Asset Analytics
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Logistics, Supply Chain and Financial Predictive Analytics

Theory and Practices

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Asset Analytics

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Cooperative/Non-cooperative Supply Chain Models for Imperfect Quality Items with Trade Credit Financing

Rita Yadav, Sarla Pareek and Mandeep Mittal

1 Introduction

Game theory is a powerful mathematical tool, which studies the capacity of the players. This theory plays a significant role in the area of supply chain connected problems whose purpose is to construct supply chain policies under numerous assumptions with different perspectives. These policies demonstrate coordination between several channels in the supply chain to get effectual outcome. Now a days, the maximum supply chain industries are using credit period policy to improve the profit of both the partners of the supply chain. The trade credit policy is generally offered to the buyer by the seller which is authorized settlement between buyer and seller for the late payment. Many researchers explored their study in this area. Hayley and Higgins [1] studied the buyer's lot size problem having a trade credit contract by assuming a fixed demand and showed that lot size is not affected by the length of trade credit period. Kim et al. [2] formulated a mathematical model to find out the optimal trade credit period with the assumption that the seller's price is fixed. Hwang and Shinn [3] showed in his study that the order quantity of the buyer fluctuates with the length of the trade credit period by considering demand, price sensitive.

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Primarily, Schwaller [4] developed *EOQ* models on defective items by counting inspection cost. Some researchers like Jamal et al. [5] and Aggarwal and Jaggi [6] considered under the fixed demand, the deteriorating item problems with permissible delay in payments, the market demand depends on the selling price of the buyer; Jaggi and Mittal [7] established policies for imperfect quality deteriorating items under inflation and permissible delay in payments. Thangam and Uthaya [8] developed an *EPQ* model for perishable items and with trade credit financing policy in which demand depends on both the variable's selling price and the trade credit period. Under permissible delay in payments, Jaggi and Mittal [9] developed an inventory model for imperfect quality items in which shortages are permitted and fully backlogged. Shortages are eliminated during the screening process and with the assumption that demand rate is less than the screening rate. Zhou et al. [10] developed a specific condition in the Supplier-Stackelberg game in which the trade credit policy not only increases the overall supply chain profit but also gives benefits to each partner of the supply chain.

Zhou et al. [11] also found a synergic economic order quantity model, in which the concept of imperfect quality, shortages with trade credit, and inspection errors are considered. Abad and Jaggi [12] deliberated supply chain model, in which the end market demand is price sensitive and with trade credit period offered by the seller to the buyer. They developed non-cooperative (Seller-Stackelberg game) and cooperative (Pareto-efficient solution) relationship between buyer and seller. Esmaeili et al. [13] also explained cooperative and non-cooperative games in which the end demand was price sensitive as well as marketing expenditure sensitive. This work was extended by Esmaeili and Zeephongscul [14] for asymmetric information game. Zhang and Zeephongsekul [15] investigated non-cooperative models with credit option by assuming the same demand function.

None of the researchers considered the effects of imperfect production on the supply chain model in the cooperative and non-cooperative environment with trade credit financing. In this paper, we develop a supply chain model for imperfect items with credit option which is offered by the seller. Seller delivers a lot to the buyer. In a delivered lot, some items may not be of perfect quality. Thus, buyer applies inspection process to separate defective items on the supplied lot. The perfect quality items are sold at selling price, and imperfect items are sold at a discounted price. In this paper cooperative and non-cooperative relationship are derived between the two players (buyer and seller) of the game. Effect of credit period is also considered when the demand is price sensitive.

This paper consists of seven sections. The first section consists of an introduction and literature review. The second section introduces notation and assumptions used in the paper. The third section formulates the noncooperative Seller-Stackelberg mathematical model, in which seller is the leader and buyer is the follower. In the fourth section, we discuss the cooperative game model with Pareto-efficient approach. In the fifth and sixth sections, we provide numerical examples with sensitivity analysis. The last section consists of conclusion part with suggestions for future research work.

2 Notations and Assumptions

2.1 Notations

The notations that are used in the proposed models are given below:

Decision variables	
c_b	Buyer's unit purchasing cost ($/unit)
Q	Order quantity of the buyer (units)
M	Credit period offered to the buyer by the seller (year)
Parameters	
A_b	Ordering cost of the buyer ($/order)
H_b	Inventory carrying cost ($/unit/unit time)
p_b	Buyer's retail price ($/unit)
T	Cycle time in years
I_s	Seller's opportunity cost of capital ($/year)
A_s	Ordering cost of the seller ($/order)
I	Inventory carrying cost ($/year)
I_e	Interest earned rate for the buyer ($/year)
I_p	Interest paid rate to the buyer ($/year)
D	Annual demand rate (unit/year)
α	Percentage of defective items delivered by the seller to the buyer
λ	Buyer's screening rate (unit/year)
c_s	Cost of defective items per unit ($/year)
C	Seller's unit purchasing cost ($/unit)
T	Required time to screen the defective items, $t = Q/\lambda$ (years)

2.2 Assumption

(1) The annual demand is price sensitive; i.e., $D = Kp^{-e}$ Abad and Jaggi [12].
(2) Shortages are not taken into consideration, and planning horizon is infinite.
(3) In each lot, there is α percentage defective items with known probability density function. In order to avoid shortage, the demand has to be less than the screened lot, Jaggi et al. [16].

(4) A specific credit period is offered by the seller to the buyer. In this paper, interest earned by the buyer, I_e, and interest paid, I_p, are considered equal, Zhang and Zeephongsekul [15].
(5) $I_s = a_1 + a_2 M$, $a_1 > 0$, $a_2 > 0$, Seller's opportunity cost of capital is assumed as linear and function of credit period [1, 2].
(6) There is no carrying cost that links with lot size as the seller considers a lot to lot strategy.

3 Mathematical Models

In this section, mathematical formulation of buyer's model, seller's model, and Seller-Stackelberg game model are explained and optimal solutions are found.

3.1 The Buyer's Model

In this model, the objective of the buyer is to determine the lot size/order quantity, Q, and selling price which maximizes his expected profit. In the lot size, Q, α Q, items are defective and sold at a discounted price c_s and $(1-\alpha)Q$ are non-defective items and are sold at a price p_b. Therefore, the total revenue of the buyer is $p_b(1-\alpha)Q + c_s\alpha Q$.

There are three possible cases (Fig. 1):

Case I: $M \le t \le T$

The interest earned by the buyer for the inventory in the time period 0 to M is $D\frac{M^2 I_e c_b}{2}$, and the interest paid by the player buyer for the inventory not sold after the credit period is $\frac{D(T-M)^2 I_p c_b}{2} + c_s I_p \alpha Q(t-M)$. Hence, buyer's total profit is expressed as $TP_{b1}(p_b, Q)$.

$TP_{b1}(p_b, Q)$ = Sales Revenue − Purchasing cost − Ordering cost − Inventory carrying cost + Interest gain − Interest paid

$$= p_b(1-\alpha)Q + c_s\alpha Q - c_b Q - A_b - \left(\frac{Q(1-\alpha)T}{2} + \frac{\alpha Q^2}{\lambda}\right)H_b + D\frac{M^2 I_e c_b}{2}$$
$$- \frac{D(T-M)^2 I_p c_b}{2} - c_s I_p \alpha Q(t-M)$$

Put $T = \frac{(1-\alpha)Q}{D}$, $t = \frac{Q}{\lambda}$, $H_b = \mathrm{I}c_b$, then buyer's profit is

$$TP_{b1}(p_b, Q) = p_b(1-\alpha)Q + c_s\alpha Q - c_b Q - A_b - \left(\frac{Q^2(1-\alpha)^2}{2D} + \frac{\alpha Q^2}{\lambda}\right)\mathrm{I}c_b + D\frac{M^2 I_e c_b}{2}$$
$$- \frac{D\left(\frac{(1-\alpha)Q}{D} - M\right)^2 I_p c_b}{2} - c_s I_p \alpha Q\left(\frac{Q}{\lambda} - M\right)$$

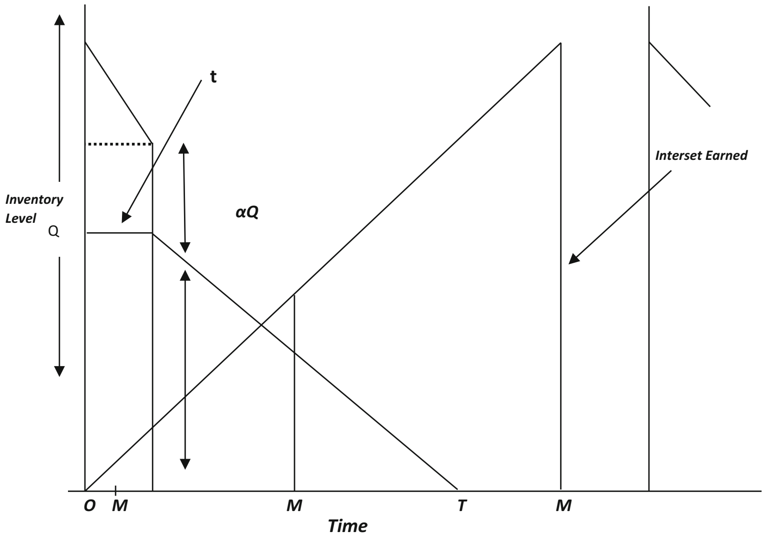

Fig. 1 Inventory system with inspection and trade credit for all the three cases $(i) M \leq t \leq T \quad (ii) t \leq M \leq T \quad (iii) M \geq T$

$$= p_b(1-\alpha)Q + c_s\alpha Q - c_bQ - A_b - \left(\frac{Q^2(1-\alpha)^2}{2D} + \frac{\alpha Q^2}{\lambda}\right)\mathrm{I}c_b + D\frac{M^2 I_e c_b}{2}$$
$$-\frac{D}{2}\left(\frac{Q^2(1-\alpha)^2}{D^2} + M^2 - 2QM(1-\alpha)I_pc_b\right) - c_sI_p\alpha\frac{Q^2}{\lambda} + c_sI_pQM\alpha$$

Thus, the buyer's total expected profit is given by

$$E\left[TP_{b1}(p_b, Q)\right] = p_bE[1-\alpha]Q + c_sE[\alpha]Q - c_bQ - A_b - \left(\frac{Q^2E\left[(1-\alpha)^2\right]}{2D} + \frac{E[\alpha]Q^2}{\lambda}\right)Ic_b$$
$$+ D\frac{M^2I_ec_b}{2} - \left(\frac{Q^2E\left[(1-\alpha)^2\right]}{2D} + \frac{DM^2}{2} - QME[1-\alpha]\right)I_pc_b - c_sI_pE[\alpha]\frac{Q^2}{\lambda}$$
$$+ c_sI_pQME[\alpha]$$
$$= -\left[\left(\frac{E\left[(1-\alpha)^2\right]}{2D} + \frac{E[\alpha]}{\lambda}\right)Ic_b + \left(\frac{E\left[(1-\alpha)^2\right]}{2D}\right)I_pc_b + c_sI_p\frac{E[\alpha]}{\lambda}\right]Q^2$$
$$+ \left[p_b(1-E[\alpha] + c_sE[\alpha] - c_b + M(1-E[\alpha]I_pc_b + c_sI_pME[\alpha]\right]Q - A_b$$
$$+ D\frac{M^2I_ec_b}{2} - D\frac{M^2I_pc_b}{2}$$

By using the concept of renewal theory used in the paper Maddah and Jaber [17], the buyer's expected total profit per cycle is

$$E\left[TP_{b1}^c(p_b, Q)\right] = \frac{E\left[TP_{b1}(p_b, Q)\right]}{E(T)}$$

$$= \frac{D}{Q(1-E[\alpha])} E\big[TP_{b1}(p_b, Q)\big]$$

$$E\big[TP^c_{b1}(p_b, Q)\big] = \frac{D}{Q(1-E[\alpha])}\Bigg[-\Bigg[\Bigg(\frac{E\big[(1-\alpha)^2\big]}{2D} + \frac{E[\alpha]}{\lambda}\Bigg)Ic_b + \Bigg(\frac{E\big[(1-\alpha)^2\big]}{2D}\Bigg)I_p c_b$$
$$+ c_s I_p \frac{E[\alpha]}{\lambda}\Bigg]Q^2 + \big[p_b(1 - E[\alpha] + c_s E[\alpha] - c_b + M(1 - E[\alpha]I_p c_b + c_s I_p M E[\alpha]\big]Q - A_b$$
$$+ D\frac{M^2 I_e c_b}{2} - D\frac{M^2 I_p c_b}{2}\Bigg] \quad (1)$$

Case II: $t \le M \le T$

In this case, interest gain due to credit balance in the time period 0 to M is $D\frac{M^2 I_e p_b}{2} + c_s I_e \alpha Q(M - t)$ and interest paid by the buyer for the period M *to* T is $\frac{D(T-M)^2 I_p c_b}{2}$. The total profit for the player buyer is expressed as T$P_{b2}(p_b, Q)$.

$TP_{b2}(p_b, Q)$ = Sales Revenue − Purchasing cost − Ordering cost − Inventory carrying cost + Interest gain − Interest paid

$$TP_{b2}(p_b, Q) = p_b(1-\alpha)Q + c_s\alpha Q - c_b Q - A_b - \left(\frac{Q(1-\alpha)T}{2} + \frac{\alpha Q^2}{\lambda}\right)\mathrm{I}c_b + D\frac{M^2 I_e p_b}{2}$$
$$+ c_s I_e \alpha Q(M - t) - \frac{D(T-M)^2 I_p c_b}{2}$$

Put $T = \frac{(1-\alpha)Q}{D}, t = \frac{Q}{\lambda}$

Thus, the total expected buyer's profit is given by

$$TP_{b2}(p_b, Q) = p_b(1 - E[\alpha])Q + c_s E[\alpha]Q - c_b Q - A_b - \left(\frac{Q^2 E[(1-\alpha)^2]}{2D} + \frac{E[\alpha]Q^2}{\lambda}\right)\mathrm{I}c_b$$
$$+ D\frac{M^2 I_e p_b}{2} + c_s I_e E[\alpha]Q\left(M - \frac{Q}{\lambda}\right) - \frac{1}{2}D\left(\frac{(1-E[\alpha])Q}{D} - M\right)^2 I_p c_b$$

Using renewal theory used in the paper Maddah and Jaber [17], the buyer's expected total profit per cycle will be

$$E\big[TP^c_{b2}(p_b, Q)\big] = \frac{E\big[TP_{b2}(p_b, Q)\big]}{E(T)}$$
$$= \frac{D}{Q(1-E[\alpha])}\big[p_b(1 - E[\alpha])Q + c_s E[\alpha]Q - c_b Q - A_b$$
$$-\left(\frac{Q^2\big[E(1-\alpha)^2\big]}{2D} + \frac{E[\alpha]Q^2}{\lambda}\right)\mathrm{I}c_b + D\frac{M^2 I_e p_b}{2} + c_s I_e E[\alpha]Q\left(M - \frac{Q}{\lambda}\right)$$
$$-\frac{1}{2}D\left(\frac{(1-E[\alpha])Q}{D} - M\right)^2 I_p c_b\Bigg] \quad (2)$$

Case III: $t \le T \le M$

For this case, the buyer's total earned interest is $D\frac{T^2 I_e p_b}{2} + p_b I_e DT(M - T) + c_s I_e \alpha Q(M - t)$ and there is no interest which is payable to the seller by the buyer. The buyer's total profit is expressed as $TP_{b3}(p_b, Q)$.

$TP_{b3}(p_b, Q)$ = Sales Revenue − Purchasing cost − Ordering cost − Inventory carrying cost + Interest gain due to credit

$$= p_b(1-\alpha)Q + c_s\alpha Q - c_b Q - A_b - \left(\frac{Q(1-\alpha)T}{2} + \frac{\alpha Q^2}{\lambda}\right)\mathrm{I}c_b + D\frac{T^2 I_e p_b}{2} + p_b I_e DT(M - T) + c_s I_e \alpha Q(M - t)$$

Put $T = \frac{(1-\alpha)Q}{D}$, $t = \frac{Q}{\lambda}$, thus the total expected buyer's profit is given by

$$\mathrm{T}P_{b3}(p_b, Q) = p_b(1 - E[\alpha])Q + c_s E[\alpha]Q - c_b Q - A_b - \left(\frac{Q^2\left[E(1-\alpha)^2\right]}{2D} + \frac{E[\alpha]Q^2}{\lambda}\right)\mathrm{I}c_b + D\frac{\frac{(E[(1-\alpha)^2])Q^2}{D^2} I_e p_b}{2} + p_b I_e D\left[\frac{(1-E[\alpha])Q}{D}M + \frac{\left(E\left[(1-\alpha)^2\right]\right)Q^2}{D^2}\right] + c_s I_e E[\alpha]Q\left(M - \frac{Q}{\lambda}\right)$$

By using the concept of renewal theory as used in the paper Maddah and Jaber [17], the buyer's expected total profit per cycle is

$$E\left[TP_{b3}^c(p_b, Q)\right] = \frac{E\left[TP_{b3}(p_b, Q)\right]}{E(T)} = \frac{D}{Q(1-E[\alpha])}\left[p_b(1 - E[\alpha])Q + c_s E[\alpha]Q - c_b Q - A_b - \left(\frac{Q^2 E\left[(1-\alpha)^2\right]}{2D} + \frac{E[\alpha]Q^2}{\lambda}\right)\mathrm{I}c_b + \frac{1}{2}\frac{(E[(1-\alpha)^2])Q^2}{D} I_e p_b + p_b I_e D\left[\frac{(1-E[\alpha])Q}{D}M + \frac{\left(E\left[(1-\alpha)^2\right]\right)Q^2}{D^2}\right] + c_s I_e E[\alpha]Q\left(M - \frac{Q}{\lambda}\right)\right] \tag{3}$$

Under the assumption $I_e = I_p$, it is found that the mathematical expression for buyer's expected profit per cycle for case I, case II, and case III is same [15] and is denoted by $E\left[TP_b^c(p_b, Q)\right]$

Demand is assumed to be price sensitive, $D = Kp_b^{-e}$

Thus, the buyer's expected total profit can be written as

$$E\left[TP_b^c(p_b, Q)\right] = Kp_b^{-e}\left[p_b - A_1 Q - A_2 - \frac{A_3}{Q}\right] - c_b A_4 Q \tag{4}$$

Let, $A_1 = \frac{E[\alpha]Ic_b}{(1-E[\alpha])\lambda} + \frac{c_s I_e E[\alpha]}{(1-E[\alpha])\lambda}$

$$A_2 = \frac{c_b}{(1 - E[\alpha])} - \frac{c_s E[\alpha]}{(1 - E[\alpha])} - \frac{c_s I_e E[\alpha]M}{(1 - E[\alpha])} - c_b I_p M$$

$$A_3 = \frac{A_b}{(1 - E[\alpha])} \quad , \quad A_4 = \frac{E\left[(1-\alpha)^2\right]}{2(1 - E[\alpha])}\left(\mathrm{I} + I_p\right)$$

Differentiating Eq. (4) with respect to p_b for a fixed Q and equating to zero. The resultant equation gives the unique value of p_b that maximize $E\left[TP_b^c(p_b, Q)\right]$. The value of p_b is given below:

$$p_b = \frac{e}{e-1}(A_1 Q + A_2 + \frac{A_3}{Q}) \tag{5}$$

where A_1, A_2, and A_3 are defined by Eq. (4)

Putting the value of p_b into Eq. (4),

$$E\left[TP_b^c\left(p_b^*(Q), Q\right)\right] = \frac{K}{e}\left(\frac{e}{e-1}\left[A_1 Q + A_2 + \frac{A_3}{Q}\right]\right)^{-e+1} - c_b A_4 Q \tag{6}$$

The first-order condition for maximization with respect to Q yields,

$$Q = \frac{k A_3 p_b^{-e}}{c_b A_4 + k A_1 p_b^{-e}} \tag{7}$$

And second-order differentiation of $E\left[TP_b^c(p^*(Q), Q)\right]$ with respect to Q

$$\frac{\partial^2}{\partial Q^2} E\left[TP_b^c\left(p_b^*(Q), Q\right)\right] = \frac{e^2}{(e-1)p_b}\left(A_1 - \frac{A_3}{Q^2}\right)^2 - 2\frac{A_3}{Q^3} \tag{8}$$

$$\text{where} \quad p_b = \frac{e}{e-1}\left(A_1 Q + A_2 + \frac{A_3}{Q}\right)$$

$\frac{\partial^2 E\left[TP_b^c(p,Q)\right]}{\partial p_b^2} > 0$, $\frac{\partial^2 E\left[TP_b^c(p,Q)\right]}{\partial Q^2} > 0$ and

$$\left[\frac{\partial^2 E\left[TP_b^c(p, Q)\right]}{\partial p_b^2}\right]\left[\frac{\partial^2 E\left[TP_b^c(p, Q)\right]}{\partial Q^2}\right] - \left[\frac{\partial^2 E\left[TP_b^c(p, Q)\right]}{\partial p_b \partial Q}\right]^2 < 0$$

It is very difficult to prove the concavity of the expected total profit function analytically.

Thus, expected total profit $E\left[TP_b^c(p_b, Q)\right]$ in Eq. (4) is concave function with respect to p_b and Q for $e \geq 1$ is shown with the help of graph (Fig. 2). Further, first and second derivatives are defined in Appendix A in the end of the paper.

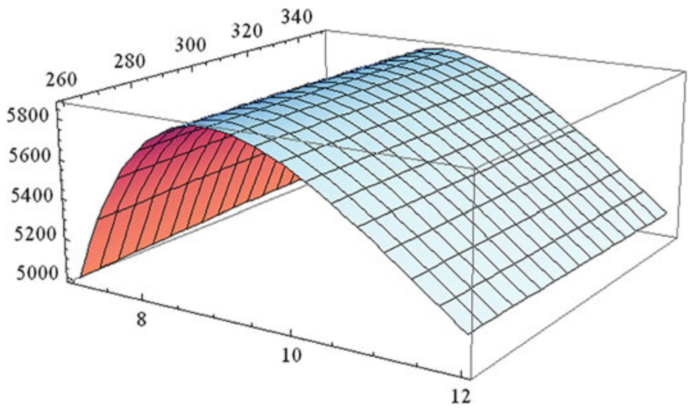

Fig. 2 Optimal buyer's total expected profit with respect to p_b and Q

3.2 Seller's Model

The seller transports products to the buyer by offering a fixed credit period, M to the buyer. The motive of the seller is to find his optimal policies which are selling price, c_b, and length of credit period, M, to maximize his expected profit. Supply chain's cycle length is $\mathrm{T} = (1-\alpha)Q/D$. The player seller considers a lot to lot strategy; thus, there is no inventory cost.

The seller's total annual profit function is

Seller profit = Sales Revenue − Purchasing cost − Ordering cost − Opportunity cost

$$\mathrm{Tp}_s(c_b, M) = c_bQ - CQ - A_s - (a_1 + a_2M)((1-\alpha)c_bM\ Q)$$

The seller's expected profit per cycle is

$$\begin{aligned} E\left[\mathrm{TP}_s^c(c_b, M)\right] &= \frac{D}{Q(1-E[\alpha])}\left[c_bQ - CQ - A_s - (a_1 + a_2M)((1-E[\alpha])c_bMQ)\right] \\ &= D\left[\frac{1}{(1-E[\alpha])}\left(c_b - C - \frac{A_s}{Q}\right) - (a_1 + a_2M)c_bM\right] \end{aligned} \tag{9}$$

For fixed c_b, the first-order condition with respect to M results in

$$\frac{\partial}{\partial M}E\left[\mathrm{TP}_s^c(c_b, M)\right] = -D[a_1c_b + 2a_2c_bM]$$

and second-order differentiation yields the result

$$\frac{\partial^2}{\partial M^2}E[\mathrm{TP}_s^c(c_b, M)] = -2a_2c_bD < 0.$$

Given the expected profit function, $E[\mathrm{TP}_s^c(c_b, M)]$ is concave for a fixed c_b

By Eq. (9), it can be easily seen that $E[\mathrm{TP}_s^c(c_b, M)]$ is linear in c_b,; therefore, the selling price of the seller is unbounded and is denoted by c_b^*, and the seller has to set the selling price by setting zero profit; i.e., $E(\mathrm{TP}_s^c(c_b, M)) = 0$, we have

$$c_{b_0} = \frac{4a_2\left(c + \frac{A_s}{Q}\right)}{4a_2 + a_1^2(1 - E[\alpha])}$$

$c_b^* = T\, c_{b_0} = T \frac{4a_2\left(C+\frac{A_s}{Q}\right)}{4a_2+a_1^2(1-E[\alpha])}$, for some $T > 1$ can be obtained through the mediation with the buyer.

3.3 The Non-cooperative Seller-Stackelberg Model

The Stackelberg model is a non-cooperative strategic game model in which both the players (seller and buyer) have interaction with each other. Among them, one player performs the leader and second player behaves as the follower. The leader moves first, and the follower moves sequentially. In this Seller-Stackelberg model, the seller player behaves as a leader, whereas buyer player behaves as follower. The seller (leader) moves first and offers credit period, M, and selling price,c_b to the player buyer. The buyer chooses his best response based on the policies given by the seller with an aim to increase his expected profit. Further, the seller chooses his best strategy for finding the optimum credit period and his selling price based on the buyer's best response.

$$\operatorname{Max} E(\mathrm{TP}_s^c(c_b, M))$$

Subject to the conditions

$$p_b = \frac{e}{e-1}(A_1 Q + A_2 + \frac{A_3}{Q}) \tag{10}$$

$$Q = \frac{kA_3 p_b^{-e}}{c_b A_4 + kA_1 p_b^{-e}} \tag{11}$$

$$\text{where} \quad A_1 = \frac{E[\alpha]Ic_b}{(1-E(\alpha))\lambda} + \frac{c_s I_e E[\alpha]}{(1-E(\alpha))\lambda}$$

$$A_2 = \frac{c_b}{(1-E[\alpha])} - \frac{c_s E[\alpha]}{(1-E[\alpha])} - \frac{c_s I_e E[\alpha] M}{(1-E[\alpha])} - c_b I_p M$$

$$A_3 = \frac{A_b}{(1 - E[\alpha])}, A_4 = \frac{E\left[(1-\alpha)^2\right]}{2(1 - E[\alpha])}\left(\mathrm{I} + I_p\right)$$

From Eq. (11), we get

$$p_b = \left(\frac{k\left(A_3 - A_1 Q^2\right)}{c_b A_4 Q^2}\right)^{1/e} \tag{12}$$

From Eqs. (10) and (12), we get

$$M = \left(\frac{c_b}{(1-E[\alpha])} - \frac{c_s E[\alpha]}{(1-E[\alpha])} - \frac{e-1}{e} p_b + A_1 Q + \frac{A_3}{Q}\right) / \left(\frac{c_s I_e E[\alpha]}{(1-E[\alpha])} + c_b I_p\right) \tag{13}$$

Putting the values of p_b and M into Eq. (9), this problem becomes

$$\text{Max}\, E(\mathrm{TP}_s^c(c_b, Q)) = \mathrm{K}\, p_b^{-e}[\frac{1}{(1-E[\alpha]))}(c_b - C - \frac{\mathrm{A}_s}{Q}) - (a_1 + a_2 M) c_b M] \tag{14}$$

where $p_b = \left(\frac{k\left(A_3 - A_1 Q^2\right)}{c_b A_4 Q^2}\right)^{1/e}$

$$M = \left(\frac{c_b}{(1-E[\alpha])} - \frac{c_s E[\alpha]}{(1-E[\alpha])} - \frac{e-1}{e} p_b + A_1 Q + \frac{A_3}{Q}\right) / \left(\frac{c_s I_e E[\alpha]}{(1-E[\alpha])} + c_b I_p\right)$$

$E[\mathrm{TP}_s^c(c_b, Q)]$ is a nonlinear objective function. Solution of the above problem can be obtained for different optimal values of Q and c_b.

4 The Cooperative Game

In the cooperative games, the interaction between different players is established. The Pareto-efficient solution is one approach for cooperative game for solving seller–buyer chain problem in which both the players make effort together with an aim to maximize their profit. This is an economic state or situation in which one player cannot make better off without making another player's worse off. In this approach, the objective is to optimize the profits of both the members by finding retailer price, p_b, selling price of the seller, c_b, trade credit, M, offered by seller and lot size, Q.

The Pareto-efficient solution concept can be obtained by maximizing the combined sum (weighted) of both the players, buyer and seller's expected profit [13].

$$E[JTP_{sb}] = \mu E[\mathrm{TP}_s^c] + (1-\mu) E[\mathrm{TP}_b^c], \quad 0 \le \mu \le 1 \tag{15}$$

$E[JTP_{sb}] = \mu D[\frac{1}{(1-E[\alpha])}(c_b - C - \frac{A_s}{Q}) - (a_1 + a_2 M) c_b M] + (1-\mu) K p_b^{-e}\left[p_b - A_1 Q - A_2 - \frac{A_3}{Q}\right] - c_b A_4 Q$, where A_1, A_2, A_3, and A_4 are defined by Eq. (4)

The first-order condition for maximizing $E[JTP_{sb}]$ with respect to c_b gives the following result

$$\mu = \frac{w_1}{w_1 - w}, \tag{16}$$

$$w = \left(\frac{D}{1 - E[\alpha])} - (a_1 + a_2 M)MD\right)$$

$$w_1 = I_p MD - D\frac{E[\alpha]IQ}{(1 - E[\alpha])\lambda} - \frac{E[(1-\alpha)^{2]}Q}{2(1 - E[\alpha])}\left(\mathrm{I} + I_p\right) - D\frac{1}{(1 - E[\alpha])}$$

And the first-order condition with respect to $Q, p_b\ and\ M$ yields the following results

$$Q = \sqrt{\left(\frac{\mu A_s D}{(1 - E[\alpha])} + (1-\mu)A_3 D\right)/(1-\mu)(A_1 D + c_b A_4)} \tag{17}$$

$$p_b = \frac{e}{e-1}\frac{(w_3(1-\mu) - \mu w_2)}{(1-\mu)}, \tag{18}$$

where

$$w_2 = \frac{1}{(1 - E[\alpha])}[c_b - C - \frac{A_s}{Q}] - (a_1 + a_2 M)c_b M, \quad w_3 = A_1 Q + A_2 + \frac{A_3}{Q}$$

$$M = \frac{1}{2\mu a_2 c_b}\left((1-\mu)\left(\frac{c_s I_e E[\alpha]M}{(1 - E[\alpha])} + c_b I_p M\right) - \mu a_1 c_b\right) \tag{19}$$

5 Numerical Examples

Example 1 Suppose $C = \$3$ units, $A_b = \$40$, $A_s = \$300$, $a_1 = 0.08, a_2 = 0.06, I_e = 0.15, I_p = 0.15$, $c_s = 5, \mathrm{I} = 0.12, e = 2.5$, $\mathrm{K} = 400000, \lambda = 125280$ unit/year. The fraction of imperfect quality item, α, uniformly distributed on $(a, b), 0 < a < b < 1$ *i.e.*, $\alpha \sim$ U(a, b) with $\alpha \sim$ U(a, b), $\mathrm{E}[\alpha] = \frac{a+b}{2}$ and can be determined with the formula $E[(1-\alpha)^2] = \int_a^b (1-\alpha)^2 f(\alpha) = \frac{a^2+ab+b^2}{3} + 1 - a - b$; the expected value of α is $E[\alpha] = 0.01$, $E[(1-\alpha)^2] = 0.980$, where $a = 0$ and $b = 0.02$. Then from Eq. (14), the results are $Q = 292$ units and $c_b = \$5.72$.Eqs. (12) and (13) yield $p_b = \$9.070$ and $M = 0.485$ years. With these results, the end demand $D = 1613$ units. The buyer's expected profit, $E\left[(\mathrm{TP}_b^c\right] = \$5631.330$, and the seller's profit $E\left[(\mathrm{TP}_s^c\right] = \$2266.440$. The cycle length $T = 0.179$ years.

Example 2 We consider that all parameters assume the same values of as defined in Example 1 except $c_s = 4$. Suppose the seller and buyer agree at $c_b = \$4.4/$unit through negotiation under the cooperative approach. Then using Eqs. (16)–(19), we obtain $\mu = 0.505$, $p_b = \$5.080$/unit, $Q = 2022$/unit, $M = 0.570$ years. The end demand D = 6877 units. The cycle length is 0.294. The seller's profit $E\left[(TP_s^c\right] = \$6725$, and the buyer's profit $E\left[(TP_b^c\right] = \$\ 5932$. It can be easily be seen from the numerical example that although selling price is less, but profits of the seller and buyer are high as compare to non-cooperative Seller-Stackelberg game because of high demand in this case. Through this approach, both players, seller and buyer, get benefited.

6 Sensitivity Analysis

Sensitivity analysis is performed to explore the impact of three parameters: fraction of defective items, α, price elasticity, e, the interest earned, I_e, on decision variable

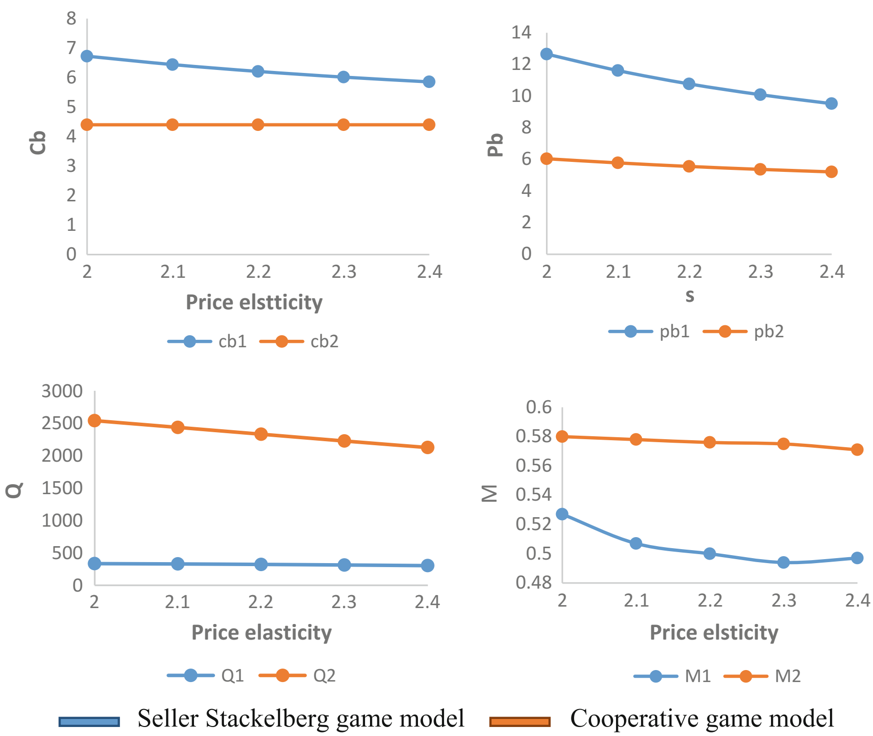

Fig. 3 Effect of e parameter on p_b, M, Q, C_b

of the players, c_b, p_b, M, Q in both the models in the Seller-Stackelberg game and cooperative game. Sensitivity analysis's results are shown in the graph.
Observations:

1. It is evident from Fig. 3 that as the value of $\boldsymbol{e}$ increases, then there are considerable decreases in selling price, less demand which results in decrease in buyer's profit. Findings suggest, there is decrease in the value of credit period, but cycle length increases which results in decrease in the demand and seller's expected profit (Seller-Stackelberg model).
2. It is depicted from Fig. 4 that when fraction of imperfect quality items increases, then order quantity increases. The buyer places orders more frequently as the imperfect rate increases. Demand depends upon the selling price, as the selling price decreases, then the demand increases (Seller-Stackelberg model).
3. It is also observed from the Fig. 5 that increase in I_e, increases credit period which implies that when interest earned by the buyer is high which results in

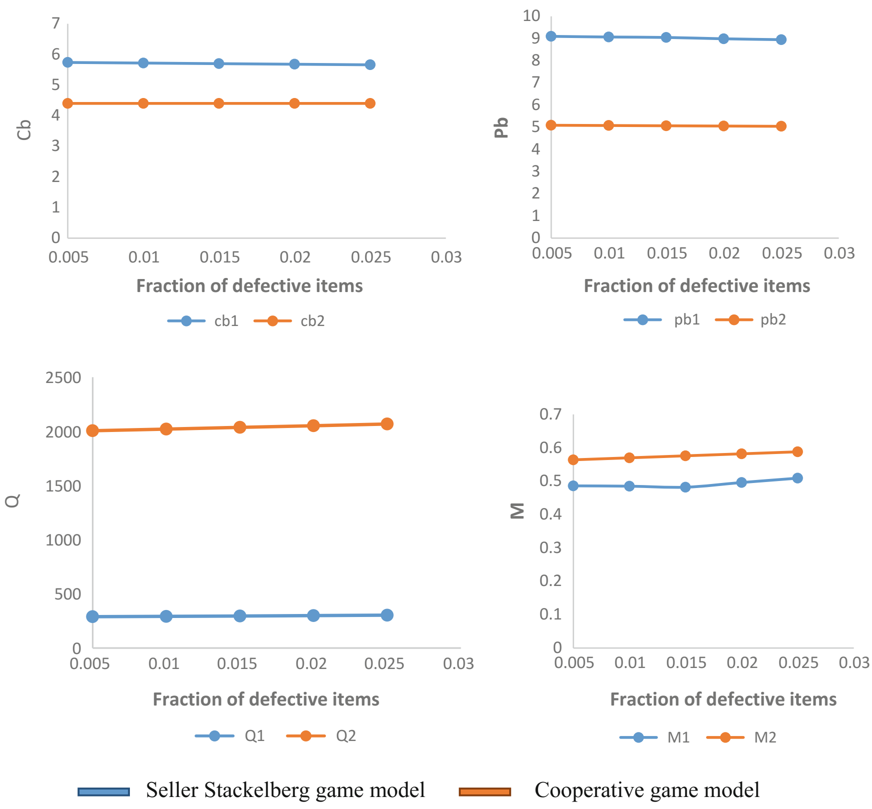

Fig. 4 Effect of α parameter on p_b, M, Q, C_b

higher expected profit. When I_e increase, order quantity decreases which results in less seller's profit (Seller-Stackelberg model).

4. It can be also seen from the Fig. 3 that as price of elasticity increase, the selling price decreases significantly and optimal order quantity decreases, which results decrement in seller's and buyer's profit (cooperative game). Selling price in the non-cooperative game is more than in the cooperative game, and optimal order quantity in the non-cooperative game is less than in the cooperative game. Findings suggest that the seller and the buyer get more profit in the cooperative game.
5. Further, Fig. 4 shows that as the expected number of imperfect quality items increases, the optimal order quantity increases, and the cycle length decreases marginally, whereas the retailer's expected profit decreases significantly (cooperative game). If we compare results of Seller-Stackelberg model with cooperative game model in this Fig. 4, we find that order quantity in the cooperative game is more than in the non-cooperative game where selling price is less in cooperative

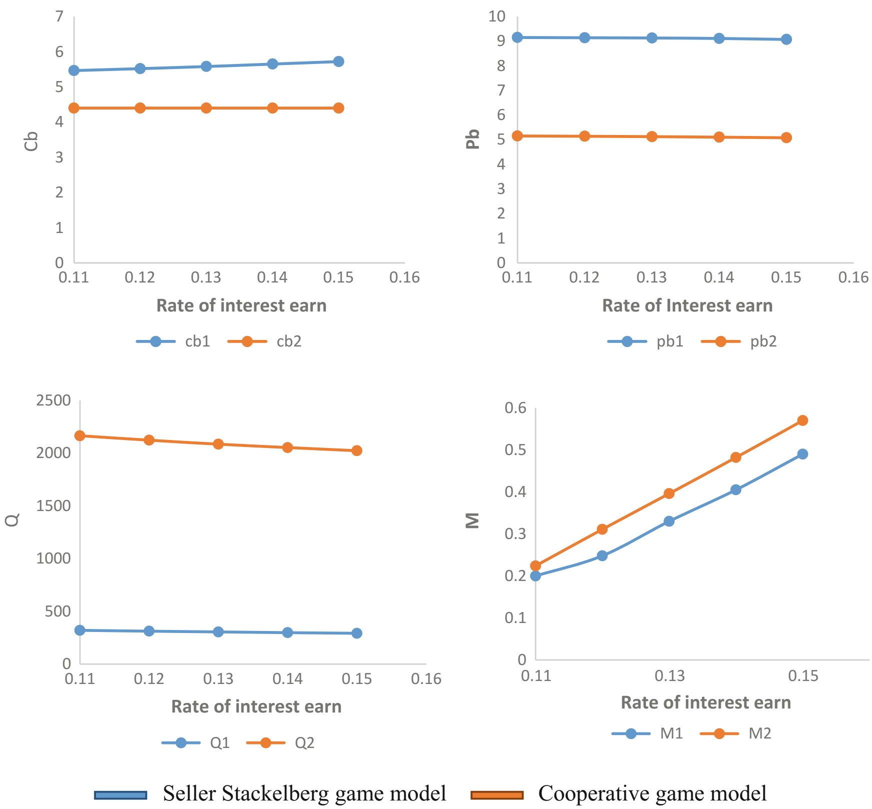

Fig. 5 Effect of I_e parameter on p_b, M, Q, C_b

game. When the number of imperfect quality items increases, the profit of both the players in the cooperative game is more than in the non-cooperative game.

6. Figure 5 investigates that order quantity and market demand in the non-cooperative game is more than in the cooperative game, whereas selling price of the buyer is less in the cooperative game.

7 Conclusions

In this study, non-cooperative and cooperative game theoretic approaches are discussed in which end demand depends on the selling price of the buyer with trade credit period offered by the seller to the buyer. Seller-Stackelberg game is discussed under non-cooperative approach, whereas in cooperative approach, Pareto-efficient solution is obtained. Sensitivity analysis is conducted to show the comparison between Seller-Stackelberg and Pareto-efficient solution concept. This study shows the effect of trade credit financing on the expected total profit of the players of the supply chain. Both the members of supply chain improve their expected profit by using credit financing policy. The study also shows that the expected profit function is impacted by the production of imperfect quality items. Further, it is also clearly observable from the numerical examples that order quantity is increasing due to effect of imperfect production of the items. The result shows that both the players would prefer cooperative game, because they got more profit in cooperative game approach than to non-cooperative game approach. Further, the presented research can be extended to supply chain under asymmetric information game environment.

Appendix 1

Expected profit function for the buyer is given by

$E\left[TP_b^c(p,Q)\right] = Kp_b^{-e}\left[p_b - A_1Q - A_2 - \frac{A_3}{Q}\right] - c_bA_4Q$, where A_1, A_2, A_3 and A_4 are defined by Eq. (4).

First and second derivative with respect to p_b, Q are given below

$$\begin{aligned}\frac{\partial E\left[TP_b^c(p,Q)\right]}{\partial p_b} &= kp_b^{-e} - kep_b^{-e-1}\left(p_b - A_1Q - A_2 - \frac{A_3}{Q}\right)\\ &= (1-e)kp_b^{-e} + kep_b^{-e-1}(A_1Q + A_2 + \frac{A_3}{Q})\end{aligned}$$

$$\frac{\partial^2 E\left[TP_b^c(p,Q)\right]}{\partial p_b^2} = e(e-1)kp_b^{-e-1} - ke(e+1)p_b^{-e-2}\left(A_1Q + A_2 + \frac{A_3}{Q}\right)$$

$$\frac{\partial E\left[TP_b^c(p,Q)\right]}{\partial Q} = kp_b^{-e}\left(-A_1 + \frac{A_3}{Q^2}\right) - c_bA_4$$

$$\frac{\partial^2 E\left[TP_b^c(p,Q)\right]}{\partial Q^2} = kp_b^{-e}\left(-\frac{2A_3}{Q^3}\right)$$

$$\frac{\partial^2 E\left[TP_b^c(p,Q)\right]}{\partial p_b \partial Q} = ekp_b^{-e-1}\left(A_1 - \frac{A_3}{Q^2}\right)$$

where A_1, A_2, A_3 and A_4 are defined by Eq. 4.

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