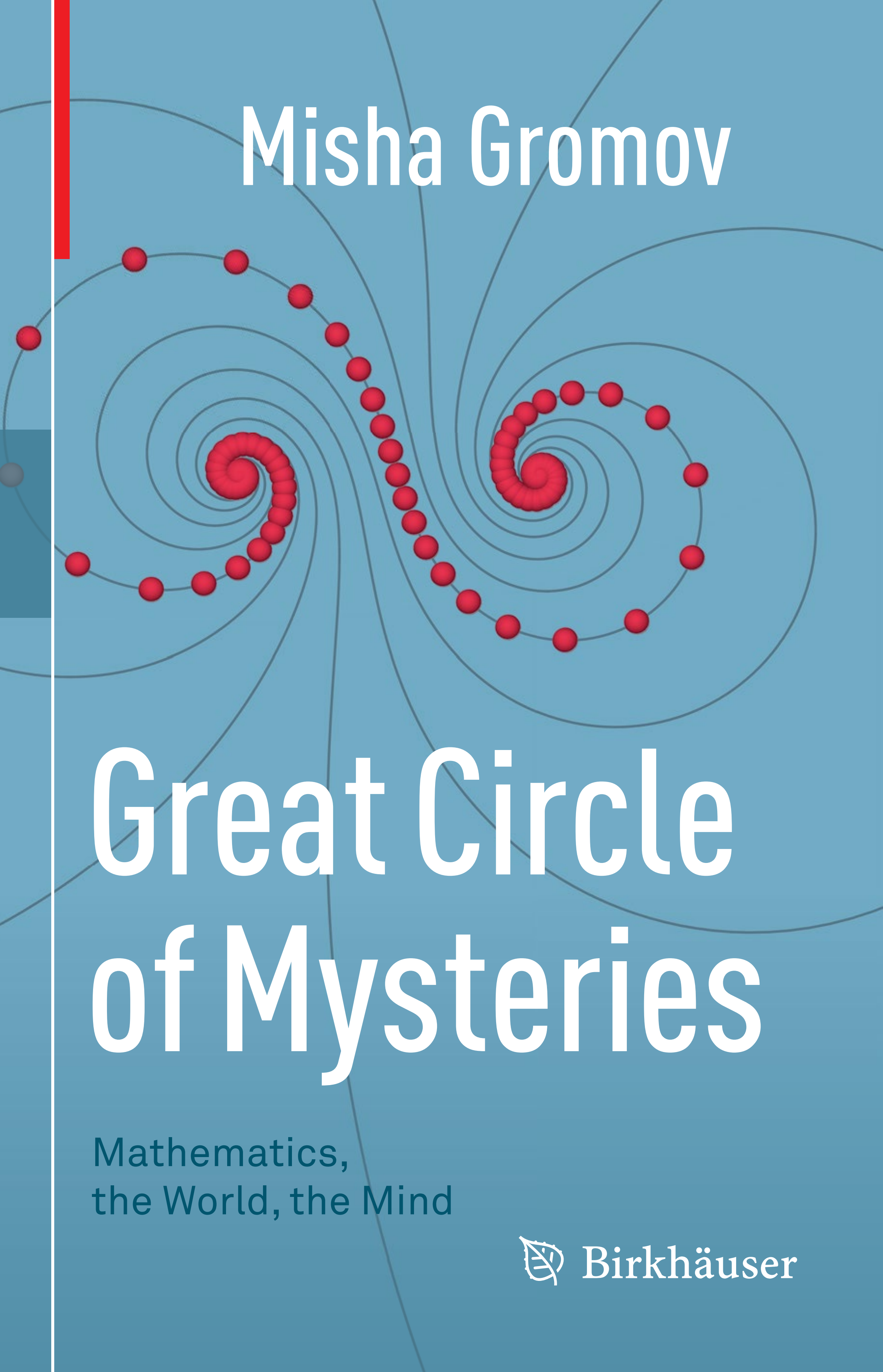

Misha Gromov

Great Circle of Mysteries

Mathematics, the World, the Mind

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Misha Gromov
Institut des Hautes Études Scientifiques
Université Paris-Saclay
Bures-sur-Yvette, France
and
Courant Institute of Mathematical Sciences
New York University
New York, USA

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Introduction

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Geometry is one and eternal shining in the mind of God.

JOHANNES KEPLER

God's plan for mathematics is beyond human reach but mathematics is the only light that can illuminate the mysteries of this world.

What we call the *science of physics* is filled with the radiance of mathematics; the fog of ignorance around what we now call LAWS OF NATURE receded in the brilliance of this physics.

But the beam of this light has barely touched the edges of the kingdom of LIFE and the face of PRINCESS THOUGHT remains hidden from us in the shroud of darkness.

And unless we know the ways of THOUGHT we cannot understand what mathematics is.

The first part of the book – *Quotations and Ideas* – is sprinkled with the ideas of those who saw sparks of light in the dark sea of the unknown. In the second part – *Memorandum Ergo* – we reflect on what in mathematics could shed light on MYSTERY THOUGHT.

Quotations and Ideas

1. Beautiful Elsewhere

Round us, near us, in depth and height,
Soft as darkness and keen as light.

ALGERNON SWINBURNE, LOCH TORRIDON

"*Mathématiques, un dépaysement soudain,*" was an exhibition organized in 2011 by *Fondation Cartier pour l'art contemporain* in Paris. It featured, among other things, *The Library of Mysteries*:

THE MYSTERY OF PHYSICAL LAWS
THE MYSTERY OF LIFE
THE MYSTERY OF THE MIND
THE MYSTERY OF MATHEMATICS

These were presented as quotes from writings of great scientists in a film made by David Lynch – an artist's visualizations of the ideas of Time, Space, Matter, Life, Mind, Knowledge, and Mathematics.

M. Gromov, *Great Circle of Mysteries*, https://doi.org/10.1007/978-3-319-53049-9_1

Michel Cassé and Hervé Chandés have persuaded me to try to do a *Naive Mathematician's* version of what David Lynch has done – to project a vision of these ideas to the invisible screen in our mind, an image illuminated by the eternal shining of mathematics rather than by the reflection of the beauty of the goddess of arts.

I knew it would be impossible but I tried anyway. Much of what I wrote was done in discussions with Giancarlo Lucchini and some of my English was corrected by Bronwyn Mahoney. Below is a modified version of what came out of it.

2. Science

Nothing exists except atoms in the void;
everything else is opinion.

DEMOCRITUS OF ABDERA (?), 460–370 BCE

All men by nature desire knowledge. Thinking is the talking of the soul with itself. ... knowledge is the one motive attracting and supporting investigators ... always flying before them ... their sole torment and their sole happiness. To perceive is to suffer.

Common sense is the collection of prejudices acquired by age eighteen.

... Science ... [*is*] *asking, not whether a thing is good or bad ... but of what kind it is?*

Science is the belief in the ignorance of experts. What we know already ... often prevents us from learning. Our freedom to doubt was born out of a struggle against authority in the early days of science.

Science is no more a collection of facts than a heap of stones is a house. A fact is valuable only for the idea attached to it, or for the proof that it furnishes. He who does not know what he is looking for will not understand what he finds. The investigator should have a robust faith – and yet not believe.

Science increases our power in proportion as it lowers our pride. Ignorance more frequently begets confidence than does knowledge. Whoever undertakes to set himself up as a judge of Truth and Knowledge is shipwrecked by the laughter of the gods.

The gods are fond of a joke – the universe is not only queerer than we suppose, but queerer than we can suppose. And the most incomprehensible thing about the world is that it is comprehensible.

There is geometry in the humming of the strings, there is music in the spacing of the spheres. A hidden beauty is stronger that an obvious one. It is godlike ever to think on something beautiful and on something new.

The most beautiful thing we can experience is the mysterious. It is the source of ... art and ... science – branches of the same tree. He to whom this emotion is a stranger ... is as good as dead: His eyes are closed.

When it comes to atoms, language can be used only as in poetry. Poetry is nearer to vital truth than history. Knowledge is limited. Imagination encircles the world.

But put off your imagination, as you put off your overcoat, when you enter the laboratory. Put it on again when you leave.

The objective reality of things will be hidden from us forever; we can only know relations. Everything we call real is made of things that cannot be regarded as real. The internal harmony of the world is the only true objective reality.

It is not nature that imposes time and space upon us, it is we who impose them upon nature because we find them convenient. ... the distinction between past, present, and future is a most stubbornly persistent illusion.

A human being is a part of a whole, called by us "universe" – a part ... restricting us to our personal desires and to affection for a few persons nearest to us. Our task must be to free ourselves from this prison by widening our circle of compassion to embrace all living creatures and the whole of nature in its beauty.

Pythagoras 1	Charles Darwin	Niels Bohr
Heraclitus 1	Claude Bernard	Albert Einstein
Plato 1	James Clerk Maxwell	John Haldane
Aristotle 1	Henri Poincaré	Richard Feynman

What these people think and how they write enlightens your mind and elevates your spirit but these thoughts have little life of their own. They do not grow, they do not transform, they do not shoot new green sprouts – they luminesce as crystals of fiery flowers frozen in eternity. They are not quite what mathematicians call *ideas*, they are halfway between *ideas* and *opinions.*[1]

Great scientific *ideas* are different – they are alive, they ignite your soul with delight, they invite you to fight and to contradict them. Unlock your spirit from the cage of the mundane, let your imagination run free, start playing with such ideas as a little puppy with its toys – and you find yourself in the world of *Beautiful Elsewhere* – that is called *Mathematics.*

[1] An opinion about X is a function, say $OP_X = OP_X(p)$, that assigns yes (agree) or no (disagree) to a person p who is dying to say what he/she thinks about X; e.g., about the existence (non-existence) of *vacuum*. Democritus did not care about specific yes/no-values of $OP_X(p)$, except for p being his best friend, maybe. But the philosopher would love to see a correlation between $OP_X(p)$ for $X =$"*vacuum exists*" and the distance from the house of p to Abdera.

3. Numbers

All the mathematical sciences are founded on relations between physical laws and laws of numbers.

JAMES CLERK MAXWELL (1856)

The numbers named by me exceed the mass equal in magnitude to the universe.

ARCHIMEDES, THE SAND RECKONER, 250 BCE (?)

Archimède … dont la vie est une époque dans l'histoire de l'homme, et dont l'existence parait un des bienfaits de la nature.

NICOLAS CONDORCET

Archimedes estimated the diameter of the universe at about 2 light-years that is $\approx 2 \cdot 10^{13}$ km – twenty thousand billions kilometers, about half, as we know today, of the distance to the nearest stars – the binary system Alpha Centauri A & B.

Then Archimedes invented an exponential representation of large numbers and evaluated the number of sand grains or rather of ≈ 0.2 millimeter poppy seeds needed to fill it at less than 10^{63} in modern notation. (I took these numbers from the Wikipedia article. In fact the $2 \cdot 10^{13}$ km cube has volume $8 \cdot 10^{57}\,\text{mm}^3$ that makes 10^{60} cubes of 0.2 mm.)

If a philosopher would not be impressed and say that

a good decision is based on knowledge and not on numbers,

Archimedes might respond that decisions may be left to our mighty rulers but that *numbers* are the guardians of our true knowledge.

Large numbers are everywhere. Even Socrates, Plato, and Aristotle would admit that *knowledge of thyself* is incomplete if you are unaware of the roughly 10^{14} (100 000 billion) bacteria in your guts – several bacteria per each cell of your own body.

(Bacteria are roughly 1 μm – one thousandth of a millimeter = one millionth of meter – in size, a few thousand times smaller than your own cells volume-wise. If you had a bacterium inside every one of your cells you would hardly notice this – you will be safely dead by that time.)

A single bacterium, if there are enough nutrients, can divide every 20–30 minutes and, in 24 hours you may have a blob $10^5\mu$m = 10 cm in size with about $2^{50} = \left(2^{10}\right)^5 = 1024^5 \approx 1000^5 = 10^6 \times 10^9$ (million billions) bacteria in it.

A schoolboy now computes:

Day 2. The blob contains $10^{15} \times 10^{15} = 10^{30}$ bacteria and is $10^{10}\mu\mathrm{m} = 10\,\mathrm{km}$ in diameter, about 1 kg of bacteria per every square meter of the surface of the Earth.

Day 4. It increases to $10^{20}\mu\mathrm{m} = 10^{11}\,\mathrm{km}$ in diameter and reaches the outermost regions of the Solar system. It will engulf the Sun ($\approx 1.5 \times 10^8\,\mathrm{km}$ from Earth), all planets including Pluto ($\approx 6 \times 10^9\,\mathrm{km}$) but not fully the orbit of Sedna at its farthest point from us ($\approx 1.4 \times 10^{11}\,\mathrm{km}$).

(Aristotle, who maintained that

the shape of the heaven is of necessity spherical,

would feel relieved if he knew that the bacteria are still contained by the *spherical heavenly shell* of the hypothetical *Oort cloud* of comets around the Sun about a light-year away.)

Day 7. The blob contains $10^{15\times 7} = 10^{105}$ bacteria and has $10^{5\times 7}\mu\mathrm{m} = 10^{26}\,\mathrm{km} \approx 10^{13}$ light-years in diameter, a hundred times the diameter of the observable universe ($\approx 10^{11}$ light-years).[2]

Bacteria have been around for billions of days, but do numbers like $10^{100\,000\ldots}$ make any sense at all? The answer is *yes and no*. They cannot be reached by counting 1, 2, 3, . . . , at least not in our space-time *continuum*, nor be represented by *collections of physical objects* of any kind. However, unrealistically large (and unrealistically small) numbers are instrumental in our treatment of NATURE'S LAWS that are manifested in observable properties of *objects* in the Universe.

How does Nature, who, as Einstein says, *integrates empirically,* manage to satisfy these laws?

Is it because she has something *much bigger* than space/time (kind of quantum fields?) at her disposal where *empirical integration* is possible?

Or is there a *secret logical something* built into Nature and she proceeds by *mathematical induction* as mathematicians do?

Or had she found a *simple logical bypass* for arriving at these laws but we cannot reach it being bound to the mental routes available to our brains?

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[2]Darwin computed the numbers of descendants of a couple of elephants and arrived at the number 15 000 000 after 500 years. This is much exaggerated, but in 5 000 years the Earth would be covered several times over by more than 10^{15} elephants and the whole Universe would not contain 10^{90} elephants after 30 000 years. (30 000 is yesterday on the Earth geological time scale, it is less than 0.02% of (about 200 000 000 years of) the evolution time of mammals on Earth.)

These questions, probably, make no sense; it is frustrating being unable to formulate a good one.

Yet, a mathematician may find a consolation in trying to estimate the number N_{can} of different logical arguments (brain routes), say in L words, that a sentient brain *can*, in principle, generate. Probably, if somebody told our mathematician what the words *can* and *in principle* signify, he/she would bound N_{can} by something like $\sim L \log L$, or even less than that – far from the number $N_{all} \sim 2^L$ of *all* such arguments, well behind what bacteria can do.

This might hurt his/her pride but then the mathematician will soon realize that numbers that linger behind his/her own logic/language beat the fastest replicating bacteria.

Indeed, *think* of Schrödinger's cat. The body of a cat is, roughly, composed of $N \approx 10^{26}$ molecules, those of water and small molecular residues in macromolecules. Suppose each molecule can be in two states. Then there are $S = 2^N \approx 10^{0.3N}$ *states of a cat*. Some of these states are judged being *alive* and some are classified as *dead*. The number, say $\mathcal{CAT}$, of possible judgements/opinions is

$$2^S = 2^{2^{10^{26}}} > 10^{10^{10^{25}}}$$

How does one decide, how can one select a sound judgement from this super-duper-universe of possibilities? Mathematicians do not understand how it works, but a cat, if he/she is alive and oblivious of math., somehow manages, makes a right choice and ... stays alive.

Some courageous people play with unimaginably greater numbers, the descendants of *Gödel's incompleteness theorem*. If you meet such a number on your path of reasoning about "real world" your logic is as good as dead.[3] Fortunately, you do not meet them in "real life" unless you call these misshapen monsters by their names.

THE MONSTER OF **SToP**. If your computer has M bits of memory, say with $M = 10^{10}$, then whatever you "ask" the computer to do, it either *stops* after $< 2^M$ steps or it goes into a cycle and runs forever. (You can use convenient *time units* instead of "steps"; the number $2^{10^{10}}$ is so big, it makes little difference if you take nanoseconds or billions of years for these units.)

Here, what we call a *question* or a *program* that you "ask" your computer to perform, is a sequence of *letters* that are the names of keys on the keyboard you have to press in order to activate this *program*.

Let us leave the "real world" and allow your computer to have an *infinite* (unbounded) memory. As in the finite case, the computer may stop after finitely many steps or run for infinite time *depending on your question*, (and on the design of the hardware and the operating system in your computer) except that *infinite* does not have to be *cyclic* for infinite memory. For example, if asked to find a file

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[3]These numbers lie at the heart of *Turing's halting theorem* and of Kolmogorov–Chaitin complexity. A whiff of either of the two is poisonous for any *scientific* theory.

named *cell-phan-nimber-Bull-Gytes* the computer either finds it and stops, or, if there is no such file, runs forever.

(A fundamental ability of your *slow* brain, not shared by the *fast* Windows search system, is an *almost instantaneous* NO SUCH FILE response to this kind of inquiry. A simple, yet structurally nontrivial, instance of this is BITTER on your tongue for almost anything chemically away from potential nutrients, with a few mistakes; e.g., SWEET for saccharin.

One may speculate together with Robert Hooke, Charles Babbage, and Alan Turing on plausible brain memory architectures that makes this possible. There is no experimental means for checking non-trivial conjectures of this kind but there is a mathematically attractive model of such a memory suggested by Pentti Kanerva, called *sparse distributed memory*.)

Pick up a (moderately large) number L and take *all those programs* in L letters for which the computer *eventually stops*. Since the number of these programs is finite ($< 100^L$ if there are < 100 different letters at your disposal for writing programs) the longest (yet, finite!) time of executing such a program, say, the time measured in years, is also *finite*; call this time **STOP**(L).

Although finite, this number of years, even for moderate L, say **STOP** $=$ **STOP**(50 000), (a program in 50 000 letters takes a dozen pages to write down) is virtually indistinguishable from infinity in a certain precise logical sense.

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Our Universe is pathetically small not only for containing anything of this size but even for holding a *writing* of an *explicit formula* or of an *explicit verbal description* of a number of such size, even if we use atoms for letters in such a writing. (The way we have just described this number does not count – distinguishing stopping times from infinity without indicating a specific "experimental protocol" for this is not what we call *explicit*.)

By comparison, $\mathcal{CAT}$ appears tiny – the corresponding exponential formula can be expressed with a few dozen binary symbols (and *physically* written down with a few thousand atoms manipulated by means of an *atomic force microscope*).

To be fully honest we must admit there was something missing from our definition of **SToP**(L).

Imagine, for example, that the memory of our computer does contain the string *cell-phan-nimber-Bull-Gytes* but it is positioned so far away that it cannot be reached in less than T time units. Since you can choose T as large as you wish, the logic of our definition inevitably makes **SToP**(L) equal infinity, even if you limit the length L of admissible programs to something quite small, say, less than one thousand letters.

Somehow one has to prohibit such a possibility, by insisting that all "memory cells" in you computer that *cannot* be reached in less than, say, 10^{10} time units are empty, nothing is written in these "far away cells". Moreover, the computer is supposed to know when it crosses the boundary of "non-empty space" and will not spend any time in searching the empty one.

On the other hand, the computer is allowed, in the course of a computation, to write/erase in these "far away cells"; this is what may eventually generate an enormous volume of occupied memory cells and make its reading excruciatingly long.

With this provision, the definition of **SToP**(L) becomes correct, it does give you something *finite*, PROVIDED you have *precisely* defined what "far away cells" and "reaching something in the memory" mean.

But can one *explicitly* describe in *finitely many* words an *infinite* memory along with a description of a search program through this memory?

A commonly accepted solution articulated by Turing, is to assign memory cells/units to *all* numbers $1, 2, 3, 4, \ldots, 1000\ldots$ with labels "*empty*"/"*non-empty*" attached to them, and with the symbols 1 and 0 written along with all "*non-empty*" marks. Then an individual step of a memory search is defined as moving from an i-cell to $i+1$ or $i-1$, where each cell is labeled "*non-empty*" after having been visited.

If you are susceptible to the magic of the word "all" and believe this truly defines the infinity of numbers, then you will have a "mathematically precise" definition of **SToP**(L).[4]

All this being said, there is something not quite right with **SToP**: it is a byproduct of the machinery of formal logic – not a *true mathematical* number. But about thirty years ago, **pretty** $\mathcal{HUGE}$ numbers were discovered; e.g., the time Hercules needs to slay Hydra in a mathematical model of this regrettable parricide. But all such $\mathcal{GIANT}$ $\mathcal{BEAUTIES}$ are incomparably smaller than the ugly looking **SToP**.

[4] There are, probably, 5–10 mathematicians and logicians in the world who think that the monsters like **SToP** are indicators of fundamental flaws in our concepts: "number", "finite", "infinite".

4. Laws

Lex 1: *Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum illum mutare.*

Isaac Newton, Philosophiæ Naturalis Principia Mathematica, 1687[5]

… les lois générales, connues ou ignorées, qui règlent les phénomènes de l'univers, sont nécessaires et constantes.

Nicolas Condorcet

So much as we know of them [the laws of nature] *has been developed by the successive energies of the highest intellects.*

Michael Faraday

The most astounding single event in the history of science, a physicist argued, was the discovery of *general relativity* in 1916 – this would have been postponed by 20–30 years if not for Einstein, twice as long as any other scientific discovery, according to our physicist's calculation on the back of an envelope.

Our physicist, a theoretician, could not confirm his calculation by an experiment, but history has done it for him and … proved him wrong.

The true logic of this world is the calculus of probabilities.

James Clerk Maxwell

[5]Historically, the *first recorded* LAW – the velocity/force formula for *motion in a viscous medium* – was stated by Aristotle. (Later versions of this were suggested by Newton, 1687, and by Stokes, 1851.) A century after Aristotle, Archimedes discovered basic laws of *statics of mechanical systems*: The laws of the lever and of equilibrium of solid bodies in liquids.

Mendel's theory of genes – units of inheritance – was published in 1866, where Mendel derived their very existence and essential properties from

the striking regularity with which the same hybrid forms always reappeared

in thousands of his experiments with pea plants.

The discovery of genes was the greatest event in biology since the discovery of *cells* by Robert Hooke in 1665 and of *infusoria* by Antonie van Leeuwenhoek in 1674.

Mendel's methodology of

combinatorial design of multistage interactive experiments

+

extracting specific structural information from statistics of observable data by mathematical means

was novel for all of science. This is why Mendel's paper was ignored by biologists for about thirty years.

When similar data were obtained and analyzed by de Vries, Correns, and Tschermak at the turn of the 20th century, biologists returned to Mendel's paper. Many, including Alfred Russel Wallace, were appalled by Mendel's ideas, while even the most sympathetic ones were confounded by Mendel's "counterintuitive" and "biologically unfeasible" algebra.

In 1908 a leading English mathematician G.H. Hardy and independently a German physician Wilhelm Weinberg spelled out this *counterintuitive* as

$$\frac{\left[(p+q)^2+(p+q)(q+r)\right]^2}{[(p+q)^2+(p+q)(q+r)]\cdot[(p+q)(q+r)+(q+r)^2]}=\frac{(p+q)^2}{(p+q)\cdot(q+r)}$$

and Mendel's Laws of Inheritance were accepted by (almost) everybody.

(Hardy called this *a mathematics of the multiplication table type*. He overlooked the mathematical beauty of *Mendelian dynamics* of the *next generation maps* M in the spaces of *truncated polynomials* – the maps that represent transformations of *distributions of alleles* in populations under *random matings*. In a simple instance, such an M applies to matrices $P=(p_{ij})$ by substituting each (i,j)-entry p_{ij} by the product of the sums of the entries in the i-row and the j-column in P,

$$(p_{ij})=P\overset{M}{\mapsto}P^{\text{next}}=(p_{ij}^{\text{next}})\text{ for }p_{ij}^{\text{next}}=\sum_i p_{ij}\cdot\sum_j p_{ij}.$$

It is amazing, albeit obvious, that $M(M(P))=\text{const}\cdot M(P)$ for const $=\sum_{ij}p_{ij}$, where this amounts to the above (p,q,r)-formula for symmetric 2×2 matrices in Hardy's notation.)

> *... are we justified in regarding them* [genes] *as material units; as chemical bodies of a higher order than molecules? ... It makes no difference in ... genetics. Between the characters that are used by the geneticist*

and the genes his theory postulates lies . . . embryonic development.

THOMAS HUNT MORGAN, 1934

In 1913, almost half a century after the publication of Mendel's paper *Versuche über Pflanzen-Hybriden*, 21-year-old Alfred Sturtevant made the next step along Mendelian lines of logic and determined *relative positions* of certain genes on one of the chromosomes of Drosophila by analyzing frequencies of specific morphologies in generations of suitably interbred flies.

Just think about it. You breed fruit flies, you count how many have particular *combinations* of certain features; i.e., you record the distribution of occurrences of the following eight ($2 \times 2 \times 2$)-possibilities:

[striped bodies]/[*yellow* bodies]
[red eyes]/[*white* eyes]
[normal wings]/[*smallish* wings]

Then you distinctly see with your mathematically focused mind's eye (even if you happened to be as color blind as Sturtevant) that the corresponding genes – abstract entities of Mendel's theory – that are associated with these features, are all lying in definite relative positions on an imaginary line, where, as for Mendel's genes, the existence of this line is derived from how *hybrid forms reappear*. In particular, you assign the "eyes gene" a position *between* the "body gene" and the "wing gene", since

[smallish wings] + [yellow bodies] "**implies**" [white eyes]

in descendants of certain parents with an abnormally high probability.

Decades later, molecular biology and sequencing technology demystified "Sturtevant's line" by identifying it with a DNA string segmented by genes, but a mathematical unfolding of Sturtevant's idea still can be seen only in dreams.

ABOUT ROBERT HOOKE, ANTONIE VAN LEEUWENHOEK, DROSOPHILA MELANOGASTER AND THE IDEA OF STURTEVANT.

Hooke's name is associated with *Hooke's elasticity law* but that was only one of many of his experimental discoveries, original concepts, and practical inventions. For example, he recognized fossils as the remains of *extinct species*, he developed an almost modern *model of memory*, he proposed (1665?) the construction of *the spring balance watch* (Huygens' description of his own construction dates to 1675) and he suggested (1684) a detailed design of an *optical telegraph with semaphores*. (The first operational system with a network of ≈ 500 stations was built in 1792 in France.)

Leeuwenhoek found out how to obtain small glass balls for the lenses of his microscopes but he made others to believe he was grinding tiny lenses day and night by hand. Besides infusoria, he observed and described crescent-shaped bacteria (*large Selenomonads*), cell vacuoles, and spermatozoa. The secret of Leeuwenhoek's microscopes was rediscovered in 1957.

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Drosophila melanogaster – the pomace fly ($\approx$ 2.5 mm long, normally with red-eyes and stripped bellies) – were introduced as a major model organism in genetics by Thomas Hunt Morgan. He and his students were counting the mutant characteristics of thousands of flies and studied their inheritance. Analysing these data, Morgan demonstrated that genes are carried on chromosomes; also he introduced the concepts of *genetic linkage* and of *crossing over*.

Sturtevant mathematically "synthesised" his string of genes from the "substrate" of results and ideas taken from the work of Morgan almost as Kepler "crystallized" elliptical orbits from Tycho Brahe's astronomical tables. Sturtevant recalls the great moment as follows.

> *I suddenly realized that the variations in strength of linkage, already attributed by Morgan to differences in the spatial separation of the genes, offered the possibility of determining sequences in the linear dimension of a chromosome. I went home and spent most of the night (to the neglect of my undergraduate homework) in producing the first chromosome map, which included the sex-linked genes y, w, v, m, and r, in the order and approximately the relative spacing that they still appear on the standard maps.*

Sturtevant's idea of (re)construction of the (a posteriori linear) geometry of the genome is similar to Poincaré's suggestion for how the brain (re)constructs the (a posteriori Euclidean) geometry of the external world from a set of samples of retinal images.

Grossly oversimplifying, an unknown geometric (or non-geometric) structure S from a given class $\mathcal{S}$ on a set X under study – be it the set of (types of) genes in the genomes (of organisms) of a given species or the set of photoreceptor cells in the retina – is represented by some probability measure on the set of subsets Y of X. What is essential, this measure is supported on $\mathcal{S}$-*simple* (*special*) *subsets* Y that admit *short descriptions in the language of* $\mathcal{S}$; this allows a reconstruction of S from relatively few samples.

This sampling is far from random. In genetics, such a $Y = Y(O)$ is the subset of genes of particular allele versions in the genome of an individual organism O, where these O are obtained via a *controlled* breeding protocol that had been specifically designed by an experimenter.

In vision, such a $Y = Y(t)$ is the set of excited photoreceptor cells in the retina of your eye at a given moment t, where more often than not the variation $Y(t)$ with t is due to a motion of an object relative to your eye; the brain, which commands the muscles that move your eye, has an ability to design/control such variations.

The hardest step in finding S is *guessing* what $\mathcal{S}$ is. After all, what is *a structure*?

Mendel's laws are no more than a Platonic shadow – a statistically averaged image of the workshop of Life on the flat screen of numbers. The molecular edifice of the cell is crudely smashed on this screen and its exquisite structure cannot be reconstructed from this image by pure thought. Hundreds (thousands?) of ingenious experiments are needed to recover the enormousness of information that had been lost.

> *... I hold it true that pure thought can grasp reality,*
> *as the ancients dreamed.* ALBERT EINSTEIN

Contrary to what we see in biology, the mathematical image of the basic machinery running the world of physics retains the finest details of this machinery. It even may seem, probably, only to our *Naive Mathematician*, that the less you know the better you understand how the Universe is run.

For example, forget about velocities, forces, accelerations. Imagine a world exclusively populated by wandering watches that have no perception of speed and force. But when two watches meet, they can recognize each other and compare their records of the intervals of times between consecutive meetings.

A watch-mathematician would sum up what he/she believes he/she "sees" in the watch-world as *self-obvious axioms* and, after pondering for a few centuries on what they imply, he/she will figure out that there is a unique simplest most symmetric *watch space* of every given dimension. It is the *Lorentz–Minkowski time-space*, that is four-dimensional in the Universe in which we happened to exist.

The mathematician will be delighted by this marvelous space-idea; yet perplexed, since his/her mental picture of the world does not explain

> *why the watches that have no physical contact apart from their meeting points remain synchronous.*

(Here, on Earth, it is not this incredible synchronisation but its violation that is regarded as paradoxical.)

But then his friend physicist conceives the idea of speed and his colleague experimentalist designs fast traveling watches. The mathematician sighs with relief: the formulas of his/her theory (that is called *special relativity* on Earth), are perfectly right and desynchronisation is clearly seen for watches traveling with mutual relative speeds close to 1. (On Earth, this 1, that is the speed of light, is elegantly expressed as 299 792 458 ... × another unit of speed the meaning of which no watch-mathematician has ever been able to grasp.)

> *Lines of force convey a far better and purer idea ...*
>
> Michael Faraday, 1833[6]

> *Thy reign, O force! is over. Now no more*
> *Heed we thine action;*
> *Repulsion leaves us where we were before,*
> *So does attraction.*
>
> James Clerk Maxwell, 1876

> *The theory of relativity by Einstein ...*
> *cannot but be regarded as a magnificent work of art.*
>
> Ernest Rutherford[7]

[6] *Faraday must have grasped with unerring instinct ... spatial states, today called fields, ...*
Albert Einstein, 1940

[7] Rutherford, named the Faraday of nuclear physics, experimentally identified alpha, beta and gamma emissions, discovered atomic nuclei and proposed protons and "nuclear electrons" for their constituents. (Neutrons were discovered and fit into the model of the nucleus about 20 years later.) *"All science is either physics or stamp collecting"* is a saying attributed to him. He, apparently, regarded chemistry as a part of physics and he did not live to see the birth of molecular biology from the seed of the Mendelian-chromosome theory of heredity.