Early Mathematics Learning and Development

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Statistics in Early Childhood and Primary Education

Supporting Early Statistical and Probabilistic Thinking

Springer

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To Sercan, Róisín and Deren who will be disappointed to find this book is not about wizards, dragons or fictional characters
&
To Stathis, Nikolas, and Athanasia for giving me the power to embrace the uncertain future with curiosity and optimism
&
Panayiotis, Christoforos and Despina for creating chances

Foreword

Educate a child according to his way:
even as he grows old he will not depart from it.
Proverbs 22, 6

In the era of data deluge, people are no longer passive recipients of data-based reports. They are becoming active data explorers who can plan for, acquire, manage, analyse, and infer from data. The goal is to use data to understand and describe the world and answer puzzling questions with the help of data analysis tools and visualizations. Being able to provide good evidence-based arguments and critically evaluate data-based claims are important skills that all citizens should have and, therefore, that all students should learn as part of their formal education.

Statistics is therefore such a necessary and important area of study. Moore (1998) suggested that it should be viewed as one of the liberal arts and that it involves distinctive and powerful ways of thinking. He wrote: "Statistics is a general intellectual method that applies wherever data, variation, and chance appear. It is a fundamental method because data, variation, and chance are omnipresent in modern life" (p. 134). Understanding the powers and limitations of data is key to active citizenship and to the prosperity of democratic societies. It is not surprising therefore that statistics instruction at all educational levels is gaining more students and drawing more attention. Today's students need to learn to work and think with data and chance *from an early age*, so they begin to prepare for the data-driven society in which they live. This book is therefore a timely and important contribution in this direction.

This book provides a useful resource for members of the mathematics and statistics education community that facilitates the connections between research and practice. The research base for teaching and learning statistics and probability has been increasing in size and scope, but has not always been connected to teaching practice nor accessible to the many educators teaching statistics and probability in early childhood and primary education. Despite the recognized importance of developing young learners' early statistical and probabilistic reasoning and conceptual understanding, the evidence base to support such a development is rare.

By focusing on this important emerging area of research and practice in early childhood (ages 3–10), this publication fills a serious gap in the literature on the design of probability and statistics meaningful experiences into early mathematics teaching and learning practices. It informs best practices in research and teaching by providing a detailed account of comprehensive overview of up-to-date international research work on the development of young learners' reasoning with data and chance in formal, informal, and non-formal education contexts.

The book is also an important contribution to the growth of statistics education as a recognized discipline. Only recently, the first International Handbook of Research in Statistics Education has been published (Ben-Zvi, Makar, & Garfield, 2018), signifying that statistics education has matured to become a legitimate field of knowledge and study. This current book provides another brick in building the solid foundation of the emerging discipline by providing a comprehensive survey of state-of-the-art knowledge, and of opportunities and challenges associated with the early introduction of statistical and probabilistic concepts in educational settings.

By providing valuable insights into contemporary and future trends and issues related to the development of early thinking about data and chance, this publication will appeal to a broad audience that includes not only mathematics and statistics education researchers, but also teaching practitioners. It is not often that a book serves to synthesize an emerging field of study while at the same time meeting clear practical needs: educate a child during his early years with powerful ideas in statistics and probability even at an informal level, and even as he grows old he will not depart from it.

It is a deep pleasure to recommend this pioneering and inspiring volume to your attention.

Haifa, Israel

Dani Ben-Zvi
The university of Haifa

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Preface

Introduction

New values and competencies are necessary for survival and prosperity in the rapidly changing world where technological innovations have made redundant many skills of the past. The expanding use of data for prediction and decision-making in almost all domains of life has made it a priority for mathematics instruction to help all students develop their statistical and probabilistic reasoning (Franklin et al., 2007). Despite, however, the introduction of statistics in school and university curricula, the research literature suggests poor statistical thinking among most college-level students and adults, including those who have formally studied the subject (Rubin, 2002; Shaughnessy, 1992).

In order to counteract this and achieve the objective of a statistically literate citizenry, leaders in mathematics education have in recent years being advocating a much wider and deeper role for probability and statistics in primary school mathematics, but also prior to schooling (Shaughnessy, Ciancetta, Best, & Canada, 2004; Makar & Ben-Zvi, 2011). It is now widely recognized that the foundations for statistical and probabilistic reasoning should be laid in the very early years of life rather than being reserved for secondary school level or university studies (National Council of Teachers of Mathematics, 2000).

As the mathematics education literature indicates, young children possess an informal knowledge of mathematical concepts that is surprisingly broad and complex (Clements & Sarama, 2007). Although the amount of research on young learners' reasoning about data and chance is still relatively small, several studies conducted during the past decade have illustrated that when given the opportunity to participate in appropriate, technology-enhanced instructional settings that support active knowledge construction, even very young children can exhibit well-established intuitions for fundamental statistical concepts (e.g. Bakker, 2004; English, 2012; Leavy & Hourigan, 2018; Makar, 2014; Makar, Fielding-Wells & Allmond, 2011; Meletiou-Mavrotheris & Paparistodemou, 2015; Paparistodemou & Meletiou-Mavrotheris, 2008; Rubin, Hammerman, & Konold, 2006). Use of appropriate educational tools (e.g. dynamic statistics software), in combination with suitable curricula and other supporting material, can provide an inquiry-based learning environment through which genuine endeavours with data can start at a very young age (e.g. Ben-Zvi, 2006; Gil & Ben-Zvi, 2011; Hourigan & Leavy, 2016; Leavy, 2015; Leavy & Hourigan, 2015, 2018; Paparistodemou & Meletiou-Mavrotheris, 2010; Pratt, 2000). Through the use of meaningful contexts, data exploration, simulation, and dynamic visualization, young children can investigate and begin to comprehend abstract statistical concepts, developing a strong conceptual base on which to later build a more formal study of probability and statistics (Hall, 2011; Ireland & Watson, 2009; Konold & Lehrer, 2008; Leavy & Hourigan, 2016, 2018; Meletiou-Mavrotheris & Paparistodemou, 2015).

Edited Volume Objectives

The edited volume will contribute to the Early Mathematics Learning and Development Book Series, a volume focused on the development of young children's (ages 3–10) understanding of data and chance, an important yet neglected area of mathematics education research. The goal of this publication is to inform best practices in early statistics education research and instruction through the provision of a detailed account of current best practices, challenges, and issues, and of future trends and directions in early statistical and probabilistic learning worldwide. Specifically, the book has the following objectives:

1. Provide a comprehensive overview of up-to-date international research work on the development of young learners' reasoning about data and chance in formal, informal, and non-formal education contexts;
2. Identify and publish worldwide best practices in the design, development, and educational use of technologies (mobile apps, dynamic software, applets, etc.) in support of children's early statistical and probabilistic thinking processes and learning outcomes;
3. Provide early childhood educators with a wealth of illustrative examples, helpful suggestions, and practical strategies on how to address the challenges arising from the introduction of statistical and probabilistic concepts in preschool and school curricula;
4. Contribute to future research and theory building by addressing theoretical, epistemological, and methodological considerations regarding the design of probability and statistics learning environments targeting young children; and
5. Account for issues of equity and diversity in early statistical and probabilistic learning, so as to ensure increased participation of groups of children at special risk of exclusion from math-related fields of study and careers.

This timely publication approaches an audience that is broad enough to include all researchers and practitioners interested in the development of children's understanding of data and chance in the early years of life. Early childhood educators can access a compilation of best practices and recommended processes for optimizing the introduction of statistical and probabilistic concepts in the mathematics curriculum. Mathematics and statistics education researchers interested in exploring and advancing early probabilistic and statistical thinking can be informed about the latest developments in the field and about relevant research projects currently being implemented in various formal and informal educational settings worldwide. Academic experts, learning technologists, and educational software developers can become more sensitized to the needs of young learners of probability and statistics and their teachers, supporting the development of new methodologies and technological tools. National and transnational education authorities responsible for setting mathematics curricula and educational policies can get useful information regarding current developments and future trends in statistics education practices targeting young learners. Teacher education institutions can utilize the book for further improvement of their teacher preparation programmes. Finally, the book can also be useful to professionals and organizations offering parent training programmes in early mathematics education.

Edited Volume Contents

The edited volume has compiled a collection of knowledge on the latest developments and approaches to probability and statistics in early childhood and primary education (ages 3–10). It has collected incisive contributions from leading researchers and practitioners internationally, as well as from emerging scholars, on the development of young children's understanding of data and chance in the prior-to-school and early school years. The contributions address a variety of theoretical aspects underpinning the development of early statistical and probabilistic reasoning and their related pedagogical implications. The authors identify current best practices, place them within the overall context of current trends in statistics education research and practice, and consider the implications both theoretically and practically. The majority of the chapters report on original, cutting-edge empirical studies, which demonstrate validated practical experiences related to early statistical and probabilistic learning. Chapters presenting interim results from innovative, ongoing projects have also been included. The volume also contains conceptual essays which will contribute to future research and theory building by presenting reflective or theoretical analyses, epistemological studies, integrative and critical literature reviews, or forecasting of emerging learning technologies and tendencies.

The book includes 17 chapters that cover a broad range of topics on early learning of data and chance in a variety of both formal and informal education contexts. The chapters have been organized into three parts covering the following themes: (a) Part I: Theory and Conceptualization of Statistics and Probability in the Early Years; (b) Part II: Learning Statistics and Probability in the Early Years; (c) Part III: Teaching Statistics and Probability in the Early Years. Each section includes chapters that discuss the above from both research and innovative practice perspectives.

Part I: Theory and Conceptualization of Statistics and Probability in the Early Years

Chapters included in Part I focus on theoretical, epistemological, and methodological considerations related to early statistics education.

In Chap. 1, Katie Makar argues that conventional approaches to early statistics education tend to undervalue young children's capacity by adopting incremental approaches (from simple to complex) that isolate and disconnect statistical concepts from purposeful activity and their structural relations with other key statistical ideas, thus making them less coherent from students' perspective. The author theorizes how contextual experiences can be a powerful scaffold for young children to engage informally with powerful statistical ideas. She introduces the theoretical notion of *statistical context structures,* which characterize aspects of contexts that can expose children to key statistical ideas and structures (concepts with their related characteristics, representations, and processes). The author claims that use of statistical context structures to create repeated opportunities for children to experience informal statistical ideas has the potential to strengthen their understanding of core concepts when they are developed later. A classroom case study involving statistical inquiry by children in their first year of schooling (ages 4–5) is included in the chapter to illustrate characteristics of age-appropriate links between contexts and structures in statistics.

Chapter 2, authored by Zoi Nikiforidou, focuses on probabilistic thinking in preschool years. It provides a critical review of key theories and models on the early development of probabilistic thinking and highlights a number of pedagogical implications while introducing probabilistic concepts in the early years. The first part of the chapter contrasts findings from the first systematic explorations of the origins of probabilistic thinking conducted by Piaget and Inhelder (1975) that had indicated young children's difficulties in differentiating between certainty and uncertainty, to the findings of more recent studies which support pre-schoolers' capacity for sophisticated informal understanding of probability concepts. The second part reviews important curriculum-related aspects in embedding probabilities in the early childhood classroom so as to set foundations for probability literacy. The argument is made that early years practice should use young children's personal experiences with probabilistic situations and their initial understandings as stepping stones for a spiral curriculum that gradually builds probabilistic thinking and reasoning through meaningful tasks and collaborative learning environments.

Part II: Learning Statistics and Probability

Part II includes chapters which explore issues pertaining to learner and learning support in the early classroom, from both research and innovative practice perspectives.

In Chap. 3, Sibel Kazak and Aisling M. Leavy explore early primary school children's emergent reasoning about uncertainty from the three main perspectives on the quantification of uncertainty: classical, frequentist, and subjective. Their focus is on children's subjective notion of probability which, although being closely related to what people commonly use for everyday reasoning, is either neglected or has minimal mention in school curriculum materials. Combining a critical literature review with an analysis of empirical data arising from small group clinical interviews with children, the authors investigate the ways in which young children reason about the likelihood of outcomes of chance events using subjective probability evaluations before and after engaging in experiments and simulations, and the types of language they use to predict and describe stochastic outcomes.

Chapter 4 by Jane Watson describes a study which explored primitive understandings of variation and expectation by seven 6-year-old children in their beginning year of formal schooling. Children worked through four interview protocols which sought to present them with meaningful contexts that would allow them to display their naïve understandings. Across the contexts, students were asked to make predictions and to create or manipulate representations of data. At no time were the words "variation", "expectation", or "data" used with the children. Collected videos, transcripts, and written artefacts were analysed to document demonstration of understanding of the concepts of expectation and variation in relation to data. Findings support Moore's (1990) and Shaughnessy's (2003) view that appreciation of variation is the foundation of all statistical enquiry and the starting point for children's engagement with the practice of statistics. The 6-year-olds in the study had virtually no trouble recognizing and discussing variation in data, despite not always being able to explain its origin. Evidence of appreciation of variation in children occurred much more frequently than evidence of appreciation of expectation. This confirms Watson's (2005) claim that, in contrast to the traditional order of introduction of measures of centre and spread in the school curriculum, dealing with variation generally develops before the ability to express meaningful expectation related to that variation.

Chapter 5, by Celi Espasandin Lopes and Dana Cox, discusses the learning of probability and statistics by young children, centred on culturally relevant teaching and solving problems with themes derived from the children's culture and their daily life context. This chapter is part of a qualitative longitudinal research project that methodologically explores the temporal dimension of experience, in order to discern human action and take into account the social practices, the subjective experiences, identity, beliefs, emotions, values, contexts, and complexity of the participants. Using some of the data collected through the longitudinal study, Lopes and Cox identify structural elements and triggers of mathematical and statistical learning from activities, based on probabilistic and statistical content, prepared by the teachers who are responsible for the learners in the class. They also identify indicators of the development of different forms of combinatorial, probabilistic, and statistical reasoning that children acquire throughout their second and third year of primary school (ages 7–8).

The next chapter (Chap. 6), by Aisling M. Leavy and Mairéad Hourigan, builds on previously conducted research on young children's statistical reasoning when engaged in core components of data modelling. It describes a study which investigated young children's approaches to collecting and representing data in a data modelling environment. The investigation involved 26 primary school children aged 5–6 years in interpreting and investigating a context of interest and relevance to them. The children engaged in four 60-min lessons focusing on data generation and collection, identification of attributes, structuring and representation of data, and making informal inferences about the results. The authors focus on the outcomes of the first lesson which engaged children in generating and collecting data arising from a story context. They use the Worthington and Carruthers (2003) taxonomy of mathematical graphics to categorize the repertoire of inscriptions or marks used by children to track and record the appearance of their data values, and explore the justifications children provided for their invented inscriptions. They conclude that when the focus of statistical investigation is on reasoning about and understanding meaningful situations, the variety of marks young children make become both a record of and an abstraction for the real event and thereby serve an important communicative function in their efforts to make sense of and communicate statistical situations.

The aim of the design-based research study described in Chap. 7 by Jill Fielding-Wells was to investigate the ways in which a statistical inquiry could be facilitated in the early statistics classroom. The study insights emerged from observation and analysis of teacher–student interactions as an experienced teacher of inquiry scaffolded a class of 5–6-year-old students to engage with ill-structured statistical problems. The chapter details the framework employed in the study for introducing statistical inquiry to these young students and then provides an overview of the study findings. Sufficient detail of the classroom context is provided to enable the reader to envisage the learning. Implications and suggestions for educators are addressed.

Chapter 8, authored by Gilda Guimarães and Izabella Oliveira, examines young students' (aged 5–9) and their teachers' knowledge regarding activities involving classification, in the context of a statistical investigation. The chapter presents the results of three different studies conducted by the authors' research group, which involved students and/or teachers of the earliest school years. The first study involved 20 kindergarten children (aged 5), the second study 48 Grade 3 children (aged 8) and 16 early grade teachers, and the third study 72 Grade 4 children (aged 8–9). Findings of these studies demonstrate that people are able, from a very young age, to classify based on a previously defined criterion and to discover a classification criterion, but that they have difficulties in creating criteria to carry out a classification. The authors justify the reasons behind children's difficulties and make suggestions as to how instruction could utilize kindergarten children's ability to classify in different situations using pre-defined criteria to help them build skills in producing their own classification criteria.

Parts III–V: Teaching Statistics and Probability: Curriculum Issues, Tasks and Materials, and Modelling

Parts III–V focuses on issues related to statistics and probability teaching and on providing insights on how to support teachers and other educators in the adoption of the new pedagogical approaches that are needed for successful statistics instruction in the early years. The part is further divided into the following three subparts: (i) Curriculum Issues, (ii) Tasks and Materials, and (iii) Modelling.

Curriculum Issues

In Chap. 9, Randall E. Groth unpacks implicit disagreements among various early childhood standards for probability and statistics regarding the roles of student-posed statistical questions, probability language, and variability in young students' learning. He considers several different sources of disagreement including beliefs about students' abilities, beliefs about teachers' abilities, robustness and influence of the research literature, and priorities for early mathematics education in the early grades. The aim of the author is to define a space in which disagreements about curriculum standards for early childhood and primary statistics are made explicit and then respectfully analysed. In considering the different sources of disagreement, Groth makes suggestions for directions that could be taken by the field so as to provide high-quality statistics education for all young learners. Suggestions are made for ways to move towards a greater degree of consensus across standards documents. At the same time, steps that could be taken to support early statistics teaching and research in absence of consensus on curriculum standards are also highlighted. Specifically, Groth suggests the use of boundary objects, which allow related communities of practice to operate jointly despite the existence of disagreement.

In Chap. 10, Carmen Batanero, Pedro Arteaga, and María M. Gea argue that statistical graphs are complex semiotic tools requiring different interpretative processes of the graph components in addition to the entire graph itself. Based on this argument and on hierarchies proposed in previously conducted research, they analyse the content related to statistical graphs of the Spanish curricula, textbooks, and external compulsory examinations taken by 6–9-year-old children. Batanero et al. investigate the types of graphs introduced in the curriculum, the type of activity demanded, the reading levels required from children, as well as the graph semiotic complexity and the task context. This analysis leads the authors to the conclusion that the expected progression in young children's learning of statistical graphs as reflected in the Spanish current curricular guidelines, the textbooks, and the external assessment is in accord with contemporary research literature recommendations for the teaching of graphs. Curricular materials introduce a rich variety of different types of graphs, activities, tasks, and contexts, with reading levels being adequately ordered in progressive difficulty in the different grades as described by Curcio (1989) and Shaughnessy, Garfield and Greer (1996), and with the graph semiotic complexity (Batanero, Arteaga & Ruiz, 2010) being age-appropriate. Nonetheless, Batanero et al. caution that, in some of the textbooks, an excessive emphasis is being placed on computation with the graph data, resulting in a very high percentage of *reading between the data* (level 2) activities when compared to *reading beyond the data* (level 3) and *reading behind the data* (level 4) activities. Due to this and other important differences between textbooks observed, Batanero et al. highlight the responsibility of teachers when selecting the most adequate book for their students.

Tasks and Materials

Chapter 11, authored by Virginia Kinnear, explores the dual role that picture storybooks can play in contextualizing a statistical problem for investigation through the provision of both an engaging context for the task and of the context knowledge children can use to find a solution to the problem. The chapter presents the results of a small study conducted with fourteen 5-year-old children in a public school in Australia. The study's theoretical perspective, Models and Modeling (Lesh & Doerr, 2003), provided a theoretical framework for task design principles. Three picture storybooks were used to initiate three separate and consecutively implemented statistical problems (as data modelling activities). The study investigated the role of the picture storybooks in initiating children's interest in the statistical context of the problem and in handling the data to solve the statistical problem. The chapter identifies the characteristics of the books that interested children and discusses how knowledge of these characteristics could be used to inform educators' selection of picture storybooks, so as to stimulate students' interest in statistical problem-solving activities. The unique challenges in identifying books for contextualizing statistical problems are also discussed.

Chapter 12 by Efi Paparistodemou and Maria Meletiou-Mavrotheris presents a study which investigated early childhood teachers' planning, teaching, and reflection on stochastic activities targeting young children (4–6-year-olds). Five early childhood teachers (all females) participated in this research, which was organized in three stages. In Stage 1, the teachers were engaged in lesson planning. They selected a topic from the national mathematics curriculum on probability and statistics and developed a lesson plan and accompanying teaching material aligned with the learning objectives specified in the curriculum. In Stage 2, they implemented the lesson plans in their classroom, with the support of the researchers. Once the classroom implementation was completed, in Stage 3, teachers were interviewed and prepared and submitted a reflection paper, where they shared their observations on students' reactions during the lesson, noting what went well and what difficulties they faced and making suggestions for improvement. The researchers analysed the design of each lesson, observed teachers implementing their lesson, and interviewed them while they reflected on their instruction. The study has provided some useful insights into the varying levels of attention teachers paid to different kinds of activities during their lesson implementation, and into the different types of instructional material they used. Findings indicate that the early childhood teachers in this study appreciated the importance of using tools and real-life scenarios in their classrooms for teaching stochastics. They had rich ideas about the context, but needed extra effort to understand the stochastical ideas hidden in the tasks. Moreover, the findings also show that early childhood teachers' attention to different aspects of probability tasks can be developed through a reflective process on their teaching.

The next chapter, by Daniel Frischemeier, addresses the following two questions: in what manner is it possible to introduce early statistical reasoning elements (in regard to analysing large data sets) in German primary school? In what manner is it possible to lead Grade 4 students to fundamental statistical activities such as group comparisons? The first part of the chapter describes the design and implementation of a teaching unit on early statistical reasoning for German primary school students in Grade 4. The teaching unit was designed and developed using the design-based research approach (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003), and it incorporated key elements of the Statistical Reasoning Learning Environment (Garfield & Ben-Zvi, 2008): focus on central statistical ideas (group comparisons), use of real and motivating data sets (class and school data), use of engaging classroom activities (cooperative learning environments), employment of multiple representation levels (enactive, ikonic, symbolic), integration of appropriate technological tools (TinkerPlots) for analysing large and real data sets and comparing groups. The second part of the chapter presents results of an empirical study which investigated how a class of 11 ($n = 11$) Grade 4 students compared groups before and after experiencing the teaching unit described in part 1 of the chapter. The results show the potential of engaging young students' sophisticated statistical reasoning with some pedagogical support at an early stage and provide some design ideas for instructional sequences to lead young children to group comparisons.

In Chap. 14, Soldedad Estrella focuses on the challenging process of representing (modelling) for pupils in the first years of school. She makes a teaching proposal which involves the exploration of a set of raw data before young children can then go on to build their own representations to reveal and provide evidence of the behaviour of the data, its patterns, and relationships. Estrella first describes some concepts that support the teaching proposal and its aim to develop statistical thinking: meta-representational competence (MRC), some components of representation, transnumeration, statistical thinking, and data sense. She then goes on to detail the experiences of three 5-year-old preschool students (from a class of 27 students) and two 7-year-old primary pupils (from a class of 38 pupils) that participated in an open-ended data organization lesson. In both classes, the lesson was jointly designed by teachers in the school (a group of four preschool teachers and a group of four second Grade 4 teachers) that participated in a professional development course on statistics education which adopted the lesson study approach. Findings from the study indicate that strengthening teachers' reflections in lesson study groups promotes the connection between theory and teaching practice, enabling teachers to innovate in the statistics classroom and to get children involved in resolving exploratory data analysis situations. The richness of participating students' productions provided evidence of essential components of data representations and of increased understanding of data behaviour acquired by the children when freely developing their own representations. The chapter presents the diverse data representations produced by the children, details components (statistical, numerical, and geometric) of the different representations, and identifies transnumeration techniques they used, which helped them to gain deeper understanding of the characteristics of a data set and its relationships.

The intent of Chap. 15 authored by Lucía Zapata-Cardona was to explore young children's counting combinatorial strategies and to reflect on how these strategies could orient teachers' actions in the classroom when teaching combinatorics in the early years. To address this goal, a convenience sample of three young children (ages 6–8) were interviewed in a home setting while solving a combinatorial task centred on the process of combinatorial counting. The task was presented in verbal form and was accompanied by some manipulatives to help children visualize, explore, model, and solve the combinatorial task. Zapata-Cardona provides a thorough description of the combinatorial counting strategies the young children activated when solving the task, so as to illustrate the kind of questions and strategies that researchers and teachers could use to challenge young children's combinatorial reasoning and make them go beyond their initial strategies. One of the main ideas revealed through the investigation of the young children's strategies was the close relationship between their combinatorial reasoning and multiplicative reasoning, leading Zapata-Cardona to the conclusion that combinatorial reasoning could be stimulated from the moment children begin to work with multiplication rather than waiting for formal combinatorial instruction which usually occurs in secondary education. The author argues that teachers' strategies to support young children's combinatorial reasoning need to be grounded upon the parallel development of multiplicative reasoning; i.e. they should support young children's exploration of combinatorial counting processes through solving different formats of multiplicative situations. The chapter ends by presenting and discussing some strategies for teachers to support and challenge young children's combinatorial reasoning as drawn from the current study and the existing research literature on combinatorial development in the early years. These strategies include interesting tasks which to children to deal with combinatorial counting situations in a playful, attractive, and familiar way, manipulatives to support the modelling and exploration of combinatorial situations, and probing questions by the teacher to focus children's attention and to challenge their reasoning.

Modelling

In Chap. 16, Maria Meletiou-Mavrotheris, Efi Paparistodemou, and Loucas Tsouccas explore the educational potential of games for enhancing statistics instruction in the early years. Acknowledging the crucial role of teachers in any effort to bring about change and innovation, the authors conducted a study aimed at equipping a group of in-service primary teachers with the knowledge, skills, and practical experience required to effectively exploit digital games as a tool for fostering young children's motivation and learning of statistics. The study took place within a professional development programme focused on the integration of games within the early mathematics curriculum (Grades 1–3; ages 6–9), which was designed based on the Technological, Pedagogical and Content Knowledge (TPACK) framework (Mishra & Koehler, 2006) and was attended by six (n = 6) teachers. Following the TPACK model and action research procedures, the study was carried out in three phases: (i) familiarization with game-based learning; (ii) lesson planning; and (iii) lesson implementation and reflection. Each of the three phases supported teachers in strengthening the connections among their technological, pedagogical, and content knowledge. At the same time, various forms of data were collected and analysed in order to track changes in teachers' TPACK regarding game-enhanced statistics learning in the early years. Findings illustrate the usefulness of TPACK as a means of both studying and facilitating teachers' professional growth in the use of games in early statistics education. They indicate that the TPACK-guided professional development programme had a positive impact on all three perspectives of the participants' experiences examined: (i) attitudes and perceptions regarding game-enhanced learning; (ii) TPACK competency for using digital games; and (iii) level of transfer and adoption of acquired TPACK to actual teaching practice.

In Chap. 17, Lyn D. English describes two investigations which revealed 8-year-olds' statistical literacy in modelling with data and chance. These two investigations, one dealing with statistics and the other with probability, were implemented during the first year of a 4-year longitudinal study being conducted across grades 3 through 6 in two Australian cities. This was the participating students' first exposure to modelling with data. Children's responses to both investigations were explored in terms of how they identified variation, made informal inferences, created representations, and interpreted their resultant models. The responses indicate that these young students were developing important foundational components of statistical literacy. Using their understanding of variation as a foundation, they were able to make predictions based on their findings and to draw informal inferences, as well as generate and interpret a range of representational models to display data. This, English argues, points to the need for early statistics education to provide more opportunities for children to engage in modelling involving data and chance in order to capitalize on, and advance, their learning potential.

Concluding Remarks

Despite the importance of developing young learners' early statistical and probabilistic reasoning, the evidence base to support such development is scarce. An urgent need exists for scholarly publications, and a broader research agenda aimed at investigating the infiltration of probability and statistics into early mathematics teaching and learning practices and experiences. Thus, by focusing on this important emerging area of both research and practice, this publication fills a significant gap in the early mathematics education literature. To the best of our knowledge, this is the first international book to provide a comprehensive survey of state-of-the-art knowledge, and of opportunities and challenges associated with the early introduction of statistical and probabilistic concepts in educational settings, but also at home. While there are several manuscripts covering various aspects of early mathematics education, no other book focuses specifically on the disciplinary particularities of early statistics learning. With contributions from many leading international experts, this book provides the first detailed account of the theory and research underlying early statistics learning. It gives valuable insights into contemporary and future trends and issues related to early statistics education, informing best practices in mathematics education research and teaching practice.

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Contents

Part I
Theory and Conceptualisation of Statistics and Probability in the Early Years

Chapter 1
Theorising Links Between Context and Structure to Introduce Powerful Statistical Ideas in the Early Years

Katie Makar

Abstract Recent literature in the early years has emphasised the benefits of introducing children to powerful disciplinary ideas. Powerful ideas in statistics such as variability, aggregate, population, the need for data, data representation and statistical inquiry are generally introduced in the later years of schooling or university and therefore may be considered too difficult for young children. However, at an informal level, these ideas arise in contexts that are accessible to young children. The aim of this chapter is to theorise important relations between children's contextual experiences and key structures in statistics. It introduces the notion of *statistical context–structures,* which characterise aspects of contexts that can expose children to important statistical ideas. A classroom case study involving statistical inquiry by children in their first year of schooling (ages 4–5) is included to illustrate characteristics of age-appropriate links between contexts and structures in statistics. Over time, engaging children in significant activities that rely on statistical context–structures can provide children with multiple opportunities to experience statistics as a coherent and purposeful discipline and develop rich networks of informal statistical concepts well before ideas are formalised. For teachers and curriculum writers, statistical context–structures provide a framework to design statistical inquiries that directly address learning intentions and curricular goals.

1.1 Introduction

Researchers have long argued that powerful mathematical ideas are accessible to young children (e.g. Alexander, White, & Daugherty, 1997; English & Mulligan, 2013; Greer, Verschaffel, & Mukhopadhyay, 2007; Perry & Dockett, 2008). Yet many approaches to teaching young children undervalue their capacity—and therefore limit their opportunities—to access powerful statistical ideas. Content is often disconnected from purposeful activity, and learning sequences tend to focus on small

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increments building from simple to complex. Incremental approaches tend to isolate and disconnect statistical ideas from their rich contextual and structural relations with other key ideas, making them less coherent from the students' perspective (Bakker & Derry, 2011).

Addressing the gap between the conviction that children can benefit from access to powerful statistical ideas and the operationalisation of this conviction is critical. How does one design age-appropriate learning experiences with complex content? In this paper, I theorise how the context of a problem can be a powerful scaffold for children to engage informally with powerful statistical ideas. The paper introduces the theoretical notion of *statistical context–structures,* which characterise aspects of problem contexts that can expose children to key statistical ideas and structures (concepts with their related characteristics, representations and processes). Using statistical context–structures to create repeated opportunities for children to experience informal statistical ideas has the potential to strengthen their understanding of core concepts when they are developed later. Exposure to informal concepts across a variety of problem contexts highlights their relationships to other core concepts, develops coherence of how statistical ideas work together, assists students to recognise contexts in which the ideas are appropriate and potentially useful, and improves the sense of relevance of statistical ideas.

The aim of this paper is to illustrate how a teacher in an early years classroom (children aged 4–5 years) used a personal problem context to informally introduce, scaffold and develop informal yet powerful statistical content. Over the course of two lessons, she used an inquiry approach and a context familiar to students to leverage initial conceptions of variability, aggregate, population, a need for data and the value of representation to record, analyse and communicate ideas about data.

1.2 Literature Review and Theoretical Framework

Statistical concepts that are isolated become atomistic and impoverished (Bakker & Derry, 2011). To develop rich statistical understandings, students must see how statistical concepts and structures are related to one another, to practices and conventions, to their prior knowledges and experiences, and their utility for solving problems. The focus of this literature review is on understanding links between students' reasoning in problem contexts and their reasoning about key structures in the discipline (mathematics or statistics).

Literature on informal learning environments has begun to establish how reasoning in context can strengthen students' valuing of mathematics and relationships between concepts. There has long been acknowledgement of a gap between students' formal and informal knowledge and reasoning (Confrey & Kazak, 2006; Raman, 2002; Sadler, 2004). Much of this is the result of teaching formal concepts before students have developed understanding of both their usefulness for solving problems and their connections to students' prior knowledge and belief structures. Because "mathematical ideas are fundamentally rooted in action and situated in activity"

Table 1.1 Mapping of statistical context–structures in Makar (2014)

Context entity	Statistical structures	Statistical context–structure and reasoning
Height	data	The measure of how tall a person is can be collected and recorded as height (cm) data
Height of a child	Single data point	A child is associated with their height data
Heights of students in the class	Aggregate	Collectively, the heights of the children in the class can be considered as an entity to investigate
Heights of children in the class differed	Variability	Because all heights in the class were not the same, the children had to grapple with how to manage the variability of the height data
Organised heights clumped in the middle	Distribution shape	When children invented ways to record and organise the data, they noticed that most heights were in the middle and fewer heights were high or low in value; this feature was stable across both classes
Typical height	Average	To find the typical height, children invented a point estimate to capture the most common height (mode) and an interval estimate to capture where "most" heights clumped. They used these estimates to predict (with uncertainty) the typical heights of children in other classrooms
Height of very tall child	Outlier	One child was substantially taller than the others and they considered this student to have atypical height. They reasoned that it was unlikely to see this height in other classes

(continued)

Table 1.1 (continued)

Context entity	Statistical structures	Statistical context–structure and reasoning
The heights of children in another class were collected and compared to their class	Sampling variability	Their surprise that the data in the class next door were similar to but different than their own class data prompted discussions about what aspects of their data were likely or unlikely to be encountered in other classes (e.g. similar values but different frequencies of each height; similar but possibly not exactly the same typical height)
The typical height of the children in one class was used to predict the typical height of children in another class and across Australia	sample-population inference	One Vietnamese child argued that her mother was considered short in Australia, but was of typical height in Vietnam. This prompted students to clarify that their classroom was not representative of other countries and that data would need to be collected from a country to find the typical heights there

(Confrey & Kazak, 2006, p. 322), learning concepts first informally as they are situated in problems allows students to build experiences over time with rich mathematical structures. These experiences with informal ideas also develop students' sense of the utility of mathematical ideas before their formalisation. "People extract information about the world more often than they are aware and that this knowledge exists in tacit form, influencing thought and behaviour while itself remaining mostly concealed from conscious awareness" (Litman & Reber, 2005, p. 440). For example, social practices (including mathematical conventions) can become adopted without the learner being conscious of what is being learned. Boekaerts and Minnaert (1999) argue that the active, non-threatening and explorative nature of informal learning can assist to develop and sustain students' learning in line with social goals and expectations elicited by the context, since "most informal learning contexts are more powerful for developing criteria for success, progress, and satisfaction, which are in accordance with the students' own need structure" (p. 542). Boekaerts and Minnaert further contend that informal learning can heighten students' valuing and learning goals because they perceive learning to be natural and spontaneous.

The theoretical framework in this chapter develops the idea of *statistical context–structures*. Statistical structures maintain consistent patterns (invariances), despite statistics being a field of variability. Statistical context–structures are conceptualised as a mapping between a connected web of statistical structures (concepts with their related characteristics, representations and processes) and contextual entities that stand in for the statistical structures, with relationships between the contextual entities corresponding to the relationships between the statistical structures. Reasoning about the contextual entities is analogous to reasoning about the statistical structures.

For example, the typical height of children in a classroom is a contextual entity that would allow students to reason about the concept of central tendency without explicitly learning about the statistical mean. Students' reasoning about the mean as a representative measure of Year 3 students' heights is still possible even though they have not formally learned what a mean is or how to calculate it. A key benefit is that their reasoning can include the relationship of the mean to other statistical concepts. A study by Makar (2014), for example, highlighted how Year 3 children (aged 7–8) reasoned about variability, distribution (shape, spread, centre, outliers) and sample-population inference as they wrestled with how to find the "typical height" of the children in their classroom. In the process, they invented and critiqued iterations of data displays of increasing sophistication resulting in a graph similar to a histogram. In this example, the children encountered multiple statistical context–structures (Table 1.1). None of the statistical structures they encountered were formalised, but by repeatedly reasoning about the context, the students gained important experiences with informal versions of advanced statistical structures on which they could later map onto the formal ideas (McGowen & Tall, 2010), while formally addressing the content for their own year level.[1] The role of the teacher was critical here to scaffold student learning through engineering learning experiences and using questioning to guide students' ideas. For example, the heights of the children in the class differed (see column 1, Table 1.1). Children were not formally taught the statistical structure "variability" (e.g. the concept of variability with its related terminology, characteristics, representations, measures and relationships with other statistical structures such as "distribution"), as this would not be appropriate content for 7–8-year-olds. Even without formally learning the statistical structure "variability" (see column 2), the children were able to work with variability in the context of managing the differing heights of the children in their class (see column 3). When children had to predict the typical height of Year 3 students in the class next door, they had to grapple with the variability of the height data in their class. Reasoning about differing heights in that context was analogous (and more age-appropriate) to reasoning about variability. The characteristics, representations and processes related to variability were, to the children, the characteristics, representations and processes needed for making sense of the differing heights.

In contrast, the mean is often taught as a calculation of a set of numbers to work out the "average" of that set. Multiple studies have highlighted how this approach has created an impoverished conceptualisation of central tendency as students neither

[1] In the Year 3 curriculum in Australia (Australian Curriculum: Mathematics, 2016), students would be expected to be able to identity an issue/question and relevant data to collect (ACMSP068), carry out a simple data investigation (ACMSP069) and interpret and compare data displays (ACMSP070).