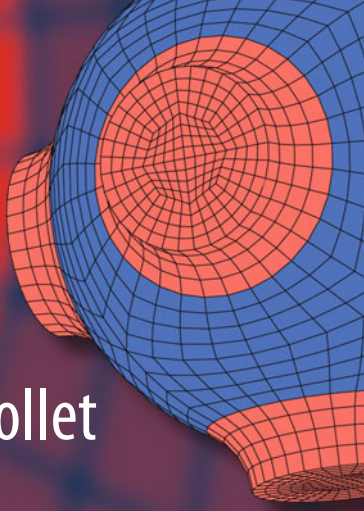
Advanced Structured Materials



Holm Altenbach · Joël Pouget
Martine Rousseau · Bernard Collet
Thomas Michelitsch *Editors*

Generalized Models and Non-classical Approaches in Complex Materials 2

Springer

Advanced Structured Materials

Volume 90

Common engineering materials reach in many applications their limits and new developments are required to fulfil increasing demands on engineering materials. The performance of materials can be increased by combining different materials to achieve better properties than a single constituent or by shaping the material or constituents in a specific structure. The interaction between material and structure may arise on different length scales, such as micro-, meso- or macroscale, and offers possible applications in quite diverse fields.

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Dedicated to the memory of a great creative spirit, G. A. Maugin

Preface

At the beginning of February 2017, the invitation letters for a special remembrance book were sent to approximately 70 friends and colleagues of the great French scientist in the field of Continuum Mechanics (or more general Continuum Physics) Gérard A. Maugin who died on September 22, 2016. As usual in such case that the response is 50% sending a kind reply that they will submit a paper, and finally one gets 15–20 papers. In the case of Gérard, the resonance was overwhelming—the editors got finally approximately 60 papers and the decision was made to publish two volumes. This is the second one including 15 papers from authors living in 13 countries following volume 1 (Altenbach, H., Pouget, J., Rousseau, M., Collet, B., Michelitsch, Th. (Eds.) Generalized Models and Non-classical Approaches in Complex Materials 1, Advanced Structured Materials Vol. 89, Springer International Publishing, 2018).

The scientific interests of Gérard are well reflected by variety of subjects covered by the contributions to this book including the following branches of Continuum Mechanics:

- relativistic continuum mechanics,
- micromagnetism,
- electrodynamics of continua,
- electro-magneto-mechanical interaction,
- mechanics of deformable solids with ferroïc states (ferromagnetics, ferroelectrics, etc.),
- thermomechanics with internal state variables,
- linear and nonlinear surface waves on deformable structures,
- nonlinear waves in continua,
- Lighthill–Whitham wave mechanics, lattice dynamics,
- Eshelbian Mechanics of continua on the material manifold,
- geometry and thermomechanics of material defects,
- material equations, and
- biomechanical applications (tissue and long bones growth).

In addition, he published several papers and books on the history of continuum mechanics. This was reason that the authors of this book have submitted so different papers with the focus on the research interests of Gérard.

We have to thank all contributors for their perfect job. Last but not least, we gratefully acknowledge Dr. Christoph Baumann (Springer Publisher) supporting the book project.

Magdeburg
Paris
February 2018

Holm Altenbach
Joël Pouget
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The original version of the book was inadvertently published without chapter 15. A correction to the book can be found at https://doi.org/10.1007/978-3-319-77504-3_16

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Chapter 1
Damping in Materials and Structures: An Overview

Yvon Chevalier

Abstract For ordinary people, mechanical damping is the attenuation of a motion over time under possible eventual external actions. The phenomenon is produced by the loss or dissipation of energy during motion and thus time. The concept of real time is therefore at the center of the phenomenon of damping and given the recent scientific contributions (of gravitational waves in 2016), the notion of space-time calls for reflections and comments. The systemic approach of the phenomenon taking into account the mechanical system, its input and output variables (generalized forces or displacements) allows a very convenient analysis of the phenomenon. We insist on the differences between a phenomenon and a system: the causality, the linearity, the hysteresis are for example properties of phenomena and not properties of system; on the other hand we can consider dissipative or non-dissipative systems. We describe some macroscopic dissipation mechanisms in structures and some microscopic dissipation at the molecular level in materials or mesoscopic dissipation in composites materials. After specifying the notion of internal forces of a system we present some classical dissipative mechanisms currently used: viscous dissipation, friction dissipation, micro-frictions. The purpose of this presentation is not to list new dissipative systems but to point out a number of errors, both scientific and technical, which are frequently committed.

1.1 Introduction

What is the damping of motion in mechanics? For common people that is the motion of a mass Which decreases with time under the eventual action of an excitation called force, the phenomenon is regarded as non-destructive, except in specific cases. This very simple concept currently uses the four general quantities of Newtonian mechanics (Isaac Newton-1638–1723 (see [33])) which are supposed to

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be independent: displacement, time, mass and force. This concept that we are going to develop is largely enough to explain and study the common phenomena in the field of engineering.

It should be emphasized, however, that the scientific revolution, which was attacked and vilified by the thurifiers (clerics, flatterers, adulators) of the various religions for three centuries, was again discussed at the beginning of the 20th century by the restricted theory of relativity and general relativity of Albert Einstein and the appearance of quantum mechanics. To try to simplify, in the field of the infinitely large variables, the parameters of the Newtonian mechanics are no longer independent: time and space depend on the reference coordinate system, mass and energy are the same entity and gravitational forces are due to the curvature of space-time. At the same time quantum mechanics is concerned with the infinitely small variables (atomic scale) and the particle-wave duality vision is probabilistic: the famous example is Schrödinger's cat (1925) which can be both dead and alive. It distinguishes 4 types of forces and three fields: electromagnetism linking electrons to the nucleus of the atom (chemistry), strong interaction linking protons and nucleus cohesion (nuclear fusion and fission), nuclear force (radiation) and gravitation. Only the first 3 actions result from a quantum field, since gravitation does not depend on a field. The theory of relativity explains the gravitation by the curvature of space-time. It should be noted that the link between the relativistic mechanics and the quantum mechanics is not yet established despite the efforts of scientists (8 Nobel prizes in physics during the last 20 years) and the technical performances of the experimental devices: CERN particle accelerator in Geneva, the laser interferometers of the centers in Europe-Italy, two in the USA-Washington and Louisiana), and the satellite observations and space probes moving in the universe.

The scientific community is booming over the last two decades and concepts resulting from theories are becoming reality: Higgs boson in 2013, gravitational waves in 2016 for example. Let us return to our preoccupation with damping in a concept of Newtonian mechanics which concerns most of the current engineering problems and where time is still the central variable, while recalling that GPS is an application of relativistic mechanics.

1.2 Mechanisms of Energy Dissipation

The attenuation of the motion of a mass over time can be analyzed from an energy point of view, which gives it a more scientific co-notation than the raw observation presented in the introduction. The mechanical energy dissipated during the movement is transformed, in heat, or else in structural modification of the environment, in electricity, etc. This leads us to consider a systemic approach to the problem which makes it possible to give an intrinsic character to the damping. Let us analyze

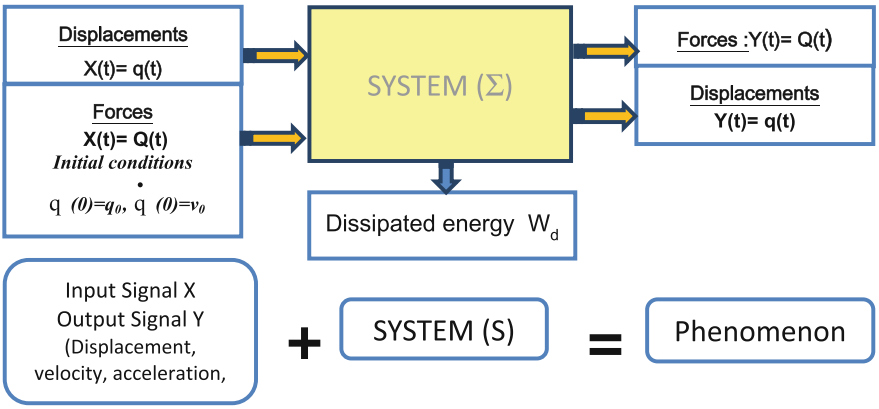

Fig. 1.1 Systemic schematics of damping and energy dissipation

the diagram above (Fig. 1.1) in which the mechanical system is called (Σ), in which the important mechanism is provided with a mass (articulated systems, solid (and/or) fluid structures, …) and is subjected to excitatory actions (input variables X). This results in a response (output variable Y). The nature of the system obviously links the input and output variables which may be scalar, vector or tensor, depending on time t and space coordinates (x, y, z). The nature of these variables provides no information in the interpretation of damping which is a temporal phenomenon which may have spatial effects in wave propagation phenomena for example. We will therefore limit ourselves to scalar variables: q(t) will be a general displacement (length, angle, deformation) and Q(t) will be general force (force, moment, stress) velocity, acceleration can also be considered. The important thing is to note the difference between "phenomenon" and "system": a phenomenon is a system equipped with its input and output variables, we may thus consider damping phenomena and dissipative systems. There is often a confusion between the properties of the phenomenon (causality, stationarity, linearity, hysteresis …) and those of the system. This energy approach is coherent because it is included in the formulation of the principle of virtual powers involving power of internal forces, power of inertia efforts (the system), and power of external forces (the phenomenon). The energy dissipation mechanisms can be schematically classified into 2 categories: macroscopic mechanisms and microscopic mechanisms.

1.2.1 Macroscopic Approach

The macroscopic side appears because the dissipation is produced on the scale of the system itself directly on the variables of input and output (force, displacement, velocity, etc.).

1.2.1.1 Viscous Dissipation

The cause of this energy dissipation is the velocity of motion. The most well-known mechanical device is the hydraulic damper (or oil-filled drilled piston) found in vehicles suspension. In this device the dissipation of energy is due to the viscosity of the oil which goes, with more or less ease, through the holed piston according to his speed. However, we must not forget the role of the spring that compensates for external forces. This simple mechanical vehicle suspension device has led to imagine more integrated systems: the idea is to concentrate the functions of stiffness (spring) and damping (damper) in the same system using the properties of rigidity and damping of materials (composite materials). The advantage is obvious: Small footprint of the device, medium good reliability of the system, good corrosion resistance, reasonable manufacturing cost. Several projects of unidirectional composite blade (glass or carbon/epoxy), which have not been completed industrially, were born in this perspective during the last 2 decades of the 20th century.

1.2.1.2 Friction Dissipation

The cause of this energy dissipation is the presence of frictional forces between two elements of the system. The normal force at the contact surface generates a tangential force which opposes the motion and the phenomenon is therefore damped. The most known device is the vehicle brake consisting of a brake housing containing a pad which rubs on a rotating disc. Compared to viscous-type dissipation, this dissipation by friction can be sudden or softer in the case of micro-friction where the two masses can be clamped in their displacements (see paragraph 1.3.4.4) This is the case for example of assemblies riveted, bolted or even glued. These previous devices are the seat of micro-displacements during external stresses and therefore of micro-frictions which are dissipative.

1.2.1.3 Magneto-Mechanic Dissipation

The cause of this energy dissipation is due to the presence of a magnetic field in which moves a conducting mass which generates eddy currents. These currents generate an own drag force, electromotive force of Laplace which opposes the movement. This concept of dissipation of energy and thus damping, is very recent compared to a pad rubbing on a wheel which is known for millennia. The first patent for electromagnetic retarder was deposited by Steckel in 1903 and realized in practice by Raoul Sarazin in 1936. These systems are known under the trade name of “Telma” and equip heavy trucks and coaches. Unlike the conventional brakes which use the friction of two masses, this braking, or this dissipation of energy,

works without contact and thus without wear of the mechanical parts. The system of damping of motion by dissipation of electric energy is a non-destructive system.

1.2.1.4 Electro-Mechanic Dissipation

The cause of this energy dissipation is due to the presence of an electric field generated by displacements of electric charges caused by external forces: piezo-electricity. If these charges can move in an electrical circuit there is dissipation of energy by Joule effect. This electric current can also excite systems of piezoelectric actuators which correct and attenuate the movement, (see [4, 27]).

1.2.1.5 Plastic Dissipation

The cause of this dissipation of energy is the plasticity of a part of the system. High external loads generate significant internal stresses. If these exceed a threshold the system is irreversibly altered (plasticity of the materials for example) but retains its integrity. The integrity of the system can be destroyed if the efforts are too large and then there is ruin. This device for absorbing energy by plastic deformation of metallic materials (see Fig. 1.2) is used, for example, in the aeronautical sector to absorb the slight shocks and is present at the front of the cockpit of the aircraft. The same principle is used in the automotive sector for absorbing shocks at low speeds: metal profiles in the shape of tubes of rectangular cross-section, attaching the front and rear automobile bumpers to the body of the vehicle, deform by buckling in the event of an impact and thus absorb kinetic energy for low speeds (of the order of 10 km/h).

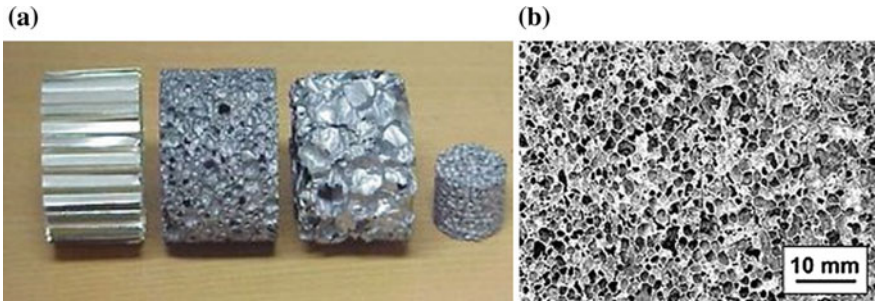

Fig. 1.2 Materials with high absorptive capacity **a** type of aluminum honeycomb **b** type of small-pore aluminum foams

1.2.2 Microscopic Approach

The mechanical dissipative system can also be studied finer by introducing smaller scales in the system to explain the macroscopic phenomenon which is the result of several micro-phenomena working in an intimate way on a smaller scale.

1.2.2.1 Atomic Scale Approach

We can schematically distinguish two close mechanisms that generate damping: one by thermomechanical effects, the other by energy effects:

- Damping in materials by thermomechanical effects
  The most well-known theory is that of the "thermoelastic peaks of Zener" (see [46]) which considers damping in metals can be interpreted by the presence of thermal diffusion phenomena which (the best-known mechanism). An increase in temperature under constant pressure always results in a local increase in volume. Vice versa, the adiabatic application of loads causes a drop-in temperature and, consequently, tends to cause a heat flow from the outside. As the temperature drop gradually relaxes, the specimen undergoes a slow increase in length and generate relaxation. This phenomenon is conditioned by the thermal diffusion coefficient which affects the heat flux. This importance of thermal conductivity was found by Kirchhoff as early as 1860 (see [20]), who noted the importance of thermal conductivity in the damping of acoustic waves. Note that damping in common metals can be neglected (less than 0.1% at ambient temperature) except for some particular ferro-magnetic alloys (Fe–Cr–Al or Mo) (see [36]) where it can reach a few per cent. These metal alloys have approximately the rigidity of steel with cushioning capacities of the polymers, they are used in military applications (submarine discretion for example). An approach also well known in, is those of "*free volumes*". Interpretation assumes that there are "empty volumes" at the atomic or molecular scale inside the material. Under the effect of temperature, forces or other physical phenomena such as moisture, for example, these volumes lose its shape and evolve according to the excitation and then tend to stabilize, with delay and according to a time of their own (material history). Compared to the present time (real time) this phenomenon generates damping and therefore energy dissipation. This interpretation has been developed by chemists concerned with the mechanical behavior of rubber materials. We can mention the work of Knauss and Emri [21, 22] in which the deformation of the free volume is due to temperature (rubber materials and polymers for example), this help to explain William, Landel, Ferry (WLF) curve, (see [13]) and the non-linear viscoelastic behavior of elastomers. In a similar way Schapery proposes that the cause of deformation of the free

volume is the stress that conditions the historical time. This gives rise to non-linear viscoelastic models reflecting the behavior of polymers (see [40, 41]). This approach by the theory of free volume has been taken up more recently by other authors to study non-linear viscoelasticity (see [14, 15]). It should be noted that the scale considered here is "large microscopic" close to those of the mechanics of continuous media.

- Damping in materials by energy effects
  This approach is energetic and based on the notion of internal variables and on the local state, this is also a microscopic point of view but does not explicitly refer to the geometric aspect. The simplest theory, "Theory of transition steps", allows us to study the influence of temperature. It is associated with the name of Eyring who analyses studies chemical reactions and the chemical kinetics (see [12]). The basic idea is that two molecules that react, to lead an activated complex, or possess a transition step, which decomposes to give final reaction products. This reaction, which comes from the theory of transition step, generates an equation which, unlike Arrhenius' law, corresponds to a theoretical model based on statistical thermodynamics (*This equation was established almost simultaneously in 1935 by Henry Eyring, G. Evans, and Michael Polanyi*). The "theory of sites" is a specific approach to damping in polymers which have an amorphous state and a crystalline state according to temperature (see [7]). The theory of sites is based on the "theory of transition steps". It applied to solid crystalline dielectrics and was extended with some success to the mechanical relaxations of polymers. This relaxation is related to the variation of free energy between the crystalline state and the amorphous state generated by the difference between two sites modified by application of a stress. There is a population change between site 1 and site 2 and this change is related to deformation. It is not difficult to imagine how this can happen at the molecular level if, for example, the motion a molecular chain involves internal rotations. Locally, the configurations of strings can be changed from a left configuration to a right configuration. The free energy difference generates a time constant identical to that of the Zener model cited above. This site model is applicable to relaxation processes showing a constant activation energy, that is to say to local motions in the crystalline regions of the semi-crystalline polymers.

1.2.2.2 Molecular Scale Approach

In this approach, the dynamics of the movement of molecules inside the material makes it possible to explain the macroscopic mechanical behavior of the material. In this perspective Rouse's theory is the most well-known (see [38]), it applies to

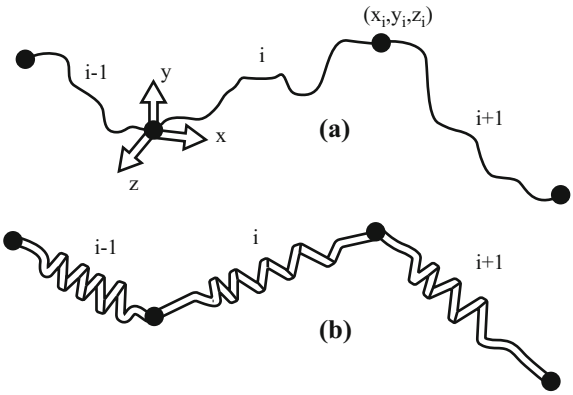

Fig. 1.3 Rouse model—**a** network of chains—**b** representation of the network by a combination of springs and shock absorbers [38, 44]

polymers. It is based on the movement of flexible insulated chains. The aim of this theory is to predict the relaxation spectrum for amorphous polymers as well as the relationship between time scale and temperature. The molecules of polymers are represented as a system of strings (sub-molecules) connected by springs whose behavior is that of a free chain on the basis of the Gaussian theory of elasticity (see Fig. 1.3). If the nodes are moved from their free equilibrium position, the motion is generated by two types of forces:

- the forces due to the friction of the chains,
- forces due to a tendency of the molecular chains to return to their state and the result on a macroscopic scale is that the behavior of the polymer is equivalent to a model of spring and shock absorbers in parallel (Kelvin-Voigt) (see [44]).

1.2.2.3 Mesoscopic Scale Approach

In an approach close to the previous ones, it is possible to envisage composite materials which have damping properties, that is to say media composed of two or more materials that are more or less damping. The scale of analysis is no longer microscopic (atoms or molecules) but intermediate between the latter and the macroscopic approach of the medium: it is called "*mesoscopic scale*". If on the macroscopic scale the composite medium is considered as homogeneous material, its behavior is determined by homogenization processes from a microscopic or mesoscopic scale (see [8, 37]). The most known case is laminated composite (see Fig. 1.4). The behavior of each ply is determined by the microscopic scale as before and the mesoscopic scale corresponds to the behavior of each ply integrated into a homogenization process (see [24]) to arrive at the macroscopic behavior.

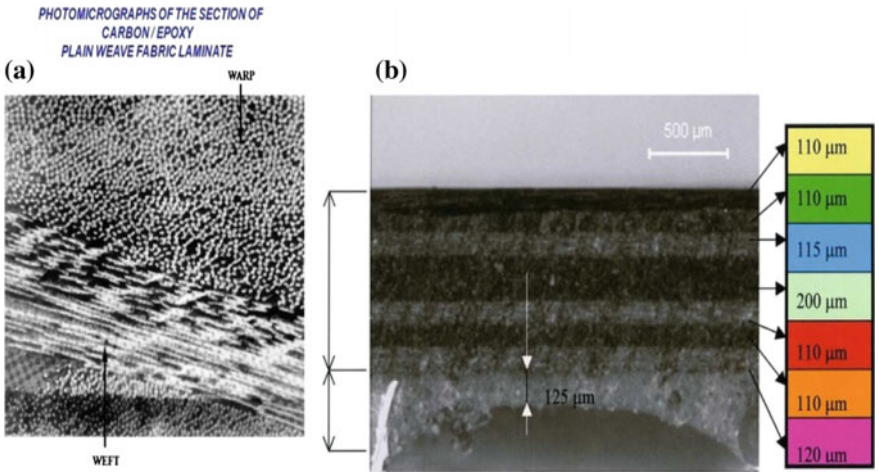

Fig. 1.4 Angle-ply carbon/epoxy composite, 8 plies: 2 × (0° × 60° × 0° × −60°)—thickness 900 µ. **a** Microscopic scale: ply—**b** mesoscopic scale: laminate structure

1.3 Modelling Energy Dissipation

The internal forces and their work in cyclic motions are examined before analyzing some models of dissipation.

1.3.1 Internal Forces

The notion of internal forces specific to a mechanical system (Σ), a thermodynamic concept, manifests only itself in reality when the system is in operation. As we have already pointed out, the energy balance of a mechanical system in operation is governed by the principle of "virtual powers", see [17, 28, 29, 39] in which work of the internal efforts is one of the elements. The real movement is a special case of the virtual movement and is expressed in general by the following equation (or equations) in temporal aspect:

$$m\ddot{q} + \Phi\left(q, \dot{q}, \ldots \overset{(n)}{q}; Q, \dot{Q}, \ldots \overset{(n)}{Q}, t\right) = Q(t) \tag{1.1}$$

in which q(t) is a generalized displacement, Q(t) a generalized effort and Φ the internal forces of the system which are sometimes called "internal frictions". These internal forces depend usually on generalized displacements and their successive

derivatives, on generalized forces and their successive derivatives according to the considered dissipation mechanism. As mentioned previously, q and Q can be scalar, vector or tensor quantities and are all causal signals (q, Q, Φ, etc.) it means they are zero for the negative values of time. This deserves undivided attention for the internal efforts Φ that exist only from the moment 0 beginning of the phenomenon: $\Phi = 0$ when q and $\dot{q}$ are zero. Caution should therefore be exercised in the analysis of aging systems whose properties change over of time.

One technique of analyzing relation (1.1) is to use the classical integral transforms, Laplace or Fourier, which are advantageous because they transform the derivatives into multiplications and the integrations into divisions. Take for example the Fourier transform of the relation of motion (1.1)

$$m\omega^2 \hat{q}(\omega) + \hat{\Phi}\left(q, \dot{q}, \ldots \overset{(n)}{q}; Q, \dot{Q}, \ldots . \overset{(n)}{Q}, t\right) = \hat{Q}(\omega) \tag{1.2}$$

Relation expressed with $\hat{q}(\omega)$ and $\hat{Q}(\omega)$, the Fourier transforms of the generalized displacements and forces q(t) and Q(t), ω is the circular frequency. The relation (1.2) is advantageous only if the Fourier transform of de Φ is expressed as a function of the Fourier transforms of q and Q (linear dependence for example). It is important to note that for any physical signal which is causal, its Fourier transform has **an even real part** and an **odd imaginary part versus circular frequency ω**. This remark must be present in any choice of frequency models. As we shall see later, relations (1.1) (temporal aspect) or (1.2) (frequency aspect), which are the most natural, allow to quantify the elementary mechanisms of energy dissipation in mechanical systems. It should be emphasized, however, that in some dissipation mechanisms the internal forces Φ are only implicitly determined and it is possible to express the generalized displacement q in the following form

$$q(t) = \Theta\left(\Phi, \dot{\Phi}, \ldots \overset{(n)}{\Phi}; Q, \dot{Q}, \ldots \overset{(n)}{Q}, t\right) \tag{1.3}$$

The dependence can be an integro-differential equation, which does not facilitate the analysis of the problem, except in the case of a linear dependence.

1.3.2 Work of Internal Forces: Cycling

The approach of the phenomenon of energy dissipation from the internal forces of the mechanical system is an analytical approach, that means the knowledge and the nature of the internal dissipation of energy is known. If the dissipation models are

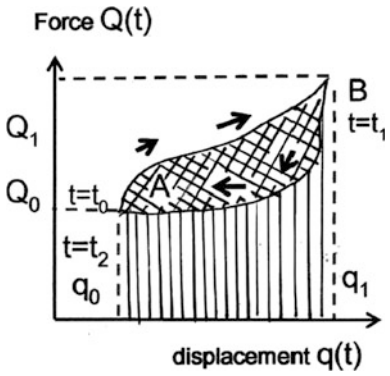

Fig. 1.5 Description of a cycle for a reversible mechanical system: q(t) and Q(t) are respectively the displacements and the generalized forces

numerous and varied (see viscous dissipation Sects. 1.3.3 and 1.3.4 below) they are left to the discretion of the user and the designer. Conversely, a cyclical approach is synthetic in the sense that it does not explicitly take into account the notion of internal efforts of the system but only their work. For example, let us describe a cycle by a system (see Fig. 1.5), the input variable being for example the generalized displacement q(t) and the output variable the generalized force Q(t). The energy balance of this cycle is as follows:

- W_F: energy supplied to the system (surface subtended by the upper curve of Fig. 1.5): Vertical stripes and hatchings
- W_R: energy recovered by the system (surface subtended by the lower curve of Fig. 1.5): Vertical stripes
- $W_D = W_F - W_R$: energy dissipated during the cycle: hatched area of the cycle (Fig. 1.5).

The commonly accepted definition *of Damping* is the "Specific damping Capacity" (SDC) Ψ and is defined as follows

$$\Psi = \frac{W_D}{W_F} = 1 - \frac{W_R}{W_F} \tag{1.4}$$

If the system is non-dissipative $W_D = 0$: the energy returned is equal to the energy supplied and thus the SDC $\Psi = 0$. The system is then thermodynamically called "elastic". This behavior is of course ideal, it is convenient in modeling and simulation, realistic in some cases, but does not correspond to the general physical reality. In a non-destructive mechanical system (excluding explosions, deflagrations, etc.) the energy recovered cannot be greater than the energy supplied and therefore

$$0 < \psi < 1 \tag{1.5}$$

In its general formulation, the SDC Ψ depends on the internal forces of the system (Σ) (relation 1.1) but also on the cycle, that means

$$\Psi[\Phi, q(t), Q(t), t_0, t_1, t_2] \tag{1.6}$$

and consequently, indirectly of q_0 and Q_1 (see Fig. 1.5). In practical aspects a number of remarks deserve to be mentioned which can simplify the analysis.

Comment 3.1 If the system is governed by a potential that means that the energy involved in going from point A to point B of the diagram (q, Q) (see Fig. 1.5) is independent of the path then the energy supplied W_F is identical to the energy recovered W_R. This implies that $\Psi = 0$.

Comment 3.2 If the phenomenon is invariant in time or "stationary" (non-aging), the evolution between times t_0 and t_1 (Fig. 1.5) does not depend on t_0 and t_1 but on the difference $t_1 - t_0$. In this case, we do not restrict the generality by taking $t_0 = 0$.

Comment 3.3 The previous scheme (Fig. 1.5) is described for a cycle but it is possible to envisage several successive cycles. The specific damping thus evolves from one cycle to another. "The Mullins effect" (see [31]) in some viscoelastic media is the best known (see Fig. 1.6).

As in many damping phenomena, when the number of cycles increases, the difference from one cycle to another is very low or nil and the notion of specific damping appears as the consequence of an intrinsic property of the system.

Comment 3.4 In the field of electricity, the notion of quality factor Q (not to be confused with forces) is sometimes used which is the inverse of the specific damping $Q = 2\pi/\Psi$.

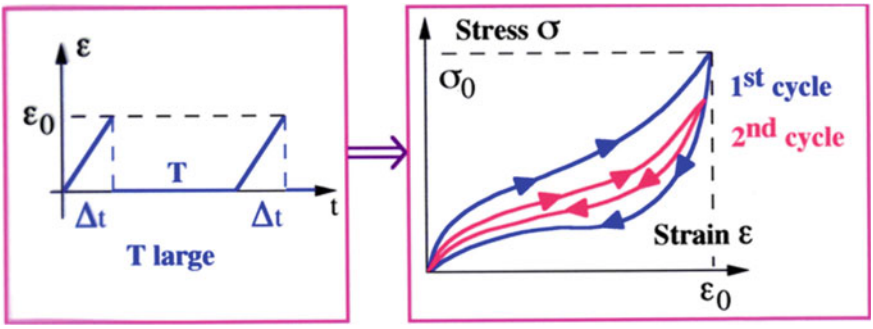

Fig. 1.6 Mullin effect [31] in viscoelastic media. q = ε (strain), Q = σ (stress)

1.3.3 Viscous Dissipation

Mechanisms related to velocity is the most commonly accepted and used because it is the most practical: In the ignorance of the dissipative phenomenon, it is often reduced to viscous damping (under the operating conditions of the system).

1.3.3.1 Linear Behavior of the Phenomenon

The phenomenon is linear (proportionality between the input and output variables (see Fig. 1.1)). The internal efforts of the system (relation 1.1) take the following generic form called "historic integral" (convolution)

$$\Phi(t) = R(0^+)q(t) + \int_0^t \dot{R}(t-\tau)\, q(\tau)d\tau = R(t)q\,(0^+) + \int_0^t R(t-\tau)\,\dot{q}(\tau)d\tau = \frac{D(R * q)}{Dt} \tag{1.7}$$

in which R(t) is the relaxation function of the system and q(t) the generalized displacement which is a causal function (or distribution) not increasing with time. The point symbolizes the temporal derivative and D/Dt the derivative in the sense of the distributions which allows a more synthetic expression of the formulation (see [10] Chapter 2), * represents is the convolution. As we mentioned this relation can be inverted (relation 1.3) and then the generalized displacement q(t) as a function of the internal forces and becomes

$$q(t) = J(0^+)\,\Phi(t) + \int_0^t \dot{J}\,(t-\tau)\Phi(\tau)\,d\tau = J(t)\,\Phi(0^+) + \int_0^t J(t-\tau)\,\dot{\Phi}(\tau)d\tau = \frac{D(J * \Phi)}{Dt} \tag{1.8}$$

In the relation (1.8) J(t) is called creep function. The creep functions and the relaxation are inverse to each other in the sense of convolutions.

Taking the Fourier transform of Eq. (1.7) we obtain the frequency aspect (relation 1.2) of the internal forces.

$$\Phi(\omega) = i\,\omega\,\hat{R}(\omega)\,\hat{q}(\omega) = F(\omega)\hat{q}(\omega) \tag{1.9}$$

$F(\omega) = i\omega\,\hat{R}(\omega) = F'(\omega) + iF''(\omega)$ is called "complex stiffness" in general or "complex modulus" in the context of viscoelasticity, i is the pure imaginary number

($i^2 = -1$). It is convenient to introduce the loss angle δ(ω) defined by its tangent (tg δ), which is the ratio of the imaginary part of the complex stiffness (loss modulus) to its real part (storage modulus), which is commonly called the "damping factor, or loss factor"

$$\eta(\omega) = tg\delta(\omega) = \frac{F''(\omega)}{F'(\omega)} \tag{1.10}$$

These models are frequently used in the field of mechanics of structures and mechanics of material.

Cycling this system makes possible to link the specific damping capacity Ψ (relation 1.4) to the system parameters and provide then energy balance of the evolution of the system. We then consider that a sinusoidal strain $\varepsilon(t) = \varepsilon_0 \sin \omega t$ generates a sinusoidal stress when the transient running has disappeared to give place to a permanent running, the cycle is of the elliptic and symmetrical type as shown in Fig. 1.7.

The choice of the supplied W_F energy leads to several expressions for the SDC Ψ as in the discussion presented by Lee and Hartmann in 1998 (see [25]). The discussion is based on the choice of W_F

$$\begin{aligned} &\psi = 2\pi \sin\delta \quad \text{Maximum potential energy: } W_F = q_0 Q_0/2 \\ &\psi = \pi \sin\delta \quad \text{Potential energy over one cycle, } W_F = q_0 Q_0 \\ &\psi = \frac{\pi \, tg\,\delta}{1 + \left(\frac{\pi}{2} + \delta\right) tg\,\delta} \quad \text{Energy involved in a cycle} \end{aligned} \tag{1.11}$$

Depending on the choice the SDC can be greater than one, which is a disadvantage.

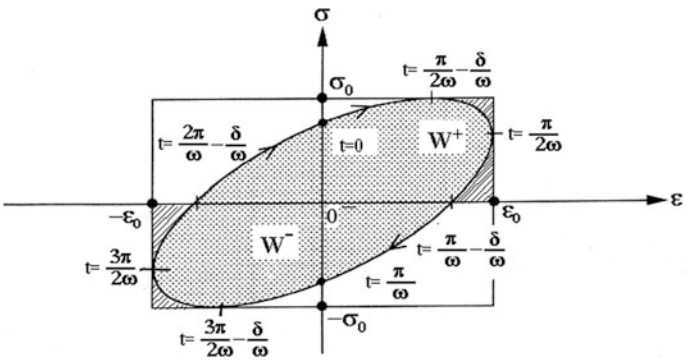

Fig. 1.7 Scheme of the cycling of a damping of the viscous type. cycle of a viscoelastic material: q = ε strain, Φ = σ stress

Discrete Bi-parametric Model (Like Voigt Model)

This model is the most common since internal forces are modelled by a spring and a damper having a linear behavior and placed in parallel. The relaxation function (see relation 1.7) is then of the form:

$$R(t) = C\,\delta(t) + KH(t) \tag{1.12}$$

The parameters C and K are respectively the damping constant and the stiffness, δ(t) is the Dirac distribution and H(t) the Heaviside unit step (H(t) = 0 for t < 0, t) = 1 for t > 0). The internal efforts take the following simple form

$$\Phi(t) = C\,\dot{q}(t) + Kq(t) \tag{1.13}$$

And the the expression of the complex stiffness thus (relationship 1.9) is then

$$F(\omega) = i\,\omega\,C + K \tag{1.14}$$

The relationship (1.14) shows that the damping (relation (1.10) evolves linearly with the circular frequency ω, which can only be realistic within a certain range of circular frequencies. This "two-parameter" model is commonly used in multidimensional formulations (simulations by finite elements for example). The complex stiffness then takes the following matrix form

$$[F(\omega)] = i\,\omega[C] + [K] \tag{1.15}$$

In which [C] and [K] are respectively the symmetric square matrices. If this formulation poses no conceptual problem, it raises some difficulties in solving the equation of motion in the frequency domain.

To solve easily numerically this matrix problem several tricks, having no real physical foundation, are proposed.

Comment 4.1 the first trick t is to introduce the notion of "*structural damping*" which consists in choosing the following complex stiffness

$$[F(\omega)] = i[C] + [K] \tag{1.16}$$

This concept has no physical reality for the simple reason that there is no real time signal whose Fourier transform has a constant and even real part and a constant and odd imaginary part.

This concept of structural damping, which is practical in design, can be explained by considering the notion of a fractional derivative of a function (see [2, 10, 42]). The complex stiffness is then equal to $[F(w)] = (i\omega)^{\alpha}[C] + [K]$ and if α is small and even close to 0, we find that the real and imaginary parts of the previous stiffness are respectively even and odd and almost constant and therefore independent of ω in a given circular frequency bandwidth.

Comment 4.2 the second trick consists in introducing the notion of proportional damping known as the "*Basile hypothesis*". If the system is conservative ([C] = 0), its resolution is done by projection of the displacement {q} on the eigen-modes of this system and in this case the matrix is diagonal. If the system is dissipative ([C] $\neq$ 0), the method consists in express the damping matrix [C] as a linear combination of the mass matrix [M] and stiffness matrix [K], then [C] = ε[K] + γ[M]. Then the matrix equation of motion is projected on the basis of the eigen-modes of the corresponding conservative system. The matrices are then diagonal which facilitates the resolution of the problem. The difficulty lies in the choice of the parameters ε and γ which have no physical reality and are simply a numerical convenience

Continuous Multi-parametric Model (Prony Series)

These models are more complex and include n parameters $(n \geq 2)$ and are used in the behavior of viscoelastic materials (see [10], Chapter 2). The relaxation function R(t) is usually expressed as a linear combination of exponentials or Prony's series

$$R(t) = K_\infty \left(1 + \sum_{p=1}^{p=n} k_p e^{-\frac{t}{\tau_p}} \right) \tag{1.17}$$

The best known of these models is that of generalized Maxwell (see [10]) but the main difficulty lies in the choice of the number of parameters which must be relatively important to correctly represent the behavior of the material (a dozen) but not too important for reasons for determination and measurement. The relation (1.17) easily leads to the complex stiffness (relation 1.9) which is a sum of rational fractions depending on ω. The loss factor varies according to the materials: on the order of a few 10^{-4} for metals, some 10^{-2} for polymers and about 10^{-1} for rubber materials at ambient temperature (20 °C).

1.3.3.2 Non-linear Behavior of the Phenomenon

The notion of nonlinear phenomenon is extremely wide and therefore very varied in representations. The most conventional approach is to admit that in its operation the system has a dissipation of viscous and linear origin and to complete the model with ingredients generating non-linearity in certain cases of motions. We therefore consider the "historic integral" generating internal forces (relations 1.7 or 1.8) by modifying it somewhat.

Schapery Model

This model is adapted to polymers and is based on the notion of free volume (see Sect. 1.2.2.1). It was developed by Shapery in 1966 [41]. The hypothesis is to assume that the real time t is modified by a number of internal variables (temperature, constraint, dilatation, moisture, etc.) and to introduce an artificial time which translates the history of material. In the Schapery model the stress σ of course generates strain but also a modification of time and the model expresses the deformation ε as follows by using the creep functions (relation 1.8)

$$\varepsilon(t) = J(0^+)\ g_1[\sigma(t)]\hat{\sigma}(t) + g_1[\sigma(t)] \int_0^t \dot{J}\ [(\varphi(t) - \varphi(\tau)] \frac{\hat{\sigma}(\tau)}{a[\sigma(\tau)]}\, d\tau \qquad (1.18)$$

In this relation (1.18)

- t is the present (or real) time and τ represents a time specific to the loading history, or historical time (classic case of linear viscoelasticity).
- $g_1[\sigma(t)]$ is a stress factor that expresses the "*nonlinear memory*" of the material. This factor is equal to 1 for the low stress levels (linear viscoelasticity) and increases approximately linearly with the stress: the slope is bounded by 0.04 and 0.05 MPa^{-1} in the case of polymers, (see [45]).
- $\hat{\sigma}(t) = \sigma(t) g_2[\sigma(t)]$ where $g_2[\sigma(t)]$ is known as the "*stiffening stresses factor*" and is equal to 1 for low stresses. It grows proportionally according to the level of stress with a slope by 0.05 and 0.06 MPa^{-1} in the case of polymers, [45].
- $\varphi(t) = \int_0^t \frac{ds}{a_\sigma[\sigma(s)]}$ is the reduced real time and s the time of memory, specific to the history of the phenomenon, a[σ] is the stress-time factor.

The Schapery model allows to describe the dissipative behavior of polymers in the case of low or high deformations or weak or high stresses for polymer materials. It can be noted that the Schapery model is formally similar to the notion of time-temperature superposition (WLF-William-Landel-Ferry, [13]), stress playing the role of temperature in the expression of reduced real time The Schapery model can be generalized by introducing time into the stress-time factor and makes it possible to obtain the behavior of the material for longer times, see [16]. Note that in the case of certain materials, ferro-magnetic alloys for example, the phenomenon of dissipation can depend on the deformation even for small deformations and generate nonlinear phenomena (see [36], Fig. 1.8).

Other kind of alloys, such as "Sonoston" (Manganese, copper, Aluminum, (see [48]), have comparable damping factors like polymers (2 to 5×10^{-2}) with Young's moduli comparable to aluminum, between 73 and 83 GPa. These alloys are used, for example, in the manufacture of submarine propellers for reasons of vibrations damping and acoustic discretion. The disadvantage is the high density,