

Gero Brockschnieder

**Asymptotics of Cubic Number Fields with
Bounded Second Successive Minimum of
the Trace Form**

Master's Thesis

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List of Algorithms

1. Compute the number of fields K with $M_2(K) \leq X$ 54
2. Compute the number of fields K with $M_2(K) \leq X$ together
with ρ_X 55