Computational Social Sciences

Urszula Strawinska-Zanko
Larry S. Liebovitch *Editors*

Mathematical Modeling of Social Relationships

What Mathematics Can Tell Us About People

Springer

Computational Social Sciences

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Contents

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Chapter 1
Introduction to the Mathematical Modeling of Social Relationships

Urszula Strawinska-Zanko and Larry S. Liebovitch

1.1 Mathematics: From Physics to Biology to Social Science

Mathematics is logically rigorous, explicit, and focused on discovering the properties that arise from the simplest axioms assumed for real or fantasized worlds. There is an unforgiving exactness in the clarity of its statements and the logically necessary conclusions that arise from them.

Human beings are just the opposite of all this. They are emotional, complex, and implicitly sculpt and are sculpted by the rich, ever-changing, and sometimes unpredictable, multifaceted worlds that swirl around outside of them and inside of them as well. So, what can mathematics possibly tell us about social relationships?

We can best appreciate the new role that mathematics is playing in understanding social relationships by tracing the history of the application of mathematics from the physical sciences, to the life sciences, and now to the social sciences.

Over the last three centuries, mathematics has been the language of physics and chemistry. Mathematics is how these sciences express their concepts in a clear way, how the theories derived from those concepts are developed, and how the logically

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necessary conclusions drawn from those theories are used to make predictions. Mathematics is so intimately woven into the fabric of these sciences that Wigner (1960) could write about *The Unreasonable Effectiveness of Mathematics in the Natural Sciences*. From this approach has come our understanding of the forces of nature: gravity, electricity-magnetism, and the nuclear forces. We have used that understanding to determine the structure and function of the things in the material world around us. This understanding has also helped us to design and build buildings, bridges, airplanes, automobiles, motorcycles, bicycles, power plants, microwave ovens, computers, telephones, cell phones, satellites, communication networks, the Internet, indoor plumbing, medical imaging devices, and (for better or worse) our engines of war.

Over the last few decades, the forefront of the application of mathematics in science has subtly shifted from physics and chemistry to biology. Mathematics is now demonstrating its value to the life sciences in four different ways:

- First, as in physics and chemistry, mathematical models can determine the logically necessary consequences of specific initial assumptions, typically those of the physical laws. For example, mathematical models of the physical interactions between atoms have been used to compute the three-dimensional shapes and motions in proteins to figure out which smaller molecules, called ligands, would bind to the protein (McCammon and Harvey 1987).
- Second, mathematical analysis can "reverse engineer" a biological system to make clear the roles of its constituent units. For example, ordinary differential equation models have shown that in building a protein in the ribosome, the two-step process of transferring amino acids from t-RNA provides an error-checking mechanism that increases the fidelity of translating the information from genes to proteins. Similar methods have also shown that the multiple binding sites needed to activate a signaling protein reduce the likelihood of accidental false signals (Alon 2006).
- Third, mathematical methods can define system-level properties and provide operational definitions to measure them from experimental data. For example, the properties of networks are particularly important in understanding gene regulation, biochemical pathways in signaling and metabolism, and the connection patterns of neurons in the brain. The mathematics of graph theory and networks has introduced new concepts to biology, such as degree distribution (how many things are connected to how many other things) and motifs (types of connections, such as "feed-forward loops," where the connection between two nodes is regulated by a third node) (Alon 2006).
- Fourth, the newest methods of data science, popularly known as "big data," can analyze very much larger amounts of data than previously used methods. Just as revolutionary as the amount of data is that these methods invert the traditional perspective of analysis, replacing a top-down approach with a bottom-up approach. The traditional mathematics of physics and chemistry is top-down. It starts with assumptions and the equations that represent them and then searches for the experimental data to determine the unknown parameters in the equations.

On the other hand, these new methods are bottom-up. They start with a blank slate. It is the data itself that is used to form the associations, links, and hierarchies through methods such as "machine learning" or "deep learning." For example, this approach is being used to organize the genetic information from cancer patients and their treatments and outcomes (IBM 2016). It could also be used to design treatments with interacting multiple drugs for more targeted results and fewer side effects (Liebovitch et al. 2007).

In summary, these methods are equations based on simple assumptions to understand the functioning of an isolated piece of a system, sets of equations to reverse engineer the essential roles of the components in a system, network analysis of complex, interconnected systems, and machine learning to let the data organize itself. These four ways of applying mathematics to biology are opening up new vistas of understanding and applications in the health sciences.

These methods have proved useful in biology because they provide new and better ways to understand a complex, interconnected, multilayered, ever-changing, and sometimes unpredictable biological world. In social science we also face a social world that is complex, interconnected, multilayered, ever-changing, and sometimes unpredictable. This strongly suggests that these mathematical methods could also tell us new things about the properties of social systems.

Over the last three centuries, mathematics has given fresh insights, first to the sciences of physics and chemistry and more recently to the life sciences of biology and medicine. The next frontier for mathematical methods is their application to social science. Now is the time to use these methods to better understand social systems, the properties of the behavior of people, and the nature of social relationships. The goal of this book is to present examples of how this can be done, but even more importantly to put on clear display the shift of culture in how to think mathematically about social relationships and use that perspective to provide new insights about them.

1.2 Mathematical Models: The Good News and the Bad News

First things first. What can mathematics do for us and what can it not do for us? The following cogent passage, complete and unedited, is from Richardson (1960), who did seminal research on predicting the weather, turbulence, and fractals and whose moral values as a Quaker led him to serve as an ambulance driver in the Great War (World War I):

> To have to translate one's verbal statements into mathematical formulae compels one to carefully scrutinize the ideas therein expressed. Next the possession of formulae makes it much easier to deduce the consequences. In this way, absurd implications, which might have passed unnoticed in a verbal statement, are brought clearly into view and stimulate one to amend the formula. An additional advantage of a mathematical mode of expression is its brevity, which greatly diminishes the labor of memorizing the idea expressed. If the state-

> ments of an individual become the subject of a controversy, this definiteness and brevity lead to a speeding up of discussions over disputable points, so that obscurities can be cleared away, errors refuted, and truth found and expressed more quickly than could have been done had a more cumbrous method of expression been pursued.
>
> Mathematical expressions have, however, their special tendencies to pervert thought: the definiteness may be spurious, existing in the equations but not in the phenomena to be described; and the brevity may be due to the omission of the more important things, simply because they cannot be mathematized. Against these faults we must constantly be on our guard. It will probably be impossible to avoid them entirely, and so they ought to be realized and admitted. (xvii-xviii)

1.3 Methods

There are many different mathematical approaches to understanding social relationships described in the following chapters. We now provide an overview from 30,000 feet of that landscape to give you a sense of the many and different approaches. This is an instructive sample, rather than a comprehensive list. Additional models are also described (qualitatively) in Chap. 4: Social Processes that Generate Fractals in Brown and Liebovitch (2010).

1.3.1 Statistics

Some social scientists hear the word "statistics" when you say the word "mathematics." Just to be clear, our use of the word "mathematics" here does not mean "statistics." Sure, statistics is mathematics, no doubt about it at all. But, how mathematics is used in the physical sciences is very different than how social scientists think about statistics. This difference may not be known to social scientists who have not had firsthand experience with the culture of physical science.

In the physical sciences, you construct your model in mathematical form from the ideas you have about what is going on in the system that you are studying. The equations you write down describe the mechanisms, specifically how this changes that. How do you know what are the mechanisms to choose that you will transform into your equations? You just make it up. Yes, we stated it just the way we meant it. You just make it up! You don't completely make all of it up, of course. You use what you already know about the system, but there will still be unknown pieces, and those, yes, you just make up. When all of your thoughts and mechanisms are transformed into equations, you use mathematics to determine the logically necessary consequences of those equations. Then you see if those results make any sense, see if they match existing experimental data, and see if they make interesting new predictions that you can test later.

Social scientists looking for statistical correlations may find this surprising. Physical scientists are looking for *mechanisms*, not *correlations*. The interpretations of the equations of the model are the mechanisms of exactly how this does that. Finding those mechanisms, whose effects match the experimental data, is the goal of physical scientists. Regression models in statistics are valuable in that they show the functional correlations between variables, but most often, those models do not explain the underlying mechanisms responsible for those correlations.

The irony here is that social scientists sometimes seem squeamish that their field is less "exact" than the "hard" sciences. In fact, the truth is the oxymoron that physical scientists use *quantitative* methods only as a way to achieve a *qualitative* understanding of the physical world. A detailed example of using quantitative physical analysis to achieve a qualitative understanding of what goes on inside your eyes is given in Liebovitch (2006).

1.3.2 Differential Equations: Ordinary, Integral, and Partial

Often in physics we want to determine how the value of x over time t is changed by some other thing y. This is represented by the differential equation, $dx/dt = y$. The time derivative, dx/dt, is the rate of change of x with time t. The most famous such equation is Newton's second law, $F = ma$, where F is force, m is mass, and the acceleration $a = d^2x/dt^2$ is the second derivative of the position in space x with respect to time t. This type of model, called an ordinary differential equation, has very broad applicability to a wide range of physical and biological problems (Alon 2006; Strogatz 1994). Since these models determine how the values of the variables evolve in time from their initial conditions, they are called "dynamical systems."

In social relationships, these dynamical systems can be used in modeling how one person influences another person. Liebovitch et al. used it to describe how the emotional state (valence) of two people changes in time. The two people could be in a conflict (Liebovitch et al. 2008) or a therapist and client in psychotherapy (Liebovitch et al. 2011; Peluso et al. 2011). The equation for each person is then $dx/dt = mx + b + f(y)$ where x is the emotional state of one person at time t, m represents the degree of memory of the previous value of x, b represents the emotional state when that person x is alone, and f(y) represents the influence of the other person y on person x. With a nonlinear influence function f(y), this relatively simple model has surprisingly interesting behaviors (Liebovitch et al. 2008). Such a model can also be extended to a much larger number of people, and Fernandez-Rosales et al. (2015) used it to study up to 2048 people interacting together. Features of some differential equation models include "attractors" that are sets of values that the variables are always drawn into and the "butterfly effect" that final states can be highly sensitive to the initial conditions. These behaviors also serve as new qualitative paradigms for social systems. Coleman et al. (Coleman 2011; Vallacher et al. 2013) used such concepts from dynamical systems to shed new light on the social relationships in intractable conflicts and how to resolve them.

If one of the variables of a person, such as their accumulated emotional baggage, depends on the sum of all its instantaneous values in time, this transforms the ordinary differential equation into an integral-differential equation which requires more sophisticated mathematical methods to solve (Murray 1989).

If the variables associated with the people are not only varying in time but are also distributed across space, then the ordinary differential equation becomes a partial differential equation (Garabedian 1964). The "partial" refers to derivatives taken with respect to time with space held constant and derivatives taken with respect to space with time held constant. Such equations appear in the study of fluids, and so the techniques that have been developed to solve for the flow of air across airplane wings provide the mathematical machinery necessary to study the interactions of people over time and space. For example, such models have been used to study the patterns of settlement and transportation routes in cities (Batty and Longley 1994).

1.3.3 Difference Equations

Continuous differential equations can be approximated by their counterparts of discrete, difference equations. The variables of the model then evolve in small, discrete steps in time, Δt, rather than continuously in time. For example, the continuous derivative dx/dt can be replaced with its discrete approximations $\Delta x/\Delta t$, where $\Delta x = x(t+\Delta t) - x(t)$. With the appropriate choice of the time steps, these discrete equations very closely approximate their continuous equations. But it is not always so simple. The deep mathematics of discrete and continuous equations are actually different. The trail of the variables with time, called the trajectories, of continuous equations can never cross; otherwise the variables would not know which way to go. But the trajectories of discrete equations are sets of disconnected points which can freely cross each other. It's rare that such subtleties alter the predictions of a model, but this is a clear warning that if you don't really understand the mathematics, you can (sometimes) get into trouble.

Gottman et al. (2005) used such difference equations to describe the social relationship between a wife and her husband. In their model, first the wife responds to her husband, then the husband responds to the wife, then the wife again responds to her husband, and their dance of alternating interactions continues.

1.3.4 Self-Organizing Critical Systems

We are used to thinking that systems will fall into and remain in their lowest energy, equilibrium state. Not true for a pile of sand. As you slowly drop sand on the top of the pile, the slope steepens until it becomes unstable, and a brief avalanche of sand cascades down part of the sandpile. These avalanches remove sand until the slope is stable, which leaves each point in the sandpile caught at that razor edge between being stable and unstable. At each moment in time and at each point in space on the sandpile, the slope of the sand is at that very edge of being *unstable*. This is the opposite of a system at its most stable equilibrium. The sandpile is called a "self-organizing critical system." It organizes itself, without outside influences, to always be poised exactly at the point between stability and instability, like water at its critical point poised between a liquid and ice (Bak 1997). These systems also generate fractals in space and time. A fractal is an object with an ever larger number of ever smaller pieces, like the ever finer branches of a tree (Liebovitch 1998). The sandpile produces an ever larger number of ever smaller avalanches. Only rarely, does a very big avalanche happen. The probability of "the big one," as correctly predicted by its fractal distribution, is much larger than would be expected from a bell curve (Brown and Liebovitch 2010). "Black swans" are not "outliers"; rather they are part of the distribution typical of these systems. These models have some of the characteristics of partial differential equations because they span both space and time; however, they are implemented in discrete and iterated steps like difference equations.

Self-organizing critical systems are a good model for any system where local stress above a threshold is then shared among its nearest neighbors and so on to their neighbors. Such models have been used to better understand physical systems such as earthquakes and forest fires. They are also useful in understanding how local social relationships generate wider structures such as the viral spread of information, traffic jams, and growth patterns in cities (Turcotte and Rundle 2002).

1.3.5 Cellular Automata

Gardner (1970) described Conway's newly created game of life. It is played out on a two-dimensional board of a grid of cells like a checkerboard. The state of each cell can be "alive" or "dead." At each step in the game, the state of each cell is determined by an updating rule that depends on the states of the eight cells in the neighborhood surrounding it. It was astounding that such a simple game produced a fantastic array of patterns of great complexity that ebbed and flowed in time. This type of model, called a "cellular automata," was further generalized to cells with a larger number of states, different types of neighborhoods, and different types of updating rules. Such simple rules could reproduce many complex patterns from those found in physical phenomena to the colorful patterns on sea shells (Wolfram 1994; Meinhardt 1995). Similar to self-organizing critical systems, cellular automata span both space and time and are implemented in discrete and iterated steps. Although there is some understanding of how the patterns depend on the updating rules and neighborhood definitions (Marr and Huett 2009), in most cases the resultant patterns can only be revealed by computer simulations.

Nowak and Vallacher (1998) used these models to study the social influence on opinions. They found that complex patterns in the geographic clustering of opinions and in the polarization of opinions could arise from simple updating rules. They described this result as "dynamical minimalism" (Nowak 2004) that some complex social patterns may arise from very simple underlying rules.

1.3.6 Non-equilibrium Thermodynamics: Complex Adaptive Systems

Traditional analysis, in both the physical and social sciences, has often fixated on discovering the nature of a system at equilibrium in an energy minimum, because that is when it is most stable. But the most interesting and important properties of systems often do not happen at such stable states. The fluctuations in the three-dimensional structure of a protein from its energy minima may be what allows ligand molecules to bind to the protein (McCammon and Harvey 1987). The dynamical history of the players in a game may determine where they wind up, which could be away from the expected minimum of game theory (Liebovitch et al. 2008). In fact, life itself is not an equilibrium state at an energy minimum. All of life on the earth is made possible by the energy in the light flowing from the sun. Plants and animals take that energy directly, or indirectly from each other, and transform it into structures and motion, and for us, thoughts. Prigogine (1980) emphasized that it is this flow of energy from the outside through a system that creates patterns in space and time by pushing the system far from local thermodynamic equilibrium.

Kedem and Katchalsky (Katchalsky and Curran 1965) developed mathematical tools to study systems out of equilibrium, yet still close to it. The properties of systems driven by outside energy to be far from equilibrium were developed by Haken (1983); Holland (1998) and Kelso (1995). They used different types of mathematical models including differential equations, networks, and cellular automata. These systems have truly fascinating properties. They can "self-organize" creating their own patterns. They include "complex adaptive systems," which can use energy to reconfigure themselves in response to the environment around them, as well as changing the environment around them. Recall the statement about human beings at the beginning of this chapter that people implicitly sculpt, and are sculpted by, the worlds outside of them and inside of them. The properties of these mathematical models resonate with those properties of real people. Therefore, qualitative insights gained from the mathematical properties of these models have proved useful in understanding social interactions and in developing useful strategies in practical applications such as in management and the delivery of healthcare (Stacey 1992; Zimmerman et al. 1998.)

1.3.7 Agent-Based Modeling

Complex patterns in a large system can arise from simple interactions between its individual units. In "agent-based modeling," these individual units are represented by agents. The agents could be people deciding to invest in the stock market or automobile drivers deciding which routes to drive in a city. Each agent has a small set of values that define its state at any one time. A small set of simple rules also describes how those values change in response to the values of the other agents. For example, each agent could decide to buy or sell their stocks based on the values of their current holdings, the average value of all other stocks in the market, and whether that average value has been rising or falling. The rules could be the same for all the agents, or there could be different types of agents with different rules. The values of the agents are then evolved in time in a computer simulation. The simulation can be used to explore how the emergent patterns and dynamics of the whole system depend on the rules of the agents or the mix of different types of agents.

This method has been helpful in understanding, for example, how individual economic decisions drive prices in the marketplace, how people in a crowded theater move toward an exit, how traffic patterns adjust to a construction delay on a highway in city, and how the sociodemographic and marriage interactions determine the kinship structures in Pakistan (Bonabeau 2002; Geller 2011).

1.3.8 Data Science: "Big Data," "Machine Learning," Artificial Neural Networks, and "Deep Learning"

The mathematical approach of physical science is top-down. You first assume what mechanisms are at work in your system. Then you translate those mechanisms into equations. Maybe you also need to add some parameters specific to your particular system. Then, you solve those equations and see if those results match other existing experimental data or make new predictions that you can test later. Data science turns this process upside down, making it bottom-up. You use lots of data; petabytes (10^{15} bytes) is now typical. The data could be where people clicked on a website, retail orders, stock prices, or locations where parking tickets were issued in New York City. For social science, the data could be Twitter feeds, Google trending searches, or location check-ins on Foursquare. Typically, there's a lot of data, and you have to deal with its volume, variety, and velocity (the 3Vs). Storing, accessing, and cleaning the data are serious issues. Special software is needed to do this, for example, Hadoop is used to store and access large amounts of data in an environment distributed across clusters of computers, non-SQL programs are used to access non-relational databases, and numerical errors, misspelled words, and missing fields such as "NA" need to be cleaned up.

Mathematical and computational methods are then used to let the data organize itself. "Machine learning" tools, such as k-nearest neighbors or support vector machines, can cluster similar data together (O'Neil and Schutt 2013), or Force Atlas 2 and Gephi (Bastian et al. 2009) can identify and segment out sub-networks of nodes (which could be people, IP addresses, or Twitter hashtags). Artificial neural networks (Amit 1989) are sets of nodes connected to each other by different weights. Data values at the output nodes represent the processing of data presented at the input nodes. The network is trained by adjusting the weights so that known inputs produce their expected output values. This is called supervised learning, but networks can also be constructed with unsupervised learning, similar to the self-organization of complex adaptive systems. Between the input and output nodes are hidden layers. In ancient times (up to 2014), there were typical one or a few hidden layers. Lately, systems with hundreds of hidden layers are being used. Artificial neural networks and other computational structures that have many iterated components or processes are capable of extracting sophisticated structures from input data. Collectively, these different techniques are now called "deep learning." For example, a video of a car moving down a road on YouTube is presented to the input nodes, and the values of the output nodes could report "automobile."

These methods provide a fresh look into many different aspects of social relationships. For example, analysis of the network of Twitter users in Baltimore, MD, shows a striking lack of connection between black and white Americans, except for a possible overlap in sports teams, which could therefore play a useful role to bridge these otherwise segregated communities (Troy 2015).

1.4 Examples

Applications of mathematics have brought significant advances in theory and methodology in all areas of social science since World War II (Wilson 2010) and arguably facilitated the evolution of the study of social issues into scientific social sciences we know today. Among the so-called Big Five traditional areas of social science (Horowitz 2006), economics was the first to embrace mathematics, with applications in anthropology, political science, sociology, psychology, and allied disciplines to follow (Newman 1956; Arrow et al. 1960).

A closer look at historical developments within each social science area reveals a common pattern with an emergence of independent fields with a mathematical focus accompanied by an establishment of specialized societies and journals. For example, in psychology, mathematical psychology, defined today broadly as a field that uses mathematical methods, formal logic, or computer simulation, emerged as an independent field of research around the mid-1950s. The *Journal of Mathematical Psychology* and the Society for Mathematical Psychology were established in 1964 and 1977, respectively, and the first edition of the *Handbook of Mathematical Psychology* was published in 1963 (see Luce et al. 1963). Interestingly, the desire to frame questions about the social issues in mathematical terms continued to spread beyond the specialized journals to become an essential ingredient of contemporary social science disciplines welcomed and appreciated by all social science journals and wider audiences.

The acceleration of the adoption of mathematical approaches in social sciences is attributed to the discovery of techniques that use mathematics in qualitative ways, as well as the development of new mathematical tools, more suitable to nonphysical sciences and especially suited to investigate complex social systems where system behavior cannot be easily predicted, such as game theory (von Neumann and Morgenstern 1945), cybernetics (Wiener 1948), information theory (Shannon et al.

1949), general systems theory (von Bertalanffy 1969), diffusion models (Coleman 1964; Granovetter 1983), and dynamical system modeling (Forrester 1973; Vallacher and Nowak 1994). Fascinating mathematical models of social relationships inspired by the above approaches investigate, for example, coordination of individuals' actions (e.g., Kelso 1995; Newtson 1994; Kulesza et al. 2015), co-regulation of emotions (e.g., Babcock et al. 2013), or synchronization of physiology (e.g., Levenson and Gottman 1985). Mathematical approaches are also used successfully to shed new light on close relationships (e.g., Gottman et al. 2002; Tesser 1980) as well as interactions by more than two individuals with a focus on, to list only a few among many application areas, structure of social relationships (e.g., Boyd 1969; Freeman 2004), social influence (Nowak et al. 1990), or peace and conflict (e.g., Coleman et al. 2007; Liebovitch et al. 2010).

Mathematical applications to social sciences have been, without a doubt, revolutionized and accelerated by the use of digital computers paving way for the emergence of computational social science (CSS), an interdisciplinary field providing a unified framework for a range of research directions inspired by the paradigms of complex adaptive systems and information processing. Representative areas integrated under the umbrella of CSS are socio-informatics, computational social simulation modeling, big data analytics, social network analysis, social complexity, and several others that emerged at the intersection of traditional social science disciplines, computer science, environmental science, and engineering sciences (see, e.g., Cioffi-Revilla 2017, Vallacher et al. 2017).

1.5 The Chapters

These chapters cover a very broad range of types of social relationships and an equally broad range of mathematical models to represent them. We hope that this provides a firm starting point for you to appreciate the value and importance of mathematical modeling in providing a new perspective and understanding of social relationships.

The illustration of applications of mathematical approaches to social relationships opens with Chap. 2 authored by John Gottman and Paul Peluso titled "Dynamical models of social interaction." In this chapter, the authors present an overview of Gottman's original dynamical systems modeling work, address some of the criticisms of those models, and outline new areas of mathematical modeling to uncover additional dynamical elements of relationships.

In Chap. 3, Gottman's approach to modeling the affective exchange between interaction partners is applied in a different social context. In the chapter titled "Quantitative video coding of therapist-client sessions," Peluso and collaborators present an empirical approach to validating a model of the relationship between a therapist and their client created to tackle important questions related to success of psychotherapy.

A more in-depth presentation of this mathematical model is offered in Chap. 4 titled “Dynamical analysis of therapist-client interactions.” Here, the investigators discuss model parameters, the graphical displays of the model, and the indices of model fit, and they do so through the use of clarifying case studies.

Subsequent Chap. 5 “Modeling psychotherapy encounters: Rupture and repair” presents key steps in the iterative process of a model development and refinement through an in-depth discussion of simulation work testing an extension to the mathematical model of therapeutic relationship.

In the chapter titled “Mathematical models as tools for understanding the dynamics and cooperation and conflict,” Michaels shows how quantitative understanding summarized in a mathematical model may be successfully generated from theoretical ideas, observation, and empirical evidence. This time, the models presented pertain to yet another area of interests to social scientists, namely, cooperative and competitive interpersonal relations.

Chapter 7 titled “A dynamical approach to conflict management in teams” extends the presentation of relevance and practice of mathematical modeling of social relationships from an interaction between two individuals to investigations of small teams.

In Chap. 8, “Modeling the Dynamics of Sustainable Peace,” Liebovitch and coauthors share details of an application of causal loop diagrams to the study of sustainable peace.

Chapter 8 titled “Capital in the first century: the evolution of inequality in ancient Maya society” takes you on a fascinating journey to the ancient Maya to enhance our understanding of how distribution of wealth evolved over time. Specifically, it discusses an application of mathematical approach to address the issue of inequality, which has always lain at the center of many social theories.

The final application presented in Chap. 10 “Can the Nash equilibrium predict the outcomes of military battles” centers on archival data and applies game theory to predict the outcome of historical military battles.

In all the chapters, the emphasis is on those aspects of the projects that show the potential of the mathematical methods presented to other social science domains beyond the specific areas featured, for example, behavioral and organizational systems.

We believe that the chapters offer a valuable and detailed practical account of efforts that are needed to prepare social science problems for an application of mathematical approaches. We also hope this volume will help and inspire readers who may not have expert knowledge of the particular research area but are interested in applying mathematical tools. We specifically asked chapter contributors to offer only as much contextual information as needed for comprehension and appreciation of the usefulness of the mathematical apparatus featured in each chapter, with the primary goal being inspiration on how it can be subsequently used by the reader in her or his own endeavors of theoretical, research, or applied nature.

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