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Piezoelectric Sensors and Actuators

Fundamentals and Applications

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Piezoelectric Sensors and Actuators

Fundamentals and Applications

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Preface

Piezoelectric devices play a major role in our everyday lives. The reason for this lies in the fact that piezoelectric materials enable an efficient conversion from mechanical energy into electrical energy and vice versa. Piezoelectric sensors and actuators represent an important subgroup of piezoelectric devices. Nowadays, the application areas of piezoelectric sensors and actuators range from process measurement technology and nondestructive testing to medicine and consumer electronics.

This book addresses students, researchers as well as industry professionals in the fields of engineering sciences, material sciences, and physics. The author aims at providing information that is important to obtain a deep understanding of piezoelectric sensors and actuators. The book additionally contains selected applications and recent developments (e.g., simulation-based material characterization), which are of great interest to science and industry.

At the beginning, we will study fundamentals of piezoelectric sensors and actuators. The fundamentals include physical basics, the principle of the piezoelectric effect and piezoelectric materials. One focus of the book relates to reliable characterization of sensor and actuator materials by combining numerical simulations with appropriate measurements. Moreover, an efficient phenomenological modeling approach for the large-signal behavior of ferroelectric materials will be presented which facilitates the operation of piezoelectric actuators. A further focus lies on piezoelectric ultrasonic transducers because they are most commonly used in applications like ultrasonic imaging and parking sensors. In this context, a nonreactive measurement approach will be detailed that allows sound field characterization in various media.

The book also deals with piezoelectric sensors and transducers in the large application area of process measurement technology. For example, we will discuss conventional piezoelectric sensors for mechanical quantities (e.g., force) as well as sensor devices for fluid flow measurements. The final part of the book concentrates on piezoelectric positioning systems and motors.

Erlangen, Germany — Stefan Johann Rupitsch

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Contents

Chapter 1
Introduction

Piezoelectric devices play a major role in our everyday lives. Currently, the global demand for piezoelectric devices is valued at approximately 20 billion euros per year. Piezoelectric sensors and actuators make a substantial contribution in this respect. At the beginning of the opening chapter, we will discuss the fundamentals of sensors and actuators. Section 1.2 addresses the history of piezoelectricity and piezoelectric materials. In Sect. 1.3, application areas as well as application examples of piezoelectricity are listed. The chapter ends with a brief chapter overview of the book.

1.1 Fundamentals of Sensors and Actuators

Sensors and actuators play an important role in various practical applications. Let us start with the fundamentals of sensors. In general, sensors convert measurands into appropriate measurement values. From the system point of view, measurands serve as inputs and measurement values as outputs of sensors. In this book, sensors will be limited to devices that convert mechanical quantities into electrical quantities. The mechanical quantities denote, thus, measurands, while the electrical quantities represent measurement values. Figure 1.1 depicts possible measurands (e.g., mechanical force) and measurement values (e.g., electric voltage).

In contrast to sensors, (electromechanical) actuators convert electrical quantities (e.g., electric voltage) into mechanical quantities (e.g., mechanical force). Hence, actuators operate in opposite direction as sensors (see Fig. 1.1). From the system point of view, electrical quantities serve as inputs, whereas mechanical quantities represent outputs of actuators.

There exist several principles to convert mechanical into electrical quantities and electrical into mechanical quantities. Some conversion principles work in both directions, i.e., it is possible to convert mechanical into electrical quantities and vice versa. Due to this fact, such conversion principles can be exploited for sensors and actu-

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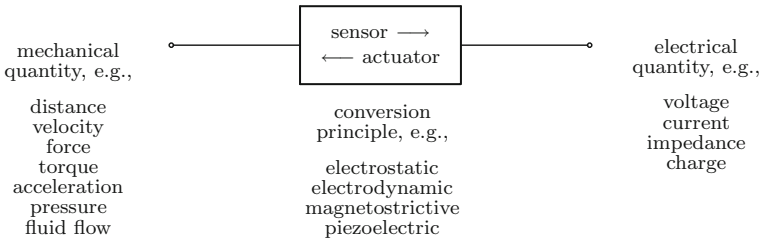

Fig. 1.1 Typical conversion principles as well as input and output quantities of sensors and actuators

ators. The most common bidirectional conversion principles are listed in Fig. 1.1. If a conversion principle allows both working directions, sensors and actuators will often be called *transducers*. As the book title implies, we will focus on the piezoelectric conversion principle. Therefore, the book deals with piezoelectric sensors and piezoelectric actuators, i.e., with piezoelectric transducers.

Apart from the conversion principle, sensors and actuators can be classified according to other aspects. Especially for sensors, one can find several classifications like active/passive sensors. Because piezoelectric sensors do not necessarily need an auxiliary energy, they belong to the group of active sensors.

1.2 History of Piezoelectricity and Piezoelectric Materials

The word *piezoelectricity* originates from the Greek language and means electricity due to pressure. Piezoelectricity was firstly discovered by the Curie brothers in 1880. They recognized that electric charges will arise when mechanical forces are applied to materials like tourmaline, quartz, topaz, and Rochelle salt. This effect is referred to as *direct piezoelectric effect*. In 1881, Lippmann deduced the *inverse piezoelectric effect* from the mathematical point of view. The Curie brothers immediately confirmed the existence of the inverse piezoelectric effect.

The first practical application of piezoelectricity was the sonar, which has been developed during the First World War by Langevin. The main component of the sonar consisted of a thin quartz crystal that was glued between two steel plates. In 1921, Cady invented an electrical oscillator, which was stabilized by a quartz crystal. A few years later, such oscillators were used in all high-frequency radio transmitters. Quartz crystal controlled oscillators are nowadays still the secondary standard for timing and frequency control. The success of sonar and quartz crystal controlled oscillators was responsible that new piezoelectric materials and new applications were explored over the next decades after the First World War. For example, the development of piezoelectric ultrasonic transducers enabled viscosity measurements in fluids and the detection of flaws inside of solids.

During the Second World War, several independent research groups discovered a new class of synthetic materials, which offers piezoelectric constants many times higher than natural materials such as quartz. The synthetically produced polycrys-

talline ceramic materials were named ferroelectrics and piezoceramic materials. Barium titanate and lead zirconate titanate (PZT) represent two well-known solid solutions that belong to the class of these materials. In 1946, it was demonstrated that barium titanate features pronounced piezoelectric properties after an appropriate poling process. The first commercial use of barium titanate was in phonograph pick-ups. The strong piezoelectric coupling in PZT was discovered in 1954. The intense research in the following decades revealed that the piezoelectric properties of PZT could be controlled by means of doping. In doing so, it is possible to produce ferroelectrically soft and ferroelectrically hard materials. While ferroelectrically soft materials are well suited for piezoelectric actuators and ultrasonic transducers, ferroelectrically hard materials provide an outstanding stability for high power and filter applications. On these grounds, PZT is most commonly used in conventional piezoelectric devices, nowadays.

Even though piezoceramic materials such as PZT feature comparatively high electromechanical coupling factors and can be manufactured in arbitrary shape, quartz crystals still play an important role in practical applications, e.g., for piezoelectric force sensors. There are several reasons for this. For instance, special cuts of quartz crystals lead to material properties that are stable over a wide temperature range as well as almost free of hysteresis. Moreover, quartz crystals can also be synthetically manufactured by the so-called hydrothermal method, which was firstly applied to artificially grow quartz in the 1940s. Aside from quartz, lithium niobate and lithium tantalate are well-known representatives of piezoelectric single crystals. Both materials play a key role in modern telecommunication systems because they often serve as piezoelectric material for surface acoustic wave (SAW) devices.

Over the past decades, the research in the field of piezoelectric materials has concentrated on various topics. Many research groups work on lead-free piezoceramic materials (e.g., sodium potassium niobate) that provide a similar performance as PZT. A further research topic concerns relaxor-based single crystals since the piezoelectric constants of such piezoelectric materials can take values, which greatly exceed those of PZT. Because microelectromechanical systems (MEMS) gain in importance, much research and development are also conducted in the fabrication of thick and thin piezoelectric films. As a last example of research topics, let us mention piezoelectric polymers like polyvinylidene fluoride (PVDF) and cellular polypropylene. If the piezoelectric polymers are produced as thin films, they can be exploited for mechanically flexible sensors and actuators.

1.3 Practical Applications of Piezoelectricity

The application areas of piezoelectricity range from process measurement technology, nondestructive testing and medicine to consumer electronics and sports. Depending on the particular application, one exploits the direct piezoelectric effect, the inverse piezoelectric effect or a combination of both. The following list contains selected applications (e.g., parking sensors) in different application areas.

- process measurement technology and condition monitoring
  - sensors for, e.g., force, torque, acceleration, viscosity
  - measurement of temperature and geometric distance
- automotive industry
  - parking sensors
  - injection systems in diesel engines
- production technology
  - ultrasonic welding
  - ultrasonic cleaning
- nondestructive testing
  - flaw detection
  - material and device characterization
- medicine
  - diagnostics, e.g., pregnancy examinations
  - therapy, e.g., kidney stone fragmentation (lithotripsy)
- consumer electronics
  - loudspeakers
  - inkjet printers
  - lens settings in cameras
- smart materials and structures
  - active noise control
  - structural health monitoring
- sports, e.g., reduction of mechanical vibrations in tennis rackets
- musics, e.g., pickup for guitars
- energy harvesting for local energy supply
- transformers

Even though this list of applications seems to be very long, it could be extended almost indefinitely.

1.4 Chapter Overview

As the title suggests, the book deals with fundamentals and applications of piezoelectric sensors and actuators. According to the list in the previous section, there exists a wide variety of applications of piezoelectricity. In this book, we will concentrate on some selected examples. Many topics refer to research activities, which have

been conducted at the Chair of Sensor Technology (Friedrich-Alexander-University Erlangen-Nuremberg) during the last ten years. Apart from the opening chapter, the book is divided into nine chapters.

Chapter 2 addresses the physical basics that are important for piezoelectric sensors and actuators. This includes fundamentals, characteristic quantities as well as basic equations of electromagnetics, continuum mechanics, and acoustics. In Chap. 3, we will study the fundamentals of piezoelectricity. The chapter starts with the principle of the direct and inverse piezoelectric effect. After thermodynamical considerations, the material law of linear piezoelectricity will be derived. Furthermore, the electromechanical coupling inside piezoelectric materials will be classified and quantitatively rated. The chapter ends with a comprehensive overview of piezoelectric materials (e.g., polycrystalline ceramic materials), which are used in practical applications.

Chapter 4 deals with the fundamentals of finite element (FE) simulations since such numerical simulations are nowadays the standard tool for the design and optimization of piezoelectric sensors and actuators. We will start with the basic steps of the FE method. Afterward, the FE method will be applied to the electrostatic field, the mechanical field, and the acoustic field. Due to the fact that piezoelectricity refers to coupling of mechanical and electric quantities, we study the simulation-based coupling of the underlying fields. This also includes the coupling of mechanical and acoustic fields, which is important for piezoelectric ultrasonic transducers.

In Chap. 5, we will discuss the characterization of sensor and actuator materials. The material characterization represents an essential step in the design and optimization because reliable numerical simulations demand precise material parameters. The chapter starts with standard approaches for material characterization. In doing so, a clear distinction between active and passive materials is carried out. Piezoelectric materials are active materials, whereas other materials (e.g., plastics) within piezoelectric sensors and actuators belong to the group of passive materials. The main focus of the chapter lies on the so-called inverse method, which has been developed at the Chair of Sensor Technology. Basically, the inverse method combines FE simulations with measurements. By reducing the deviations between simulation and measurement results, the material parameters get iteratively adjusted in a convenient way. The inverse method is exploited to identify material parameters and properties of selected active and passive materials.

Piezoceramic materials will show a pronounced hysteretic behavior if large electrical excitation signals are used during operation. Chapter 6 details a phenomenological modeling approach, which allows the reliable description of this large-signal behavior. Before the so-called Preisach hysteresis operator is introduced, we will briefly study various modeling approaches on different length scales. Since the Preisach hysteresis operator consists of weighted elementary switching operators, two different weighting procedures are given. The chapter also addressed generalized Preisach hysteresis models, which have been developed at the Chair of Sensor Technology. For instance, the generalization enables the consideration of mechanical stresses that are applied to a piezoceramic material. Finally, we discuss the inversion of the Preisach hysteresis model. The inversion will be of utmost importance when the Preisach hysteresis operator is used for hysteresis compensation.

Chapter 7 treats ultrasonic transducers that exploit piezoelectric materials. The chapter starts with a semi-analytical approach for calculating sound fields and transducer outputs. The approach is based on the so-called spatial impulse response (SIR) of the considered ultrasonic transducer, e.g., a piston-type transducer. Among other things, the SIR is utilized to determine the spatial resolution of spherically focused transducers. Afterward, we will study the general structure and fundamental operation modes of single-element transducers, transducer arrays, and composite transducers. A further section concerns a simple one-dimensional modeling approach that allows analytical description of basic physical relations under consideration of the internal transducer structure. At the end of the chapter, several examples for piezoelectric ultrasonic transducers will be presented.

Practical applications of ultrasonic transducers often demand the characterization of the resulting sound fields. That is the reason why Chap. 8 deals with appropriate measurement principles. The chapter starts with conventional measurement principles such as hydrophones and Schlieren optical methods. The subsequent sections exclusively concentrate on the so-called light refractive tomography (abbr. LRT), which has been realized at the Chair of Sensor Technology. This optical measurement principle enables nonreactive and spatially as well as temporally resolved investigations of sound fields that are generated by piezoelectric ultrasonic transducers, e.g., a cylindrically focused transducer. Exemplary results for sound pressure fields in water and air will be shown. Moreover, LRT is applied to study the wave propagation of mechanical waves in an optically transparent solid.

Piezoelectric sensors are frequently employed for the measurement of physical quantities. In Chap. 9, we will study typical setups of such sensors and their application in the process measurement technology. At the beginning, piezoelectric sensors for the quantities force, torque, pressure, and acceleration are detailed. This includes commonly used readout electronics such as charge amplifiers. Subsequently, a method will be presented which enables the simultaneous determination of thickness and speed of sound for solid plates. The underlying measurement principle is based on ultrasonic waves and has been developed at the Chair of Sensor Technology. The chapter also addresses fluid flow measurements that exploit ultrasonic transducers. We will discuss typical measurement principles as well as a recently suggested modeling approach, which allows efficient estimation of transducer outputs for clamp-on ultrasonic flow meters. At the end, a mechanically flexible cavitation sensor is presented that has been developed at the Chair of Sensor Technology.

The last chapter addresses piezoelectric positioning systems and piezoelectric motors. We will start with piezoelectric stack actuators, which provide much larger strokes than piezoelectric single elements. Because the large strokes call for large electrical excitation signals, Preisach hysteresis modeling from Chap. 6 is applied for a mechanically prestressed stack actuator. The subsequent section details an amplified piezoelectric actuator that was built up at the Chair of Sensor Technology. We will also study model-based hysteresis compensation for a piezoelectric trimorph actuator, which can be used for positioning tasks. The end of the chapter concerns linear and rotary piezoelectric motors.

Chapter 2
Physical Basics

Piezoelectric sensors and actuators connect different physical fields (e.g., electrostatic and mechanical field). With a view to studying the behavior of piezoelectric devices, the fundamentals of those physical fields are indispensable. Therefore, this chapter addresses the physical principles that are important for piezoelectric sensors and actuators. Section 2.1 deals with electromagnetics, especially with the electric field. In Sects. 2.2 and 2.3, the basics of continuum mechanics and acoustics are described, respectively.

2.1 Electromagnetics

In this section, the so-called Maxwell's equations as well as the relevant constitutive equations are introduced allowing the complete description of electromagnetic fields. We will discuss the electrostatic field, which represents a special case of electromagnetic fields. Section 2.1.3 details interface conditions for the electric field between two media exhibiting different material properties. Finally, the lumped circuit element approach is explained that can be exploited to efficiently solve electromagnetic field problems. Further literature concerning the electromagnetic field can be found in [1, 8, 9, 17].

2.1.1 Maxwell's Equations

James Clerk Maxwell published for the first time the full system of partial differential equations, which describe the physical relations in the electromagnetic field [13, 14]. His work relies on previous research performed by Ampère, Gauss, and Faraday. The

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Table 2.1 Expressions utilized in Maxwell's equations (2.1)–(2.4)

Notation	Description	Unit
$\mathbf{H}$	Magnetic field intensity; vector	$\mathrm{A\,m^{-1}}$
$\mathbf{E}$	Electric field intensity; vector	$\mathrm{V\,m^{-1}}$
$\mathbf{B}$	Magnetic flux density (magnetic induction); vector	$\mathrm{V\,s\,m^{-2}}$; T
$\mathbf{D}$	Electric flux density (electric induction); vector	$\mathrm{A\,s\,m^{-2}}$; $\mathrm{C\,m^{-2}}$
q_e	Volume charge density; scalar	$\mathrm{A\,s\,m^{-3}}$; $\mathrm{C\,m^{-3}}$
$\mathbf{J}$	Electric current density; vector	$\mathrm{A\,m^{-2}}$
$\mathbf{M}$	Magnetization; vector	$\mathrm{V\,s\,m^{-2}}$; T
$\mathbf{P}$	Electric polarization; vector	$\mathrm{A\,s\,m^{-2}}$; $\mathrm{C\,m^{-2}}$

four Maxwell's equations[1] in differential form are given by (time t; Nabla operator $\nabla = [\partial/\partial x, \partial/\partial y, \partial/\partial z]^\mathrm{t}$)

$$\text{Law of Ampère:}\quad \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \tag{2.1}$$

$$\text{Law of Faraday:}\quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{2.2}$$

$$\text{Law of Gauss:}\quad \nabla \cdot \mathbf{D} = q_\mathrm{e} \tag{2.3}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{2.4}$$

with the expressions listed in Table 2.1. The general form of Maxwell's equations contains the displacement current ($\partial \mathbf{D}/\partial t$) in (2.1) and is, therefore, also applicable in the high-frequency domain of electromagnetic fields. However, for the low-frequency domain (quasi-static case), the wavelength λ of the resulting electromagnetic waves is large compared to the dimensions of conventional electromagnetic devices. That is the reason why (2.1) can be simplified to $\nabla \times \mathbf{H} = \mathbf{J}$.

Several properties of electromagnetic fields can be deduced from Maxwell's equations. The most important findings are listed below.

- Law of Ampère: An electric current density $\mathbf{J}$ generates a magnetic field. The directions of the magnetic field lines relate to the direction of $\mathbf{J}$ according to the so-called right-hand rule (see Fig. 2.1a).
- Law of Faraday: A magnetic flux density $\mathbf{B}$ that is changing with respect to time induces an electric voltage in a conductive loop (see Fig. 2.1b).
- Law of Gauss: Electric charges are the source of the electric field (see Fig. 2.1c).
- Fourth Maxwell's equation: The magnetic field ($\mathbf{B}$, $\mathbf{H}$) is solenoidal and, therefore, the magnetic field lines are closed (see Fig. 2.1d). Furthermore, magnetic charges do not exist.

[1] To achieve a compact form of the subsequent equations, the arguments for both position $\mathbf{r}$ and time t (i.e., $\bullet(\mathbf{r}, t)$) are mostly omitted. Note that this is also done for continuum mechanics as well as for acoustics.

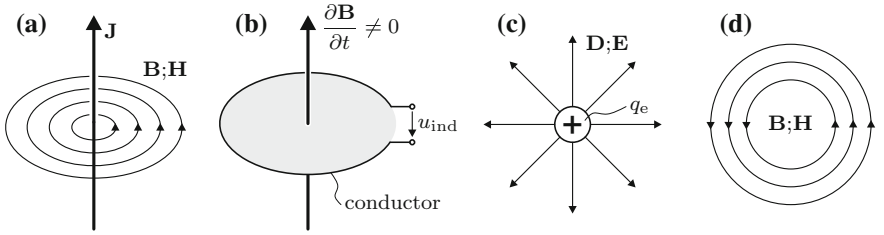


Fig. 2.1 Basic interpretations for **a** Law of Ampère, **b** Law of Faraday (induced electrical voltage u_{ind}), **c** Law of Gauss ($q_{\mathrm{e}} > 0$), **d** fourth Maxwell's equation

Table 2.2 Expressions utilized in constitutive equations (2.5)–(2.7)

Notation	Description	Unit
γ	Electric conductivity; scalar	$\Omega^{-1}\,\mathrm{m}^{-1}$
$\mathbf{v}$	Velocity of the volume charges q_{e}; vector	$\mathrm{m\,s}^{-1}$
μ	Magnetic permeability; scalar	$\mathrm{V\,s\,A}^{-1}\,\mathrm{m}^{-1}$
μ_0	Magnetic permeability of vacuum $\left(4\pi \cdot 10^{-7}\right)$; scalar	$\mathrm{V\,s\,A}^{-1}\,\mathrm{m}^{-1}$
μ_{r}	Relative magnetic permeability; scalar	–
ε	Electric permittivity; scalar	$\mathrm{A\,s\,V}^{-1}\,\mathrm{m}^{-1}$
ε_0	Electric permittivity of vacuum $\left(8.854 \cdot 10^{-12}\right)$; scalar	$\mathrm{A\,s\,V}^{-1}\,\mathrm{m}^{-1}$
ε_{r}	Relative electric permittivity; scalar	–

In addition to Maxwell's equations, the modeling of media in the electromagnetic field requires constitutive equations, which cover the materials' behavior. For a homogeneous and isotropic material, the constitutive equations read as

$$\mathbf{J} = \gamma(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \tag{2.5}$$

$$\mathbf{B} = \mu\mathbf{H} = \mu_0\mathbf{H} + \mathbf{M} = \mu_0\mu_{\mathrm{r}}\mathbf{H} \tag{2.6}$$

$$\mathbf{D} = \varepsilon\mathbf{E} = \varepsilon_0\mathbf{E} + \mathbf{P} = \varepsilon_0\varepsilon_{\mathrm{r}}\mathbf{E} \tag{2.7}$$

with the expressions listed in Table 2.2. Note that for anisotropic materials, such as piezoceramics, the electric and magnetic material properties (e.g., ε) cannot be completely assigned by single scalar quantities but demand tensors of rank ≥ 2. Table 2.3 contains the electric conductivity γ, the relative magnetic permeability μ_{r} as well as the relative electric permittivity ε_{r} of selected media.

2.1.2 Electrostatic Field

In the static case, both electric and magnetic quantities do not depend on time. Electric charges do not move and energy is neither transported nor converted. As a result,

Table 2.3 Electric conductivity γ, relative magnetic permeability μ_r, and relative electric permittivity ε_r of selected media

Media	γ in Ω^{-1} m^{-1}	μ_r	ε_r
Copper	$59 \cdot 10^6$	1	–
Iron	$10 \cdot 10^6$	>300	–
Tungsten	$18 \cdot 10^6$	1	–
PVDF*	10^{-11}	1	6
Polyethylene	10^{-13}	1	2.4
Water	$5 \cdot 10^{-3}$	1	80

* Polyvinylidene fluoride

Maxwell's equations and the constitutive equations can be divided into an electric and a magnetic subsystem. For the so-called electrostatic field (electric subsystem), the relations between the electric quantities can be described with

$$\nabla \times \mathbf{E} = 0 \tag{2.8}$$

$$\nabla \cdot \mathbf{D} = q_e \tag{2.9}$$

$$\mathbf{D} = \varepsilon \mathbf{E} \, . \tag{2.10}$$

Since the electric field intensity $\mathbf{E}$ is irrotational (see (2.8)), it can be expressed by the so-called electric scalar potential V_e

$$\mathbf{E} = -\nabla V_e \, . \tag{2.11}$$

Note that (2.8)–(2.11) will also be appropriate for quasi-static electric fields if the resulting magnetic quantities are still negligible. This is the case for materials exhibiting small relative magnetic permeabilities μ_r.

2.1.3 Interface Conditions for Electric Field

At an interface of different media, the quantities of electric and/or magnetic fields may be altered. To study this for the electric field, we assume an interface between two isotropic homogeneous materials showing different electric permittivities (ε_1 and ε_2; Fig. 2.2a). The starting point to derive the interface conditions is Maxwell's equations. The third Maxwell's equation (Law of Gauss; (2.3)) in integral form is given as (volume Ω, surface Γ, surface vector $\mathbf{\Gamma}$)

$$\int_\Omega (\nabla \cdot \mathbf{D}) \, \mathrm{d}\Omega = \oint_\Gamma \mathbf{D} \cdot \mathrm{d}\mathbf{\Gamma} = \int_\Omega q_e \mathrm{d}\Omega \tag{2.12}$$

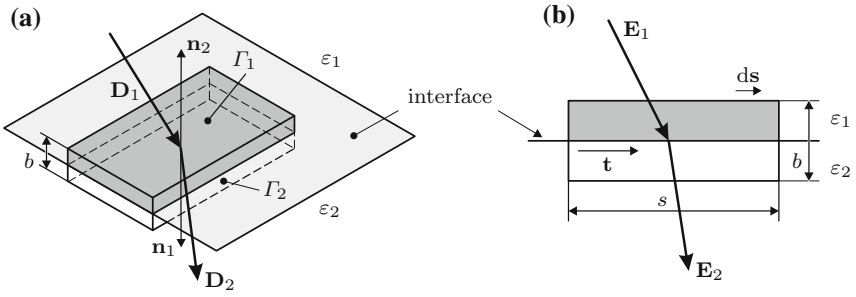


Fig. 2.2 Interface of different media: **a** continuity of electric flux density **D**; **b** continuity of electric field intensity **E**

where the divergence theorem has been applied. Using the relation $\mathrm{d}\Omega = b\,\mathrm{d}\Gamma$ and the assumption $b \to 0$ results in

$$\lim_{b\to 0} \int_{\Omega} q_\mathrm{e} \mathrm{d}\Omega = \int_{\Gamma} \sigma_\mathrm{e} \mathrm{d}\Gamma \ . \tag{2.13}$$

Here, σ_e denotes the surface charge. Furthermore, (2.12) can be rewritten to

$$\lim_{b\to 0} \oint_{\Gamma} \mathbf{D} \cdot \mathrm{d}\mathbf{\Gamma} \ \longrightarrow \ \mathbf{D}_1 \cdot \mathbf{n}_1 + \mathbf{D}_2 \cdot \mathbf{n}_2 = \mathbf{n}_1 (\mathbf{D}_1 - \mathbf{D}_2) = D_{1\mathrm{n}} - D_{2\mathrm{n}} \tag{2.14}$$

with the normal vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ at the material interface. The expressions $D_{1\mathrm{n}}$ and $D_{2\mathrm{n}}$ indicate the normal components of $\mathbf{D}_1$ and $\mathbf{D}_2$, respectively. The combination of (2.13) and (2.14) yields the continuity relation for the electric flux density $\mathbf{D} = [D_\mathrm{n}, D_\mathrm{t}]^\mathrm{t}$

$$D_{1\mathrm{n}} = D_{2\mathrm{n}} + \sigma_\mathrm{e} \ . \tag{2.15}$$

Assuming a negligible magnetic field, the second Maxwell's equation (Law of Faraday; (2.2)) in integral form becomes (closed contour $\mathcal{C}$)

$$\int_{\Gamma} (\nabla \times \mathbf{E}) \cdot \mathrm{d}\mathbf{\Gamma} = \oint_{\mathcal{C}} \mathbf{E} \cdot \mathrm{d}\mathbf{s} = 0 \tag{2.16}$$

where Stoke's theorem has been applied. For the material interface shown in Fig. 2.2b and $b \to 0$, we can simplify (2.16) to

$$\lim_{b\to 0} \oint_{\mathcal{C}} \mathbf{E} \cdot \mathrm{d}\mathbf{s} \ \longrightarrow \ \mathbf{E}_1 \cdot \mathbf{s} - \mathbf{E}_2 \cdot \mathbf{s} = s\mathbf{t} \cdot (\mathbf{E}_1 - \mathbf{E}_2) = E_{1\mathrm{t}} - E_{2\mathrm{t}} = 0 \ . \tag{2.17}$$

Consequently, the tangential component E_t of the electric field intensity $\mathbf{E} = [E_\mathrm{n}, E_\mathrm{t}]^\mathrm{t}$ is continuous at the interface of two materials.

By performing similar steps, the continuity relations at a material interface for the magnetic quantities ($\mathbf{B}$, $\mathbf{H}$) and for the electric current density $\mathbf{J}$ can be deduced.

2.1.4 Lumped Circuit Elements

As previously discussed, we will be able to simplify Maxwell's equations when the device dimensions are much smaller than the wavelength of electromagnetic waves. A further simplification can be performed if either the electric or the magnetic field dominates. The application of the resulting equations is, however, oftentimes still too complicated for various practical situations. Therefore, an alternative approach is commonly utilized yielding reliable approximations of electromagnetic fields. The approach is based on three lumped circuit elements[2]: (i) resistor R, (ii) inductor L, and (iii) capacitor C (graphic symbols in Table 2.4) [1]. While the inductor relates to the magnetic field, the capacitor belongs to the electric field. The inductor and capacitor measure the ability to store magnetic energy and electric charges, respectively. By means of the resistor, we can describe conversions of electromagnetic energy into energy in other physical fields, e.g., kinetic energy in the mechanical field.

The relation between the physical field quantities ($\mathbf{E}$; $\mathbf{D}$; $\mathbf{B}$; $\mathbf{J}$) of electromagnetic fields and the lumped circuit elements (R; L; C) is defined as

$$R = \frac{\int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{E} \cdot \mathrm{d}\mathbf{s}}{\int_A \mathbf{J} \cdot \mathrm{d}\mathbf{A}} = \frac{U}{I} \qquad \text{unit } \Omega \tag{2.18}$$

$$L = \frac{\int_A \mathbf{B} \cdot \mathrm{d}\mathbf{A}}{\int_A \mathbf{J} \cdot \mathrm{d}\mathbf{A}} = \frac{\Phi}{I} \qquad \text{unit H} \tag{2.19}$$

$$C = \frac{\oint_S \mathbf{D} \cdot \mathrm{d}\mathbf{A}}{\int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{E} \cdot \mathrm{d}\mathbf{s}} = \frac{Q}{U} \qquad \text{unit F} \,. \tag{2.20}$$

Here, U, I, Φ as well as Q stand for scalar quantities that are frequently applied in conjunction with lumped circuit elements in electrical engineering. The electric current $I = \int_A \mathbf{J} \cdot \mathrm{d}\mathbf{A}$ (unit A) relates to electric charges flowing through the area A. The expression $U = \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{E} \cdot \mathrm{d}\mathbf{s}$ (unit V) denotes the electric potential difference (electric voltage) from position $\mathbf{r}_2$ and $\mathbf{r}_1$. $\Phi = \int_A \mathbf{B} \cdot \mathrm{d}\mathbf{A}$ (unit V s) is the magnetic flux through the area A and $Q = \oint_S \mathbf{D} \cdot \mathrm{d}\mathbf{A}$ (unit A s) the electric charge enclosed by the

[2] Usually, these lumped circuit elements are time-invariant. Exceptions are configurations changing their geometry with respect to time.

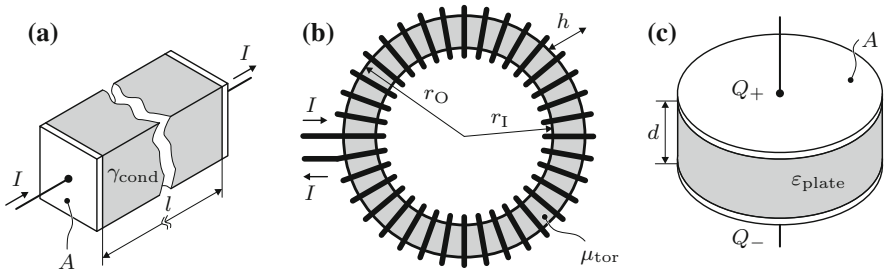


Fig. 2.3 **a** Electric conductor (length l; area A) with homogeneous conductivity γ_{cond}; **b** toroidal core coil (number of winding N_{wind}; inner radius r_{I}; outer radius r_{O}; height h) with relative magnetic permeability μ_{tor}; **c** plate capacitor (plate distance d; area A) containing dielectric medium with relative permittivity $\varepsilon_{\text{plate}}$

surface $\mathcal{S}$. Equation (2.18) represents the so-called *Ohm's law*, which is one of the most famous equations in electrical engineering.

For simple configurations such as a conductor, a toroidal core coil (number of windings N_{wind}), and a plate capacitor (see Fig. 2.3), we can approximate the lumped circuit elements with

$$\text{conductor:} \qquad R_{\text{cond}} = \frac{l}{\gamma_{\text{cond}} A} \tag{2.21}$$

$$\text{toroidal core coil:} \quad L_{\text{tor}} = N_{\text{wind}}^2 \frac{\mu_0 \mu_{\text{tor}} h}{2\pi} \ln\left(\frac{r_{\text{O}}}{r_{\text{I}}}\right) \tag{2.22}$$

$$\text{plate capacitor:} \quad C_{\text{plate}} = \frac{\varepsilon_{\text{plate}} \varepsilon_0 A}{d} \,. \tag{2.23}$$

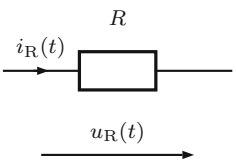
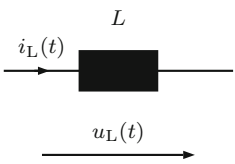
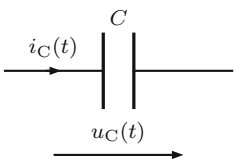
The expressions l, A, h, r_{O}, r_{I}, and d are geometric dimensions of the components. γ_{cond}, μ_{tor}, and $\varepsilon_{\text{plate}}$ refer to the decisive material properties. For several configurations of complex geometries, similar approximations can be found.

In contrast to the simplified Maxwell's equations describing purely electric or purely magnetic fields, the lumped elements approach is also applicable in case of spatially separated components of these fields, e.g., a coil in combination with a capacitor. To efficiently investigate the behavior of such combinations with respect to time, an electric circuit containing the lumped elements and sources of electric energy (voltage source and/or current source) is analyzed. For such electric circuits, Kirchhoff's current law and Kirchhoff's voltage law have to be fulfilled at any time

$$\text{Kirchhoff's current law:} \qquad \sum_{k=1}^{n} I_k = 0 \tag{2.24}$$

$$\text{Kirchhoff's voltage law:} \qquad \sum_{k=1}^{n} U_k = 0 \,. \tag{2.25}$$

Table 2.4 Fundamental relations and common graphic symbols for lumped elements resistor R, inductor L, and capacitor C

resistor	**inductor**	**capacitor**
		
$u_x(t) = f(i_x(t), x)$ for $x \in \{R; L; C\}$:		
$i_{\mathrm{R}}(t) \cdot R$	$L\dfrac{\mathrm{d}i_{\mathrm{L}}(t)}{\mathrm{d}t}$	$\dfrac{1}{C}\displaystyle\int_{t_0}^{t} i_{\mathrm{C}}(t)\,\mathrm{d}t + u_{\mathrm{C}}(t_0)$
$i_x(t) = f(u_x(t), x)$ for $x \in \{R; L; C\}$:		
$\dfrac{u_{\mathrm{R}}(t)}{R}$	$\dfrac{1}{L}\displaystyle\int_{t_0}^{t} u_{\mathrm{L}}(t)\,\mathrm{d}t + i_{\mathrm{L}}(t_0)$	$C\dfrac{\mathrm{d}u_{\mathrm{C}}(t)}{\mathrm{d}t}$
complex impedance $\underline{X}_x(\omega)$ of $x \in \{R; L; C\}$: (imaginary unit $\mathrm{j} = \sqrt{-1}$; angular frequency $\omega = 2\pi f$)		
$\underline{X}_{\mathrm{R}} = R$	$\underline{X}_{\mathrm{L}}(\omega) = \mathrm{j}\omega L$	$\underline{X}_{\mathrm{C}}(\omega) = \dfrac{1}{\mathrm{j}\omega C}$
impedance $X_x(s)$ of $x \in \{R; L; C\}$ in the Laplace domain (complex frequency variable $s = \sigma + \mathrm{j}\omega$)		
$X_{\mathrm{R}}(s) = R$	$X_{\mathrm{L}}(s) = sL$	$X_{\mathrm{C}}(s) = \dfrac{1}{sC}$

Kirchhoff's current law states that at any node of an electric circuit, the sum of the electric currents flowing into the node is equal to the sum of electric currents flowing out of the node. Kirchhoff's voltage law states that the directed sum of the electric voltages U_k around any closed network is zero.

The relation between the time-dependent quantities electric voltage $u(t)$ and electric current $i(t)$ also plays a crucial role in performing circuit analysis. Table 2.4 contains these relations for the different lumped elements. In addition to differential and integral equations in the time domain, the complex impedances $\underline{X}_x(\omega)$ of the lumped circuit elements in the frequency domain as well as those $X_x(s)$ in the Laplace domain are listed. Complex impedances in the frequency domain facilitate the analysis of electric circuits for sinusoidal excitations of frequency $f = \omega/2\pi$, while the approach based on the Laplace domain can also be utilized for certain transient excitation signals [11]. In doing so, the excitation signals have to be transformed into the

frequency domain or Laplace domain, respectively. When the (complex) impedances of the components are known, we will then be able to study electric circuits comprising various lumped elements with similar approaches as for resistor networks. However, to obtain electric voltages as well as electric currents with respect to time, appropriate inverse transforms are indispensable, i.e., transform from the Laplace domain to the time domain [4].

At this point, it should be mentioned that a real device cannot be described completely by means of a single lumped element. Reliable modeling requires, strictly speaking, superposition of several lumped elements. Nevertheless, in a certain frequency range, we can approximate the device behavior by a distinct combination of a few lumped elements.

2.2 Continuum Mechanics

Piezoelectric materials are able into convert mechanical into electrical energy and vice versa. Because the mechanical deformations in such solids are mostly rather small during operation, we can describe the mechanical field by linear relations. In this section, the fundamental equations for linear continuum mechanics as well as essential quantities (e.g., mechanical strain) for the mechanical field are detailed. Thereby, a deformable solid body (elastic body) is considered. We start with Navier's equation linking the mechanical stress to both, inner volume forces and time-dependent body displacements. The mechanical strain and the constitutive equations for a deformable solid body will be explained in Sect. 2.2.2 and in Sect. 2.2.3, respectively. At last, we discuss different elastic wave types, which might occur in solid bodies. Further literature concerning continuum mechanics can be found in [2, 3, 18, 20].

2.2.1 Navier's Equation

Navier's equation is a fundamental equation in continuum mechanics. In order to derive this equation, we assume a deformable solid body of arbitrary shape at rest with prescribed volume forces $\mathbf{f}_V$ (given body force per unit of volume; unit $\mathrm{N\,m^{-3}}$; e.g., gravity forces) and a support at equilibrium. Hence, the sum of all mechanical forces as well as of all mechanical torques equals zero. If a small part is cut out of the deformable solid body, forces will have to be applied to the cutting planes to still guarantee equilibrium (*Euler–Cauchy stress principle*). These forces correspond to the inner forces of the deformable solid body. Due to the fact that the applied forces are distributed across the cutting planes, it is reasonable to introduce mechanical stresses, which are defined as force per unit area.

In a first step, we cut out a cubical-shaped small part with cutting planes aligned in parallel to the Cartesian coordinate system (see Fig. 2.4a). The expressions $\mathbf{T}_x$, $\mathbf{T}_y$,

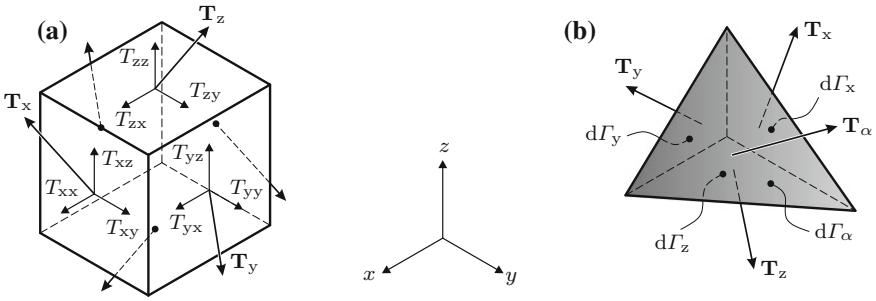


Fig. 2.4 **a** Mechanical stresses applied to cutting planes of a cubical-shaped part of deformable solid body; **b** mechanical stresses applied to tetrahedral element exhibiting oblique surface

and $\mathbf{T}_z$ denote the mechanical stresses (vector quantity; unit $\mathrm{N\,m^{-2}}$) on the cutting planes, with the index referring to the direction of their normal vector, respectively.

The stress vectors can be split up into scalar components that relate to the Cartesian coordinate system. In doing so, the stress vectors read as

$$\mathbf{T}_x = T_{xx}\mathbf{e}_x + T_{xy}\mathbf{e}_y + T_{xz}\mathbf{e}_z \tag{2.26}$$

$$\mathbf{T}_y = T_{yx}\mathbf{e}_x + T_{yy}\mathbf{e}_y + T_{yz}\mathbf{e}_z \tag{2.27}$$

$$\mathbf{T}_z = T_{zx}\mathbf{e}_x + T_{zy}\mathbf{e}_y + T_{zz}\mathbf{e}_z \tag{2.28}$$

with the unit vector $\mathbf{e}_i$ pointing in direction i. For the scalar components T_{ij}, index i refers to normal vector's direction of the cutting plane and j stands for the direction in which the stress acts. According to this notation, T_{xx}, T_{yy} as well as T_{zz} are normal stresses and T_{xy}, T_{xz}, T_{yx}, T_{yz}, T_{zx} as well as T_{zy} stand for shear stresses.

In a second step, we consider an infinitely small deformable body of tetrahedral shape with three surfaces ($\mathrm{d}\Gamma_x$, $\mathrm{d}\Gamma_y$, and $\mathrm{d}\Gamma_z$) orientated in parallel to the Cartesian coordinate plane (see Fig. 2.4b). If a mechanical force is applied to the oblique surface $\mathrm{d}\Gamma_\alpha$, the equilibrium state will require forces acting on the remaining surfaces

$$\mathrm{d}\Gamma_x\mathbf{T}_x + \mathrm{d}\Gamma_y\mathbf{T}_y + \mathrm{d}\Gamma_z\mathbf{T}_z - \mathrm{d}\Gamma_\alpha\mathbf{T}_\alpha = 0\,. \tag{2.29}$$

Since the unity normal vector $\mathbf{e}_\alpha$ of the oblique surface can be defined as (Cartesian components n_i)

$$\mathbf{e}_\alpha = n_x\mathbf{e}_x + n_y\mathbf{e}_y + n_z\mathbf{e}_z\,, \tag{2.30}$$

the surface elements $\mathrm{d}\Gamma_x$, $\mathrm{d}\Gamma_y$, and $\mathrm{d}\Gamma_z$ can be written as

$$\mathrm{d}\Gamma_x = \mathrm{d}\Gamma_\alpha n_x \qquad \mathrm{d}\Gamma_y = \mathrm{d}\Gamma_\alpha n_y \qquad \mathrm{d}\Gamma_z = \mathrm{d}\Gamma_\alpha n_z\,. \tag{2.31}$$

Combining this with (2.29) and subsequently with (2.26)–(2.28) yields

$$\begin{aligned}\mathbf{T}_\alpha &= \mathbf{T}_\mathrm{x} n_\mathrm{x} + \mathbf{T}_\mathrm{y} n_\mathrm{y} + \mathbf{T}_\mathrm{z} n_\mathrm{z} \\ &= \left(T_{\mathrm{xx}} n_\mathrm{x} + T_{\mathrm{xy}} n_\mathrm{y} + T_{\mathrm{xz}} n_\mathrm{z}\right) \mathbf{e}_\mathrm{x} + \left(T_{\mathrm{yx}} n_\mathrm{x} + T_{\mathrm{yy}} n_\mathrm{y} + T_{\mathrm{yz}} n_\mathrm{z}\right) \mathbf{e}_\mathrm{y} \\ &\quad + \left(T_{\mathrm{zx}} n_\mathrm{x} + T_{\mathrm{zy}} n_\mathrm{y} + T_{\mathrm{zz}} n_\mathrm{z}\right) \mathbf{e}_\mathrm{z} \,.\end{aligned} \tag{2.32}$$

It is possible to achieve a compact form by exploiting the so-called *Cauchy stress tensor* $[\mathbf{T}]$ of rank two

$$[\mathbf{T}] = \begin{bmatrix} T_{\mathrm{xx}} & T_{\mathrm{xy}} & T_{\mathrm{xz}} \\ T_{\mathrm{yx}} & T_{\mathrm{yy}} & T_{\mathrm{yz}} \\ T_{\mathrm{zx}} & T_{\mathrm{zy}} & T_{\mathrm{zz}} \end{bmatrix} \,. \tag{2.33}$$

The mechanical stress acting on the oblique surface $\mathrm{d}\Gamma_\alpha$ of the tetrahedral can then be expressed by (transpose t)

$$\mathbf{T}_\alpha = [\mathbf{T}]^\mathrm{t}\, \mathbf{e}_\alpha \,. \tag{2.34}$$

As already mentioned, in case of the equilibrium state, the sums of both mechanical forces and mechanical torques for the deformable solid body at rest are zero. Thus, the equation for translation (surface Γ; surface vector $\mathbf{\Gamma}$; volume Ω of the body)

$$\oint_\Gamma [\mathbf{T}]^\mathrm{t}\, \mathrm{d}\mathbf{\Gamma} + \int_\Omega \mathbf{f}_\mathrm{V} \mathrm{d}\Omega = 0 \tag{2.35}$$

and the equation for rotation (position vector $\mathbf{r}$ of a point in the body)

$$\oint_\Gamma (\mathbf{r} \times [\mathbf{T}])\, \mathrm{d}\mathbf{\Gamma} + \int_\Omega (\mathbf{r} \times \mathbf{f}_\mathrm{V})\, \mathrm{d}\Omega = 0 \tag{2.36}$$

have to be fulfilled. From these two equations, it follows that the equation describing the equilibrium state for an infinitely small part of the deformable solid body at rest is given by

$$\nabla [\mathbf{T}] + \mathbf{f}_\mathrm{V} = 0 \,. \tag{2.37}$$

The entries of the Cauchy stress tensor $[\mathbf{T}]$ feature the properties

$$T_{\mathrm{xy}} = T_{\mathrm{yx}} \qquad T_{\mathrm{xz}} = T_{\mathrm{zx}} \qquad T_{\mathrm{yz}} = T_{\mathrm{zy}} \,. \tag{2.38}$$

Consequently, $[\mathbf{T}]$ is symmetric and the nine tensor entries can be reduced to six entries. According to *Voigt notation*, it is convenient to introduce the stress vector $\mathbf{T}$