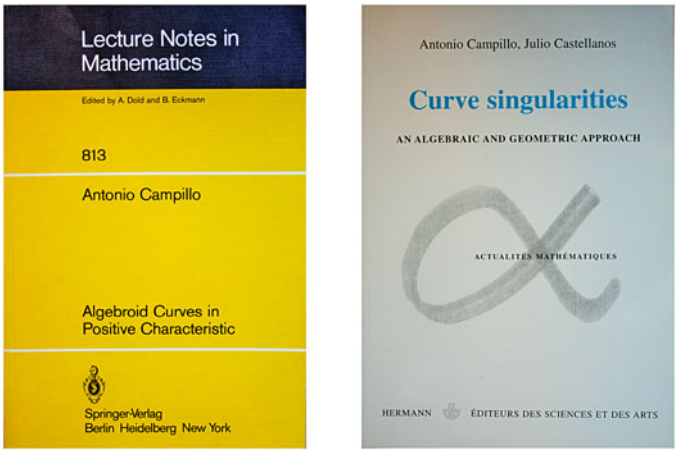

Fig. 4 Algebroid curves in positive characteristic, [1]; curve singularities, [79]

singularities of irreducible algebroid (or complex analytic) curves, in both positive and zero characteristics. [...] Here the central tool is the Hamburger-Noether expansion [...] In addition to the extension to positive characteristic, this book is useful because of the careful comparison that it develops among the different systems of invariants attached to a branch singularity: Hamburger-Noether, Puiseux, sequence of multiplicities of infinitely near points, semigroup of valuations, ... [...] There is also a very useful link, after Abhyankar's recent work, between the Hamburger-Noether expansion and the theory of the maximal contact of a branch with other singular branches [...] The last chapter studies three possible definitions of equisingularity of non-planar branch."[8]

It is worth mentioning that the Hamburger-Noether Expansions, which are the main tool used in this treatise, have been implemented by M. Lamm and C. Lossen as the package hnother.lib of the computer algebra system SINGULAR.[9]

The purpose of the book [79] is clearly described in its introduction: "One of the reasons for this book is that space curve singularities are still not classified by a good notion of equisingularity, so it becomes useful to clarify possible ways to focus on such a question. [...] Other reasons are that curve singularities also appear as a fundamental tool in recent developments concerning subjects such as arc spaces, resolution of singularities, complete ideals or valuations." In words of A. Dimca in the review MR1986115 (emphasis added), "The authors use the Arf closure relative to divisorial valuations *to define an extension of Puiseux exponents for general singularities*. Primary exponents come from valuation associated to arc space components. Secondary exponents are related to some Rees valuations." It may be worth remarking that part of this book amounts to a unified presentation of the papers [80–82] by the authors, the latter (*On Puiseux exponents for higher dimensional singularities*) included in the volumen in memory of Ruth Michler.

[8]Ce livre très clair, définissant tous les concepts qu'il utilise, constitue une excellente introduction à l'aspect algébrique de la classification des singularités des courbes algébroïdes (ou analytiques complexes) irréductibles, en charactéristique positive comme en caractéristique zéro. [...] Ici l'outil central est le développement de Hamburger-Noether [...] en plus de l'extension à la caractéristique positive, ce livre est utile par la comparison soigneuse qu'il contient entre les différents systèmes d'invariants attachés à une singularité de branche: Hamburger-Noether, Puiseux, suite de multiplicités de points infiniment voisins, semigroupe des valuations, [...] On trouve aussi le lien très utile après les travaux récents de Abhyankar entre le développement de Hamburger-Noether et la théorie du contact maximal d'une branche avec d'autres branches singulières [...] Le dernier chapitre étudie trois définitions possibles de l'équisingularité des branches non planes.

[9]W. Decker, G.-M. Greuel, G. Pfister, and H. Schönemann: SINGULAR 4-1-0 —*A computer algebra system for polynomial computations*, 2016. http://www.singular.uni-kl.de

5.2 Equisingularity and Deformations

Our general considerations above about the memoir [1] emphasized its pioneering character as a systematic study of equisingular families over fields of any characteristic and its status as a fundamental reference in the field. Here we will have a closer look on some of its more innovative features, and in particular on the three notions of equisingularity for space curves introduced in the last chapter.

The notion of equisingularity is characterized in terms of the completion of the local ring that is associated to the singularity (in particular it is invariant under change of coordinates and of parameterization), and numerically through several equivalent invariant systems: sequences of multiplicities of strict transforms of blowing-ups, characteristic exponents, maximal contact values, and semigroup of values.

In the case of twisted algebroid curves, three quite natural types of data are advanced as possible criteria for equisingularity:

1. The equisingularity class of a generic plane projection;
2. Sequences of multiplicities of strict transforms by blow-ups centered at closed points;
3. Semigroup of values.

More specifically, two irreducible algebroid curves are said to be, using the terminology of [1], *equisingular* E.s.1 if the equisingularity classes of their generic plane projections coincide [1, Definition 5.2.10]; E.s.2, if they have the same process of resolution (same multiplicity sequences) [1, Definition 5.3.1]; E.s.3, if their semigroups of values are equal [1, Definition 5.4.1]. For plane curves, these three definitions coincide, but they are different for space curves [1, Remark 5.4.4], and, as the author remarks, "none of them may be considered as a better definition than the other ones" [1, page 141]. On the other hand, the three notions of equisingularity extend naturally to curve singularities with several branches, but replacing the multiplicity sequence by the multiplicity tree.

Another important paper of A. Campillo concerning Hamburger-Noether expansions is [83]. In it, he defines a notion of Hamburger-Noether expansion over rings R (not only over fields), generalizing for them most of the results in [1] and obtaining some applications to equisingular deformation theory. In particular he proves that, for a fixed equisingularity type, there exists an equisingular family $X \to Y$ of $\mathbb{Z}$-schemes such that any equisingular family of algebroid curves (corresponding to that type) over any field K can be induced from $X \to Y$.

The notions of equisingularity E.s.1 and E.s.2 can be characterized by means of *saturation* and *Arf closure*, respectively. The concept of saturation was originally defined by Zariski in the 1960s using topological ideas (coverings); later, Pham

and Teissier[10] gave an equivalent definition for the analytic case following the line of Grothendieck. In [2], Campillo introduced a third notion of saturation that is equivalent to the notions of Zariski and Pham-Teissier, but it is also valid in positive characteristic, fully extending and considerably simplifying the two previous theories. Both notions, saturation and Arf closure, have good algebraic characterizations in terms of the local algebra R of the singularity; more specifically, they can be translated in terms of the semigroup of values of R, leading up to two numerical equisingularity invariants of types E.s.1 and E.s.2, respectively: the semigroup of the saturation of R and the semigroup of the Arf closure of R. The paper [84] is devoted to develop arithmetical properties of the concept of saturation in arbitrary characteristic.

In [80], Arf closure and saturation are introduced with respect to a finite family of valuations. The paper [85] develops a geometrical theory of the Arf closure with respect to divisorial valuations for algebraic varieties of arbitrary dimension over arbitrary fields. Other papers related with Arf closure and Arf invariants associated with valuations are [81] and [82].

The aim of the paper [75] is to study equisingular deformations of plane curve singularities over an algebraically closed field of arbitrary characteristic. If this characteristic is zero it is known that the base space of an arbitrary deformation admits a unique maximal subspace over which the deformation is equisingular, that is, it admits a unique equisingularity stratum. However, if the field has positive characteristic, the situation may be different in special cases and they are described in this paper.

The paper [74] is computational and, in particular, an algorithm to compute the equisingularity stratum and its tangent space is given. About this paper, Arvid Siqveland says the following (MR2339830): "The way the deformation categories are described through the deformation functors is particularly interesting and illustrates a somewhat different technique. The proofs [...] illustrate beautiful and clever mathematics. Also, examples of occurring pathologies are given explicitly and the paper gives a really good introduction to the subject. The article should be read by anyone who is interested in a deep understanding of the geometry of deformation theory."

5.3 Toric Geometry

In *Syzygies of affine toric varieties* [63], a purely combinatorial method is given for computing the syzygies of the describing ring R of an affine toric variety. The working of the method is particularly illustrated in dimension one, and also on

[10] F. Pham, B. Teissier: *Fractions lipschitziennes d'un algèbre analytique complexe et saturation de Zariski*. Actes du Congrès International des Mathematiciens, Nice (1970), Tome 2, Gauthier-Villars, Paris (1971).