

Advances in Industrial Control

Pushkin Kachroo
Kaan M. A. Özbay

Feedback Control Theory for Dynamic Traffic Assignment

Second Edition

AIC

 Springer

Advances in Industrial Control

Series editors

Michael J. Grimble, Glasgow, UK

Michael A. Johnson, Glasgow, UK

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Feedback Control Theory for Dynamic Traffic Assignment

Second Edition

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“You know my methods, Watson.”
Sherlock Holmes quote from
The Crooked Man
by Sir Arthur Conan Doyle

Series Editors' Foreword

The series *Advances in Industrial Control* aims to report and encourage technology transfer in control engineering. The rapid development of control technology has an impact on all areas of the control discipline. New theory, new controllers, actuators, sensors, new industrial processes, computer methods, new applications, new philosophies..., new challenges. Much of this development work resides in industrial reports, feasibility study papers, and the reports of advanced collaborative projects. The series offers an opportunity for researchers to present an extended exposition of such new work in all aspects of industrial control for wider and rapid dissemination.

The authors Pushkin Kachroo and Kaan M.A. Özbay published the first Edition of their monograph *Feedback Control Theory for Dynamic Traffic Assignment* (ISBN 13: 978-1-85233-059-0) in the *Advances in Industrial Control* monograph series in 1999, some 18 years ago. This second edition is a testament to the longevity and continuing relevance of their work in this field and also of our monograph series; the series has been active now for 25 years and has published over 170 industrial control monographs.

Since the 1999 version of *Feedback Control Theory for Dynamic Traffic Assignment*, there have been unparalleled technological developments in automated transport infrastructure systems and even more advances in the instrumentation, control, and intelligent autonomy of automotive vehicles of all types. The interaction between these two aspects of transportation systems grows ever more important in making traffic systems safer, more efficient, and in reducing their environmental impact.

The first edition of the monograph had eight chapters and had the format of a course textbook with a clearly stated objectives section opening each chapter and the chapter body text closing in each case with the sections: Summary, exercises comprising questions and problems, then finally chapter references. The second edition of the monograph now has some 10 chapters that follow the same useful “course textbook” structure as the earlier edition. Comparing the table of contents of the two editions, there is a rearrangement of some material and the introduction of new material in the second edition. This reflects the authors' experience of

teaching their original material to students and peer researchers and also presents new research that has occurred in the intervening years.

The technical flavor of the second edition continues to occupy the “academic high ground” of the first edition. The volume sets out the “fundamentals” of traffic flow modeling and the related control problems. The newly added Chap. 2 gives a valuable survey of mathematical models and techniques for the traffic assignment problem. Chapter 10, also an addition in the second edition, makes an apposite contribution on the traffic routing problem in the presence of congestion pricing policies. These policies have been introduced in many major world cities in an attempt to control traffic flow and reduce the environmental pollution caused by the exhaust emissions of excessive numbers of vehicles in urban city areas. The main chapters examine the feedback control problems using fuzzy methods, semigroup theory, and lumped parameter methods to name just three of the techniques presented. However, overall, it is the technical comprehensiveness and theoretical depth that is the hallmark of this distinctive monograph that was seminal to this technical field. In fact, the monograph can also function as a reference text and a course textbook for this particular set of control problems.

Since the appearance of the first edition and with the unstoppable technological progress in transport and vehicular systems, the field has received increasing interest from the control community. This has manifested itself in the now frequent sessions on these topics appearing at the leading control conferences. A typical session might cover improved control for automobiles (for example, cruise control, autonomous car parking routines), driverless cars, cooperative control for (autonomous) fleets of vehicles, and automated traffic control and management systems. This is a very active field of control engineering that continues to grow. The second edition of this monograph by Pushkin Kachroo and Kaan M.A. Özbay continues the record of contributions to this important transportation control literature by the *Advances in Industrial Control* monograph series.

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Preface to the Second Edition

It is a great privilege to have the opportunity to present the second edition of our book *Dynamic Traffic Routing using Feedback Control*. When we wrote the first edition, not many people in the field were looking at using feedback control theory to solve traffic control problems (apart from very few researchers such as Dr. Markos Papageorgiou). As we mentioned in the original preface, “We were inspired by the initial work of Markos Papageorgiou in using feedback control theory for traffic control problems.” The field has developed in this direction a great deal since, and many control area researchers and engineers are more involved in the field.

In this edition, we have made some specific changes:

1. Chapter 2 is completely new and has been added to give a mathematical overview of the vast field of dynamic traffic assignment (DTA) starting from its static roots and going all the way to complete PDE-based models and new travel time scalar fields. The chapter is taken from the Springer publication:

Pushkin Kachroo, and Neveen Shlayan, “Dynamic traffic assignment: A survey of mathematical models and technique,” *Advances in Dynamic Network Modeling in Complex Transportation Systems* (Editor: Satish V. Ukkusuri and Kaan Özbay) Springer New York, 2013. 1–25.

2. Chapter 10 is also a completely new chapter that has been added as an application of traffic routing using feedback control based congestion pricing. It is based on the following paper:

Kachroo P, Gupta S, Agarwal S, Özbay K., “Optimal Control for Congestion Pricing: Theory, Simulation, and Evaluation,” *IEEE Transactions on Intelligent Transportation Systems*. 2017 May;18(5):1234–40.

3. Chapter 3 which is on traffic flow theory has been drastically enhanced with many new sections, and its graphics, plots, and other features have been greatly modified in order to make them more polished and complete.

4. The book has been completely retypeset in LaTeX . That has made the look and feel more elegant. Moreover, most graphics have been completely redone, and new ones were created. Writing has also changed in many sections throughout the book.

We would like to thank Oliver Jackson, Meertinus Faber, and Nisha S. Keeran from Springer for the immense help that provided us during the preparation of this book.

Las Vegas, USA
New York, USA
January 2017

Pushkin Kachroo
Kaan Özbay

Preface to the First Edition

Traditionally, traffic control problems have been studied from the planning perspective. However, due to the advancements in microtechnology in terms of availability of various sensors, actuators, processors, and communication hardware, it has become possible to perform real-time control of traffic. It is important that we design and analyze these closed-loop systems from the perspective of feedback control theory, since these systems are in fact feedback control systems.

This book can be read by researchers and students with different backgrounds. We have attempted to provide as much basic information as possible. However, if we provided all the information to make this book completely self-sufficient, the size of the book would at least double. The references given at the end of each chapter should provide ample material as background material and also as topics of further research. This book should be of interest to people from traditional civil engineering, from control engineering (electrical, computer and mechanical engineering), and from applied mathematics. Readers from transportation background might read the book following a different sequence than other readers. For example, a reader with transportation background might skim through chapter 2, but readers lacking that background would read that chapter more carefully.

We were inspired by the initial work of Markos Papageorgiou in using feedback control theory for traffic control problems. The Center for Transportation Research (CTR) at Virginia Tech. provided us with the ideal opportunity to study the design of feedback controllers for various traffic control problems, which include traffic routing/assignment, ramp metering, and signalized intersection. Professor Antoine G. Hobeika was the director of CTR when we started the research. After that, Ray D. Petthel supported our research a great deal as the interim director, following which Professor Thomas A. Dingus continued the support as he took over the directorship of CTR. The support for the basic research out of which this book evolved came through many projects at CTR mostly funded by the Research Center of Excellence (RCE) from Federal Highway Administration (FHWA). Virginia Department of Transportation

(VDOT) has provided research support and the opportunity to study its working traffic control systems, which proved very useful. In 1997, Professor Michael W. Van Aerde, the creator of INTEGRATION traffic simulation software, Dr. Hesham A. Rakha, and Dr. Wei H. Lin joined CTR. Their immense knowledge and experience has been beneficial for the first author of this book. From FHWA, we would like to acknowledge the encouragement which was provided by Dave G. Gibson, and from VDOT, Mr. Jim Robinson, and Mr. David Gehr. Our collaboration with University of Virginia gave us some important perspective on traffic operation, and Dr. Brian Smith and Dr. Gary Allen provided that.

The first author has learnt a great deal from his association with his friends and colleagues Professor Joseph A. Ball and Professor Martin V. Day from the department of mathematics at Virginia Tech. He hopes that one day he too can acquire the same analytic skills possessed by the two. Dr. Mehdi Ahmadian has been a great friend who has helped me through different times and provided encouragement.

Most importantly, for the first author, the support given to him by his mentors at University of California at Berkeley cannot be overemphasized. His advisor Professor Masayoshi Tomizuka has provided a foundation for conducting research, for which he is indebted to him. Professor Pravin Varaiya with his formidable expertise in traffic systems, communication systems and power systems, to name just a few, has become a role model for the first author.

The authors would like to acknowledge the help that Trina F. Murphy gave in reviewing and finalizing this book. We strongly believe that she is one of the best in her field.

This book is a compilation and enhancement of the work presented by the authors in the following papers:

1. Pushkin Kachroo, Kaan Özbay, Sungkwon Kang, and John A. Burns, "System Dynamics and Feedback Control Problem Formulations for Real Time Dynamic Traffic Routing," *Mathl. Comput. Modelling* Vol. 27, No. 9-11, pp. 27-49, 1998.
2. Pushkin Kachroo and Kaan Özbay, "Solution to the User Equilibrium Dynamic Traffic Routing Problem using Feedback Linearization," *Transportation Research: Part B*, Vol. 32, No. 5, pp. 343-360, 1998.
3. Pushkin Kachroo and Kaan Özbay, "Fuzzy Feedback Control for Real-Time Dynamic Traffic Routing: User Equilibrium Model Formulations and Controller Design," *Transportation Research Record* 1556, 1996.
4. Pushkin Kachroo, and Masayoshi Tomizuka, "Chattering Reduction and Error Convergence in the Sliding Mode Control of a Class of Nonlinear Systems," *IEEE Transactions on Automatic Control*, vol. 41, no. 7, July 1996.

5. Pushkin Kachroo, and Masayoshi Tomizuka, "Integral Action for Chattering Reduction and Error Convergence in Sliding Mode Control," American Control Conference, Chicago, 1992.
6. Pushkin Kachroo, Kaan Özbay, and Arvind Narayanan "Investigating the Use of Kalman Filtering Approaches for Origin Destination Trip Table Estimation," Proceedings of IEEE Southeastcon '97, Blacksburg, VA, April 12-14, 1997.
7. Pushkin Kachroo, and Kaan Özbay "Feedback Control Solutions to Network Level User-Equilibrium Real-Time Dynamic Traffic Assignment Problems," Proceedings of IEEE Southeastcon '97, Blacksburg, VA, April 12-14, 1997.
8. Pushkin Kachroo and Kaan Özbay, "Sliding Mode for User Equilibrium Dynamic Traffic Routing Control", Proceedings of IEEE Conference on Intelligent Transportation Systems ITSC'97, Boston, 1997.
9. Pushkin Kachroo and Kumar Krishen, "Feedback Control Design for Intelligent Transportation Systems," Proceedings of Third World Conference on Integrated Design and Process Technology, July 6-9, 1998, Berlin.

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Blacksburg, USA
Piscataway, USA
1999

Pushkin Kachroo
Kaan Özbay

¹The material reprinted with permission of Virginia Department of Transportation in the first edition has been removed in the second edition and most of the material from paper 1 is now in chapter 4. Furthermore, the material from papers 6 and 9 is not used in the second edition.

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Pushkin Kachroo

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Abbreviations

FAST	Freeway and Arterial System of Transportation, Las Vegas
FHWA	Federal Highway
HJB	Hamilton–Jacobi–Bellman Equation
ITS	Intelligent Transportation Systems
LWR	Lighthill–Whitham–Richards Model
NDOT	Nevada Department of Transportation
NYDOT	New York Department of Transportation
NYU	New York University
ODE	Ordinary Differential Equations
PDE	Partial Differential Equations
PID	Proportional Integral Derivative Control
UNLV	University of Nevada Las Vegas
USDOT	U.S. Department of Transportation
V2I	Vehicle to infrastructure communication
V2P	Vehicle to pedestrian communication
V2V	Vehicle to vehicle communication
V2X	Vehicle to V, I, or P communication

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Part I

Introduction to Feedback Control and Dynamic Traffic Assignment

This introductory part of the book consists of the following two chapters.

1. Introduction (Chap. [1](#)), and
2. Dynamic Traffic Assignment: A Survey of Mathematical Models and Techniques (Chap. [2](#)).

Chapter [1](#) gives the introduction to the use of feedback control methodology to traffic problems, whereas Chap. [2](#) provides a brief mathematical introduction to the dynamic traffic assignment problem and some solutions including feedback control based.

Chapter 1

Introduction

Outline

The remainder of this chapter is organized as follows. Section 1.1 presents the meaning and significance of dynamic traffic routing and how it is used. It presents its various applications as well as different objectives in its design. Section 1.2 presents different parts of a control system. Section 1.3 shows the motivation and some details about the design of real-time dynamic traffic routing and assignment. Literature review is covered in Sect. 1.5. Specific steps and issues involved in control design are detailed in Sect. 1.5, and finally conclusions are provided in Sect. 1.6.

1.1 Dynamic Traffic Routing

Dynamic traffic routing (DTR) refers to the process of dynamically diverting traffic at a junction. Static diversion would be the case when the amount of traffic to be diverted has been precalculated and does not change with time. Being dynamic implies that the values change with time as the traffic conditions change. Figure 1.1 shows a sample site where dynamic traffic routing could be deployed. The figure shows two highway routes between the cities of North Las Vegas (node n_1) and Henderson (node n_2) in Nevada, USA. If there is congestion in one of the routes, then the travel time on that route will increase. Hence, more traffic should be diverted onto the other route. In general, if the travel time is the same on both routes, we can claim that this traffic system is working well. Many traffic controllers presented in this book try to keep equal travel time as one of the objectives of the control. This objective is called *user equilibrium* since users try to or would like to emulate that kind of route choice behavior to obtain maximum benefit. Another objective would be to obtain *system optimal* performance, which means that for the total traffic network, the overall travel time created by using the specific choice of traffic diversions at all traffic nodes is optimal (implying that it is less than the total travel time created by any other choice of diversion strategy).

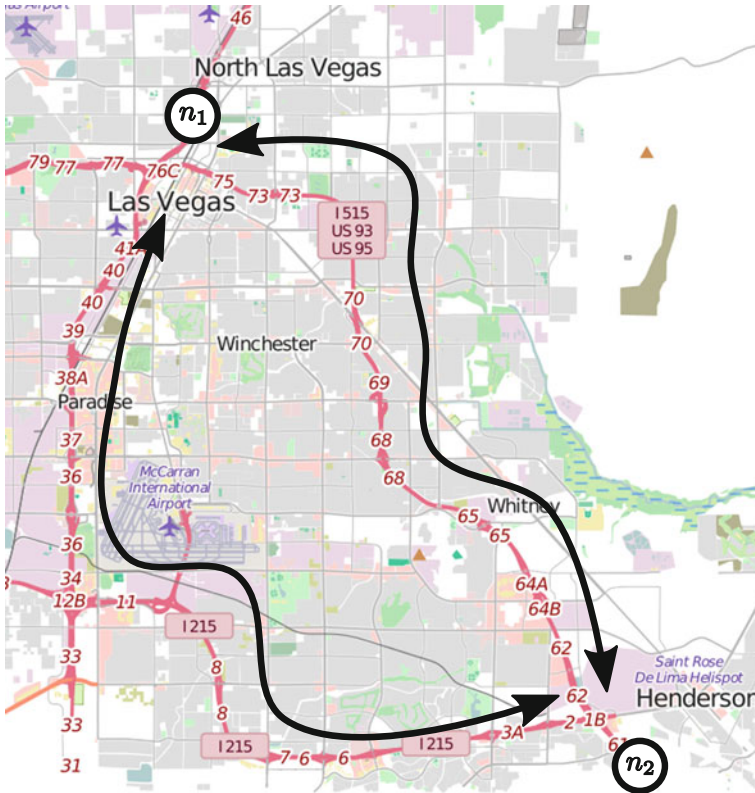


Fig. 1.1 A sample site for diversion (Las Vegas)

Traffic assignment has many applications. It is used in the following three ways in traffic applications:

Transportation Planning Transportation planning process includes travel demand analysis, travel forecasting, trip generation, trip distribution, mode choice, and traffic assignment. Transportation planning process might be used to see the impact of a proposed new road, or new changes to an area, etc.

Simulations Traffic simulations are used to evaluate various traffic control measures and are also used to analyze the impact of transportation planning. Traffic simulations usually depend on the given origin–destination (O–D) travel demands and based on those, traffic assignment is performed on various routes for all O–D pairs.

Real-time Traffic Control Real-time traffic control refers to the actual control of real traffic in real-time in order to affect the traffic behavior and performance.