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Statistical Physics of Synchronization

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ISSN 2191-5326 ISSN 2191-5334 (electronic)
SpringerBriefs in Complexity
ISBN 978-3-319-96663-2 ISBN 978-3-319-96664-9 (eBook)
https://doi.org/10.1007/978-3-319-96664-9

Library of Congress Control Number: 2018949061

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

To my beloved parents,
Late Subir Kumar Gupta and Geetanjali Gupta

Shamik Gupta

To the memory of my mother, Maria

Alessandro Campa

To Antonella

Stefano Ruffo

Foreword

In 1965, a bright undergraduate at Cornell University named Arthur Winfree undertook an experimental study for his senior thesis. He wired 71 neon tube oscillators together into a contraption that he called the firefly machine. At a time when the theory of nonlinear oscillations was largely confined to two and three oscillators, Winfree was venturing out to study dozens of them. To allow the oscillators to feel one another's influence, he connected them all to a common terminal through a small capacitor, so that each oscillator interacted equally with all the others, and he grounded that terminal through a larger variable capacitor. This setup enabled him to easily adjust the coupling strength between the oscillators.

He found that in the absence of coupling, the neon tubes blinked on and off in an uncoordinated, incoherent fashion. That was to be expected; their natural periods varied by about 10%. As Winfree slowly dialed up the coupling, the oscillators remained incoherent until it reached a critical coupling strength. Above that threshold, all the neon tubes began discharging in unison, much like the famous congregations of synchronously flashing fireflies of Thailand, Malaysia, and other parts of Southeast Asia. Winfree had discovered the sudden onset of synchronization in a population of nonlinear oscillators.

Winfree's inspiration had always been biology—not just fireflies with their rapid flash rhythms, but the much slower rhythms in sleep and wake and body temperature of mammals, the nearly 24-hour rhythms known as circadian rhythms. The early 1960s were the heyday of research into circadian rhythms. With his firefly machine, Winfree opened up a theoretical avenue for studying such rhythms.

At the time, Winfree was a college student majoring in engineering physics. His training in solid-state theory led him to approach the question of biological synchronization from a perspective that only a physicist would have. He realized that an infinite-range approximation, in which each oscillator interacted equally with all the others, offered the best hope of making progress on this daunting, nonlinear, nonequilibrium, many-body problem. That was why he coupled all the oscillators through a common capacitor. He was doing the electronic counterpart of mean-field theory.

Next, Winfree abstracted his firefly machine into a set of differential equations that he could simulate on the university's mainframe computer. At the time, computer simulations were a rarity in science. One had to go to a computing center and feed punch cards into a room-sized behemoth. To simplify the differential equations, Winfree assumed that his model oscillators were weakly coupled, compared to their attraction to their limit cycles in state space. He realized intuitively that under that assumption, each oscillator could be represented by its phase alone as it moved along its limit cycle; amplitude variations could be neglected. In a now-celebrated paper published in *Journal of Theoretical Biology* in 1967, Winfree showed that his mathematical model could do what his firefly machine had done: it could spontaneously synchronize. As the coupling strength between the oscillators was increased, or as the variance of oscillators' natural frequencies was decreased, the oscillators abruptly switched from an incoherent, desynchronized state to an ordered state in which a macroscopic fraction of the system was locked in frequency. In this 1967 paper, he explicitly noted a remarkable connection to thermodynamic phase transitions. He wrote:

> Disguised in the literature of solid-state physics under an interchange of spatial for temporal coordinates, the phenomenon of ferroelectric crystallization is strikingly analogous: the oscillators are replaced by a population of electric dipoles at crystal lattice points; the orientation of their phase vectors […] becomes the angular orientation of dipoles under a communally-generated electric field, to which they contribute […] according to orientation; the spread of synchronized phases […] due to the spread of natural frequencies […] becomes the distribution of dipole angles due to thermal buffeting; and the threshold [of synchronization] is mirrored in the Curie temperature for ferroelectric transition.

About a decade later, the Japanese statistical physicist Yoshiko Kuramoto reformulated Winfree's work and recast it as a beautifully elegant system of differential equations, now known as the Kuramoto model. Using an ingenious self-consistency argument, and retaining Winfree's assumptions of a mean-field model of phase-only oscillators, but using the more tractable form of coupling between the oscillators, Kuramoto was able to find his synchronization transition analytically and to calculate the extent of order above the synchronization threshold.

In the half a century since Winfree's landmark work, the study of collective synchronization has mainly been approached through nonlinear dynamics and computer simulation. The connection to statistical physics, though always present, has tended to play a subordinate role. The present monograph rectifies this situation. Shamik Gupta, Alessandro Campa, and Stefano Ruffo do a wonderful job of summarizing earlier work on the Kuramoto model and enlarging it to embrace the insights of statistical physics, using concepts like H-theorems, Fokker–Planck equations, and the breakdown of detailed balance. The problems they tackle are difficult and fascinating, both from the standpoint of nonlinear dynamics and from that of statistical physics, because of their nonequilibrium and many-body character. Furthermore, the authors explore the effects of inertia, always an important physical consideration, but one that has been given relatively little attention in the nonlinear dynamics literature. This is a very valuable addition to the literature of dynamical systems and nonequilibrium statistical physics. I hope you'll enjoy reading it as much as I did.

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Preface

A remarkable phenomenon common in nature is that of spontaneous synchronization, whereby a large population of coupled oscillating units of diverse frequencies spontaneously evolve to operate in unison. Such a cooperative effect commonly occurs in physical and biological systems over length and time scales of several orders of magnitude. Examples are flashing of fireflies, animal flocking behavior, audience clapping in concert halls, pedestrians on footbridges, and a variety of experiments involving electrochemical and electronic oscillators, metronomes, Josephson junctions, and laser arrays. Besides its necessity in firing of cardiac cells that keeps the heart beating and life going, synchrony is desired in man-made systems, e.g., in parallel computing, whereby computer processors must coordinate to finish a task on time, and in electrical power grids, whereby generators must run in synchrony to be locked in frequency to that of the grid. Synchrony could also be hazardous, e.g., in neurons, leading to impaired brain functions in, e.g., epilepsy. Collective synchrony among oscillators has attracted immensely the attention of physicists and applied mathematicians, and finds applications in many fields, from quantum electronics to electrochemistry, from bridge engineering to social science, and others.

Synchronizing systems may be viewed from two contrasting perspectives, namely, that of dynamical systems theory and statistical physics. To summarize in one sentence the characterizing aspects of the two perspectives, one could say that in the former, spontaneous synchronization occurs as a bifurcation in the dynamical behavior of the system as a function of the strength of interaction between the oscillating units, while in the latter, it represents a phase transition between different forms of statistical distribution of the dynamical variables of the system constituents. Until now, the approach based on dynamical systems theory has received much more attention. This could be partly due to the fact that synchronizing systems have mostly been investigated using models not belonging to any class of Hamiltonian systems, the latter constituting the prominent subject of study in mainstream statistical physics. The use of mainly models with first-order dynamics (only very recently are models with second-order dynamics being studied) has been one other reason for the abundance of studies employing tools of dynamical systems theory.

Viewed from the perspective of statistical physics, the following characteristics of synchronizing systems may be noted. Presence of long-range interactions in synchronizing systems allows the use of mean-field models, which may be seen as a major simplifying feature for extensive analytical treatments. The mean-field analysis becomes exact in the limit of a very large number of units (in particular, in the thermodynamic limit, which is naturally achieved in synchronizing systems) for systems where the interaction is the same between every pair of constituents. The latter feature is not always prevalent in real systems, as there are cases where the interaction, although long-ranged, decays slowly with the distance between the constituents; nevertheless, also in this case, the mean-field analysis is a very useful first approximation, and corrections can in principle be evaluated systematically. Another essential feature of synchronizing systems is the presence of diverse natural frequencies. In the language of statistical physics, diverse frequencies may be interpreted as quenched disordered random variables; the randomness implies the necessity to average observable quantities over the distribution of natural frequencies. Probably, the most notable feature of synchronizing systems is the fact that the stationary states to which the dynamics settles to after a transient are not equilibrium ones (in technical terms, such states do not satisfy detailed balance). Thus, synchrony is necessarily a nonequilibrium phenomenon, which therefore cannot be described by equilibrium statistical mechanics. There is as yet no theory akin to the latter that can treat and make predictions on general terms for nonequilibrium systems, thus necessitating the study of representative model systems so as to gain valuable insights into the physics of synchronizing systems. Summarizing, synchronizing systems involve the study of statistical physics of long-range systems with quenched random variables settling into nonequilibrium steady states. This brief monograph aims to present from this perspective a study of synchronizing systems.

Extensive studies of synchronizing systems over the years have led to the introduction of novel theoretical concepts in nonlinear science such as the chimera states. Chimeras are broken-symmetry states occurring in identical, symmetrically coupled oscillator ensembles in which synchronized and desynchronized subpopulations coexist. These states have been observed in a variety of experimental situations involving, e.g., chemical and mechanical oscillators. Dynamical phenomena such as chimeras have been studied analytically using the approach of dynamical systems theory. Our focus in this monograph is on statistical physics approach to synchronization, and interpreting chimeras, etc., within this approach is still largely an open issue. Hence, we will not dwell on such dynamical phenomena, interesting in their own right, in this brief monograph.

It is a great pleasure to warmly thank a number of colleagues for fruitful and enjoyable discussion and collaboration on topics discussed in this monograph: Eduardo G. Altmann, Julien Barré, Freddy Bouchet, Lapo Casetti, Pierfrancesco Di Cintio, Thierry Dauxois, Stefano Gherardini, Maxim Komarov, Haggai Landa, David Métivier, Giovanna Morigi, David Mukamel, Hyunggyu Park, Arkady Pikovsky, Antonio Politi, Max G. Potters, Alessandro Torcini, Hugo Touchette, and Lucas Wetzel. SG is thankful to his parent organization, the Ramakrishna Mission Vivekananda University, for providing a conducive ambiance for writing this book.

Finally, we acknowledge the kind support of Springer-Verlag, Berlin. We are especially thankful to Aldo Rampioni and Kirsten Theunissen for their encouragement, assistance, and patience.

Book Homepage: Comments on the book may be sent to these email addresses: alessandro.campa@iss.it; shamik.gupta@rkmvu.ac.in; ruffo@sissa.it. Misprints and errors in the book will be posted on the webpage https://sites.google.com/site/shamikguptaphysics/syncbook.

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May 2018

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