ICME-13 Monographs

Lynda Ball · Paul Drijvers
Silke Ladel · Hans-Stefan Siller
Michal Tabach · Colleen Vale *Editors*

Uses of Technology in Primary and Secondary Mathematics Education

Tools, Topics and Trends

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Chapter 1
Introduction

Lynda Ball, Silke Ladel and Hans-Stefan Siller

Abstract The use of technology in mathematics education, which encompasses the use of both classical and digital technologies, has a long and broadly discussed tradition. The potential impact of technology on what and how students learn (e.g. Fey et al. in Computing and mathematics. The impact on secondary school curricula. National Council of Teachers of Mathematics, Reston, VA, 1984) is an issue which has existed for decades and there is now a growing corpus of studies which provide insight into the role of technology in mathematics education (see for example, Blume and Heid in Research on technology and the teaching and learning of mathematics: volume 2 cases and perspectives. IAP, Charlotte, NC, 2008; Drijvers et al. in Uses of technology in lower secondary mathematics education: a concise topical survey. Springer, Cham, 2016; Heid and Blume in Research on technology and the teaching and learning of mathematics: volume 1 research syntheses. IAP, Charlotte, NC, 2008; Hoyles and Lagrange in Mathematics education and technology–rethinking the terrain. Springer, New York/Berlin, 2010; Moyer-Packenham in International perspectives on teaching and learning mathematics with virtual manipulatives. Springer International Publishing, Switzerland, 2016). Consideration of the impact of technology on the teaching and learning of mathematics has been the topic of considerable research and continues to be of interest as researchers investigate the potential of technology-enabled mathematics education. For these reasons, it is not surprising that technology use was the focus of three Topic Study Groups (TSGs 41, 42 and 43) at the 13th International Congress on Mathematical Education (ICME), held in Hamburg in 2016.

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These three TSGs focused on the uses of technology (i.e. digital tools) in mathematics education across the spectrum of school mathematics from primary to upper secondary. The aim of this book is to provide an overview of some of these studies related to the use of technology across this age spectrum, as well as point towards future directions for the use of technology in school mathematics. As the field of technology use is a very broad one, the three Topic Study Groups had different foci related to working and using digital tools in teaching and learning of mathematics. The next section provides insight into the intended foci of each of the three TSGs as this foregrounded the work of the three groups at ICME 13.

Keywords Teaching mathematics · Learning mathematics · Technology
School mathematics · Digital tools

TSG 41 focused on "Uses of technology in primary mathematics education (up to age 10)". This TSG noted that although many types of digital technology and environments have been available for primary education since before the turn of the century, individual drill and practice software and interactive tools for exposition appeared to be prevalent in many primary classrooms where technology is used (Attard & Orlando, 2016; Bate, 2010; Loong, Doig, & Groves, 2011; Zuber & Anderson, 2013). Today, all over the world, young children bring their experience with hand-held and other technology into the classroom and in recent years, these have included tools to communicate in the cloud. In the context of this experience, two questions were raised:

1. Do primary teachers keep up with digital natives?
2. Which types of technology use are emerging to enrich and foster mathematics learning at primary school?

Taking these questions into account, TSG 41 focused on the issues of 'use of technology', 'key success factors' and 'innovations' in the context of primary mathematics education, with children up to 10 years old, namely:

- *How do schools and teachers around the world, and in differently advantaged communities, use technology to enrich mathematics learning at primary level?*
- *Which factors contribute to successful and sustained use of technology in primary settings?*
- *Which innovations in digital technology for education enable primary children to inquire, problem solve and think mathematically and to share their learning?*

TSG 42 focused on "Uses of technology in lower secondary mathematics education (age 10–14)", with this age range bridging primary and secondary schooling in many countries. TSG 42 considered technology-related issues from both a learner and teacher perspective, focusing on four themes. The four themes and associated research questions shown below initiated the work of TSG 42:

- *Evidence for effect—What are the research findings about the benefits for student learning of the integration of digital tools in lower secondary mathematics education?*
- *Mathematics education in 2025—What will lower secondary mathematics education look like in 2025, with respect to the place of digital tools in curricula, teaching and learning? How can teachers integrate physical and virtual experiences to promote deep understanding of mathematics?*
- *Digital assessment—What are features of appropriate online assessment of, for and as learning?*
- *Communication and collaboration—How can digital technology be used to promote communication and collaborative work between students, between teachers, and between students and teachers? What are the potential professional development needs of teachers integrating digital tools into their teaching, and how can technology act as a vehicle for such professional development activities?*

TSG 43 focused on "Uses of technology in upper secondary mathematics education (age 14–19)". The TSG focused on four themes:

- *Theoretical Aspects. New technologies can create new kinds of activities and new forms of interactions between learners and teachers hence the need to examine current theory for developing and analyzing the implementation of new technologies from cognitive and epistemological perspectives.*
- *Role of Emerging Technologies. For example, how tablets, smartphones, Virtual Learning Environments, Augmented Reality environments, and haptic technologies might mediate new forms of access to mathematics.*
- *Interrelations between technology and the mathematics taught at this age level.*
- *Teacher Education. New challenges and opportunities for teachers to reflect on their practices and how they develop with the use of new technologies.*

One key point to be considered in any discussions about technology in mathematics education is that access to technology does not, of itself, result in improved teaching and learning. Therefore, the topic study groups on technology are crucial to highlight findings from a range of international perspectives, as well as look forward to future research directions. Consideration of the themes and research questions across the three topic study groups highlights the evolutionary nature of research into digital technology and the need for future research in this area. The following section outlines the chapters in this book, discussed in three sections which align with the three topic study groups.

Topic Study Group 41 was concerned with primary mathematics education up to age 10 and Chaps. 2–8 focus on the use of technologies and digital tools in this age range. In Chap. 2 Moyer-Packenham et al. present the results of a study that examines changes in the performance and efficiency of young children's learning as they engaged with several touch-screen virtual manipulative mathematics apps. They found that changes in the children's learning could be explained by the content alignment of the apps, as well as having similarity in the structure of the apps used for assessment and for learning. This suggests that technical familiarity could be a consideration when a teacher is choosing an app to develop or assess a student's understanding. Tucker also focused on touch-screen apps in Chap. 3 applying the Modification of Attributes, Affordances, Abilities, and Distance (MAAAD) for learning framework to evaluate potentialities of apps.

Larsen et al., in Chap. 4, found that the purposeful use of screencasting apps supported mathematical discourse and has the potential to impact teacher practices. In Chap. 5 Voltolini questions the bonus brought by technology in situations that link digital and material tools. The author shows how the duo of a digital and a material tool supports the learning processes of children with regards to processes of assimilation and adaptation.

Larkin and Milford (Chap. 6) provided an analysis of 53 apps that support geometry learning to promote discussion about the use of apps in primary mathematics education. In Chap. 7 Walter investigated students' use of a physical 'twenty frame' and the 'twenty frame' tablet-app for a group of students with special learning needs. The different approaches used by different children suggested that potential learning gains may not be achieved by all students, using either physical twenty-frames or the given app. The structure of an app was identified as a potential inhibitor to development of understanding in this case. Calder and Murphy (Chap. 8) also reported on the affordances of an app, Math Shake, and the potential for reshaping learning experiences in primary-school mathematics. While their results show the importance of the affordances of mobile technologies for students' learning, they also show that the teacher's pedagogical approach is influential.

These seven papers focus on different aspects related to the use of technology to enhance mathematics teaching and learning in primary education, but each paper shows the great potential that technologies hold—if used in a useful way.

Drijvers et al. (2016) provided a topical survey to stimulate the work of topic study group 42 at ICME 13; this international perspective included a survey of research findings and future directions for lower secondary mathematics (ages 10–14) in the context of a technological age. Evidence for effect, to assess whether technology has been shown to improve student outcomes, was examined. In addition, the role of the teacher, as well as the role of technology in summative and formative assessment, was considered. The potential for communication and collaboration enabled through technology provides two challenges—how to capitalize on technologies to promote this communication and collaboration and the resultant professional development needs for teachers who are teaching in these contexts. Finally, the topical survey attempted to look ahead to mathematics education in 2025, providing a vision for a technology-rich mathematics education. The presence of technology has provided researchers and teachers with opportunities to re-conceptualize mathematics education at lower secondary education, including a rethinking of goals for curriculum, assessment, teaching and learning (Drijvers et al., 2016).

Chapters 9–14 in this book provide insight into the TSG 42 themes focusing specifically on evidence for effect, assessment and communication. Chapters 9 and 10 provide reviews of quantitative and qualitative studies related to technology in lower secondary mathematics. Drijvers (Chap. 9), provides a review of quantitative studies related to technology use and student achievement. Although significant positive effects are reported, with moderate effect sizes, the question is posed about whether quantitative studies provide the detail about how technology can benefit students' learning of mathematics. In Chap. 10, Heid reviews qualitative literature related to mathematics learning, highlighting the important role that these studies play in probing students' mathematical work and in illustrating that technology use has the potential to enrich a student's mathematical experience. Qualitative studies provide detail into both what has been observed in student work and why this might be the case, thus providing reasons for observed changes in mathematical understanding.

Maschietto, in Chap. 11, provides one case study which highlights the interplay between classical and digital technologies. In this study the Pythagorean Theorem is explored in a laboratory setting with access to both classical and digital technologies. The chapter highlights the cognitive processes evident through kinesthetic experiences with the machines, as well as the role of the teacher in orchestrating the classroom to promote these processes. In Chap. 12, Ball and Barzel focus on an overview of communication in the presence of digital technology. The focus on communication follows on from the previous chapter, where use of a laboratory approach involved communication mediated by technology. Communication in the presence of technology has been categorized by Ball and Barzel as communication through, with and of technology and the ways that this communication can promote conceptual, procedural and metacognitive knowledge is elaborated and illustrated. This provides a lens through which to consider how the development of different types of mathematical knowledge can be supported through the affordances of different types of technology.

Chapters 13 and 14 focus on assessment in the presence of digital technology; this fosters consideration of the ways that technology can assist in assessment and how use of assessment information can inform teaching. In Chap. 13 Grugeon and colleagues analyse results from a study on the use of Pépite, an online assessment and teaching tool which provides information for teachers about students' reasoning and thus can support planning for differentiation in the classroom. The focus here is on formative assessment where the technology provides an analysis of students' algebraic reasoning. In Chap. 14, Dick proposes a prototype for an assessment system that utilizes both a computer algebra system (CAS) and a dynamic geometry environment (DGE) with the goal of assessment being carried out automatically within the system. Both assessment focused papers discuss systems where assessment can be carried out within technology and they provide insight into future possibilities for technology-assisted assessment. These chapters highlight the importance of consideration of online assessment systems that provide teachers with information about students' understanding to inform teaching. This formative assessment can assist teachers in targeting teaching to improve students' outcomes. These six papers serve to highlight that there are still many considerations that need to be addressed with regards to technology use in lower secondary mathematics.

The work in TSG 43 was prompted by a topical survey by Hegedus et al. (2016); this publication identified four challenging themes that impact the use of technology in upper-secondary mathematics education:

- Technology in secondary mathematics education: Theory
- The role of new technologies: Changing interactions
- Interrelations between technology and mathematics
- Teacher education with technology: What, how and why?

Chapters 15–25 each address one theme and focus on either the use of DGE or CAS. Across these chapters, DGE was the most used technology, particularly when the research focus was related to process orientated observations such as exploring or modeling. Some chapters also reported DGE studies investigating the teaching of specific mathematical concepts or skills. In contrast, CAS (handheld calculators, as well as software) was used only for research topics concerning proofs and justification.

By looking at the papers of TSG 43 from a meta-perspective one will be able to recognize several research foci related to taxonomies for orchestration of students' work with technology; the ways that *research informs teachers' knowledge and professional development to optimize students' learning with technology; new opportunities for interactions between teachers and students in the presence of technology and the role of teachers in these interactions.*

Using digital tools in education with the aim to experiment can be identified in two ways. On the one hand the main aim could be the promotion of mathematical thinking and design of educational digital resources, such as in Traglová et al. (Chap. 15). Concerning the issue of implementing new technologies (such as a Wii) or technology using sensors, there is potential to explore changes in the ways that students learn. Ng and Sinclair (Chap. 16) investigate the use of innovative approaches, such as a 3D drawing pen for the learning of functions and calculus, where mathematics moves from the traditional 2-D (such as on paper) to 3D. Ferrari and Ferrara (Chap. 17) suggest that these types of resources can only be produced within an innovative socio-technological environment and therefore requires collaboration by a community of mathematics teachers, computer scientists and researchers in mathematics education.

When working with technology one must be aware that representing, documenting and reflecting are key issues in the context of technological learning environments, as discussed in Chaps. 18–24. For example, Beck in Chap. 18, analyzed written notes of students who worked with CAS in upper secondary mathematics and noted the potential to promote discussion about communication of mathematical working. Interactions with digital tools and technological learning environments seems to be advantageous, as outlined by Moreno-Armella and Brady (Chap. 19). Donevska-Todora (Chap. 20) discusses a framework for developing deep understanding of concepts in linear algebra. Greefrath and Siller (Chap. 21) study the extent to which the systematic application of the dynamic geometry software GeoGebra supports "Mathematical Modelling" and Misfeldt and Jankvist, in Chap. 22, investigate the role of CAS in text books. Trgalová and Tabach (Chap. 23) describe existing ICT standards at the international and national levels, arguing that these standards are too general. In Chap. 24 Bowman proposes that graphing calculators as daily tools can enrich the mathematical learning of students.

In Chap. 25 Thurm provides empirically based recommendations for teacher education. A common concern in the TSG 43 chapters was the necessity for more research about teaching with technology to inform teacher professional development and this issue is evident throughout this book.

The issues associated with teaching and learning mathematics with technology are multi-faceted and the chapters in this book have highlighted some current research and theoretical perspectives in primary and secondary mathematics education. With technology evolving at a fast rate there is a need for qualitative, quantitative and theoretical studies to provide analysis of the benefits of current technologies, but also to drive new questions as we look towards the future of mathematics teaching and learning in the presence of existing and new technologies.

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Chapter 2
Using Video Analysis to Explain How Virtual Manipulative App Alignment Affects Children's Mathematics Learning

Patricia S. Moyer-Packenham, Kristy Litster, Emma P. Bullock and Jessica F. Shumway

Abstract In this inquiry, researchers sought to understand changes in young children's learning by examining their performance and efficiency while they engaged with a variety of touch-screen virtual manipulative mathematics apps. We were particularly interested in understanding how the alignment of the apps selected for two different learning sequences might contribute to these changes. A total of 100 children, ages 3–8, participated in interviews. Researchers examined the interviews using a frame-by-frame video analysis to interpret children's interactions with six different mathematics apps on iPads in a clinical interview setting. Results revealed improvements in children's mathematics performance and efficiency between the pre and post assessment apps. Apps that were content aligned and structurally aligned, within each of the learning sequences, helped to explain the changes in children's learning.

Keywords Virtual manipulative · Mathematics apps · Touch screen
Video analysis · Content and structural alignment

2.1 Purpose

Mathematics apps, that contain virtual manipulatives, have become a popular tool and an effective way of supporting children's mathematics learning. Originally, virtual manipulatives were designed as mouse-driven apps for the computer. Since the release of the first iPad in 2010, touch-screen devices have become wide spread platforms for personal and educational use. There are now thousands of mathe-

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matics *apps* (i.e., applications for mobile devices with a touch screen; Gröger, Silcher, Westkämper, & Mitschang, 2013) available for download in online stores. Not all apps have the same quality or value as is evident in the evaluations of apps that have appeared in the literature (Boyer-Thurgood, 2017; Schrock, 2011; Walker, 2010).

The purpose of this project was to utilize frame-by-frame video analysis to examine young children's interactions with virtual manipulative mathematics touch-screen apps. Specifically, we were interested in how app alignment contributed to changes in children's learning. In this study, we identified two types of app alignment: *content* alignment and *structural* alignment. We examined how these two aspects of app alignment contributed to changes in children's learning.

2.2 Research Perspective

Virtual manipulatives (first defined in 2002 by Moyer, Bolyard, & Spikell) are defined as: "an interactive, technology-enabled visual representation of a dynamic mathematical object, including all of the programmable features that allow it to be manipulated, that presents opportunities for constructing mathematical knowledge" (Moyer-Packenham & Bolyard, 2016, p. 13). Today, there are thousands of virtual manipulatives, with representations of mathematical objects, currently available or under development that can be used with a touch-screen interface (e.g., iPads). The current research on virtual manipulative mathematics apps includes a variety of results on learning outcomes.

2.2.1 Mathematics Apps and Learning Outcomes

The use of touch-screen apps can improve students' mathematics performance. Barendregt, Lindström, Rietz-Leppänen, Holgersson, and Ottosson's (2012) study with 87 five-, six-, and seven-year-olds found that using the subitizing iPad app, Fingu, as part of their practice supported an increase in children's computation abilities with addition and subtraction. In another study, Kermani and Aldemir (2016) designed and implemented mathematics interventions for at-risk preschoolers using iPad apps with a focus on properties of number (i.e., counting and subitizing). They found significant differences in learning between the 25 iPad intervention children and the 25 control children in a traditional classroom intervention. Kiger, Herro, and Prunty (2012) looked at the use of iPod Touch devices as supplemental practice tools for children to use at home. They found that the mobile learning interventions led to a statistically significant difference in performance for the intervention group over children who used the standard curriculum materials. Bakker, van den Heuvel-Panhuizen, and Robitzsch (2015) added new insights to the role of home and school in children's learning. They examined the effects of home and school use of virtual manipulatives with 719 second graders. They found that children who used the app at home after an in-school debrief had significant differences in multiplicative reasoning (i.e., skip counting) over children who used the app just at home or just at school.

These studies show that using mathematics apps on mobile devices can have a positive impact on young children's learning; however, they do not explain *why* they have an impact. This is an important point related to the research in this paper, because through video analysis of children's interactions with apps, we hoped to identify possible indicators that explained children's learning.

2.2.2 Defining Two Types of App Alignment

App alignment may play a role in children's mathematical learning. For the purposes of this study, we defined two types of app alignment: *content* alignment and *structural* alignment. We defined content alignment as the degree to which the specific mathematics topics contained in an app were aligned with the specific mathematics topics contained in each of the other apps in the interview sequence. For example, if one app focused on counting 1–10 blocks and another app focused on identifying the numeral that named the number of blocks from 1 to 10, we would say that the apps were closely aligned in terms of content because they both focused on developing the skill of counting a group of objects from 1 to 10. However, if one app focused on counting 1–10 blocks and another app focused on identifying the place value of a digit in a three-digit number, we would say that the apps were *not* closely aligned in terms of content because one app is developing the skill of counting while the other app is developing an understanding of place value.

We defined structural alignment as the degree to which objects and tasks contained in an app were aligned with the objects and tasks contained in each of the other apps in the interview sequence. For example, if one app displayed a group of squares of different sizes and children were asked to order the squares from largest to smallest, and another app displayed a group of rods of different sizes and children were asked to order the rods from longest to shortest, we would say that the apps were closely aligned in terms of structure because they both contained objects of different sizes and the tasks in both apps asked the child to seriate the objects. However, if one app focused on placing a number on a number line and another app focused on creating a numerical representation for a three-digit number given orally, then we would say that the apps were *not* closely aligned in terms of structure, because one app has a number line as the object with a task of placing the number on the line while the other app has place value cards as the object with the task of creating a numeral with the cards.

2.2.3 Potential Learning Benefits of App Alignment

In this study, we hypothesized that the content alignment of the four apps in the interview sequence would be important for children's learning. In prior research, Edwards Johnson, Campet, Gaber, and Zuidema (2012) suggested that teachers should consider alignment between the activity and the target mathematical content. Their research, using clinical interviews with children in Grades 2–5, found that virtual manipulatives with features that were aligned with mathematical content and procedures reinforced target concepts and addressed children's common error patterns. For example, one error pattern they noted was that children thought that 5 tens and 4 ones equaled 9. The virtual base ten blocks supported the development of place value concepts by allowing students to convert ten unit blocks into one unit of ten and emphasized the meaning of digits in the tens and ones place (p. 203). This shows the potential importance of aligning the mathematical content of each of the apps that children use when they are learning a specific mathematical topic if we want to support children's learning of that topic.

We hypothesized that the structural alignment of the four apps in the interview sequence would be less important for children's learning, because of the research that shows that being able to translate among a variety of mathematics representations supports learning (Lesh, Landau, & Hamilton, 1983). Therefore, if the structure of the apps is not aligned, this simply means that the child is exposed to a variety of different representations (i.e., different objects and different tasks) of the same mathematical topic, which should support learning. While there is little research that directly looks at the structural alignment of apps, there are related findings that may provide some insight about structural alignment. For example, Uttal et al. (2013) reported on the alignment of tests for transfer. They conducted three experiments to examine transfer from: (1) written or physical manipulative instructional methods to written tests, (2) written or physical manipulative instructional methods to physical manipulative tests, and (3) standard and distinctive physical manipulative instruction to written tests. They concluded that posttest performance depended on whether the learning method matched the testing method and suggested that relational similarities may help children transfer learning. In related research, Segal (2011) examined the structural congruence of gestures in direct touch and mouse click applications. Her study compared four different digital conditions: (1) direct touch interface with a congruently mapped application, (2) direct touch interface with an incongruently mapped application, (3) mouse-click interface with a congruently mapped application, and (4) mouse-click interface with an incongruently mapped application. Congruence was defined as matching the gesture children would complete when using a physical manipulative (e.g., turning) to the gesture children used with a virtual manipulative (e.g., swiping to turn vs. tapping to turn). Findings suggested that direct touch interfaces with a congruent mapping of gestures increased student efficiency and accuracy. While these two studies did not directly address structure, their results may provide some insights on how structural alignment may be important.

2.2.4 The Complexity and Diversity of App Features and Structures for Learning

Five categories of affordances were identified in a meta-analysis by Moyer-Packenham and Westenskow (2013): "focused constraint, creative variation, simultaneous linking, efficient precision, and motivation" (p. 35). These five categories are common among virtual manipulatives that have been shown to have positive impacts on mathematics learning. In addition, touch-screen devices, such as iPads, have interactive properties that afford learning opportunities. For example, Segal (2011) found significant differences in haptic modality (mouse vs. touch screen) in that iPads encouraged less guessing, better accuracy, and efficiency when compared with the same app on a computer. This means that app features and device modalities may not affect all children in the same way. In fact research has confirmed these differences. For example, Barendregt et al.'s (2012) Fingu app, intended to develop conceptual subitizing skills, helped different children develop different skills in subitizing. Baccalini-Frank and Maracci (2015) examined preschoolers' number sense with multi-touch devices and found that each app had different characteristics which fostered the development of various aspects of number sense. Children's prior achievement levels also seem to impact their learning with mathematics apps. For example, Moyer-Packenham and Suh (2012) found that low achievers accessed the step-by-step procedures features of fraction apps, while high achievers accessed the evident patterns afforded by the apps. Researchers have also reported that different children access app features in different ways. For example, Moyer-Packenham et al. (2015a) reported that children's access to helping and hindering features (or affordances) in mathematics apps influenced the children's progress. The children who accessed the helping affordances were more likely to progress between the pre and post assessments. These studies imply that the complexity of app features and the diversity of app structures affects different children in different ways.

This paper seeks to contribute to an understanding of why some app experiences help children to progress while others do not by using a frame-by-frame video analysis as a way to identify possible features that may explain children's learning in similar content topics (i.e. counting, subitizing, skip counting) and across different content topics (i.e. seriation, quantities, place value). We were specifically interested in understanding how learning apps that were content aligned and structurally aligned explained changes in children's learning.

2.3 Research Question

While the research base on virtual manipulative mathematics apps is growing, there is a need for further investigation into how content- and structurally-aligned apps may play a role in changes in children's learning performance and efficiency.

This study examined the following research question: How do content-aligned and structurally-aligned virtual manipulative mathematics apps contribute to changes in children's learning performance and efficiency? In this study, learning *performance* was defined as a change in accuracy between the pre- and post-assessment tasks that children completed using virtual manipulative touch-screen apps. Learning *efficiency* was defined as changes in the speed with which children completed the pre- and post-assessment tasks, after completing a variety of learning tasks using virtual manipulative touch-screen apps. Based on the findings of Edwards Johnson et al. (2012), our hypothesis was that aligning the pre- and post-assessment apps with the two learning apps, in terms of their mathematical content, would increase the likelihood of positive changes in children's performance and efficiency.

2.4 Methods

2.4.1 Research Design

To answer the research question, we used an explanatory mixed methods design. We collected and analyzed quantitative and qualitative data and then merged the results to answer our mixed methods research question (Creswell & Plano Clark, 2011; Tashakkori & Teddlie, 2010). The rationale for this design was to obtain complementary data on the same topic to better understand the research problem. We collected the video data for this paper in one of our previous research projects (Moyer-Packenham et al., 2015b). We then used these video data in several different analyses focusing on different research questions, such as the research question in this paper.

In this study we coded videos of children's interactions with a pre-app, two learning apps, and a post-app. We quantitized the learning performance and efficiency data from the pre- and post-assessment activities and explored these data using SPSS. We used qualitative methods to analyze how children's interactions with the apps might explain their outcomes for learning performance and efficiency, which allowed a holistic overall interpretation.

2.4.2 Participants

A total of 100 children (Preschool, ages 3–4, N = 35; Kindergarten, ages 5–6, N = 33; Grade 2, ages 7–8, N = 32) participated in this study. They were recruited using informational brochures and letters distributed to local public and charter elementary schools, the university campus lab school, and the university campus preschools. The demographics of the children were: Asian (1%), Caucasian (89%), Hispanic (2%), and Mixed Race (8%). One-third (34%) of children's parents

Fig. 2.1 Preschooler interacting with an iPad app under the direction of an interviewer in the clinical interview room

reported them receiving free- or reduced-lunch services at school (indicating low socio-economic status). The parents of the participating children completed surveys and reported children's prior iPad use and experiences with technology. Parents reported on the use of touch-screen devices in the home with 11% having more than five touch-screen devices, 78% with between one and four, and 8% with none. Thirteen percent of the children had access to their own touch-screen device at home. Parents reported that the children used the touch-screen devices every day (45%), 4–5 days per week (2%), 1–3 days per week (40%), and never (10%). Figure 2.1 shows a preschooler interacting with an iPad.

2.4.3 Data Sources

We used four instruments to collect data during the study: pre- and post-assessments (to document mathematics accuracy and speed), GoPro video recordings of the iPad screen, wall-mounted video recordings of children and the interviewer, and observation protocols.

The pre- and post-assessment apps used in this study focused on two mathematical content topics for each age-level group. The preschool children (ages 3–4) were assessed on seriation and counting content. The kindergarten children (ages 5–6) were assessed on quantities and subitizing content. The Grade 2 children (ages 7–8) were assessed on place value and skip counting content. The same mathematics app was used for the pre- and post-assessments on each mathematical content topic for each age-level group. To determine mathematics performance (i.e., accuracy), we identified the number of tasks the child completed correctly on the pre-assessment and the number of tasks the child completed correctly on the post-assessment. To determine efficiency (i.e., speed), we identified the time it took the child to complete the tasks on the pre-assessment and the time it took the child to complete the same tasks on the post-assessment. Speed of completion can show several things about the child's learning while using a mathematics app: (1) familiarity and confidence with the mathematics content, (2) familiarity and confidence with the features and tools in the app, or (3) a desire to complete the tasks quickly without regard to the content of the app. By viewing the interview videos to understand the child's overall interactions with the app, we could determine why children became faster or slower when they completed the pre- and post-assessment tasks. The mathematics content topics of seriation, subitizing, counting, skip counting, and place value were selected for study with young children because these concepts are critical foundations to later mathematics learning. Learning the count sequence, object counting, learning cardinal ideas, understanding the seriation of numbers, and skip counting are interrelated counting ideas that serve as the gateway to young children's developing counting strategies and understanding patterns that make up the place value number system. Current research indicates the existence of consistent relationships between counting, number relationships and basic operations, and later mathematics achievement (Jordan, Glutting, & Ramineni, 2010).

Two video views were important sources of data for the project: GoPro video recordings and wall-mounted video recordings. Each child was equipped with a wearable GoPro camera that was positioned to capture an up-close view of their interactions on the touch-screen iPad device. This video recording process captured all of the on-screen motions of the mathematics objects and tasks initiated by the children. It also captured audio interactions between the child and the interviewer as well as audio interactions between the child and the iPad. The wall-mounted video recordings captured a broad view of the child, the interviewer, the iPad, and all actions and interactions that occurred during the interviews. The second video source served as a back-up for the data collected by the GoPro camera and as a broader perspective of the child's actions that were outside the GoPro camera view and away from the iPad.

The final data source was an observation protocol. One observer watched the interview from an observation booth and recorded notes on the interview. Schubert (2009) suggests that the development of these protocols be based on current theories related to the phenomenon of interest and the researcher's own experience with observing the phenomenon. In line with that recommendation, we used the mathematics education literature to focus our attention on how the children interacted with features of the mathematics apps.

2.4.4 Procedures and Data Collection

Parents brought their children to a research building on a university campus. Children participated in individual clinical interviews in an early childhood education research building equipped with two-way mirrors, audio observation rooms, and built-in video cameras. The view that observers had from the observation room is pictured in Fig. 2.2. Prior to each interview, researchers collected information from the parents of the participating children, completed the consent form, and answered questions. During interviews, children used interactive mathematics apps on iPads. The research team had experts with experience in conducting mathematics clinical interviews with young children.

Table 2.1 displays the interview order for each of the mathematics apps used with each age-level group in the study. The research team selected three apps to further preschoolers' (ages 3–4) learning of seriation and three apps to further preschoolers' learning of counting. The team selected three apps to further kindergartens' (ages 5–6) learning of combining amounts and three apps to further kindergarteners' learning of building and representing numbers. Finally, the team selected three apps to further second graders' (ages 7–8) learning of base-10 place value and three apps to further second graders' learning of skip-counting. Screen shots of each of the apps are displayed in Tables 2.3, 2.4, and 2.5 by age level.

Fig. 2.2 A view of the clinical interview room showing observers watching an interview from the observation room

Table 2.1 List of mathematics apps and interview order for each age-level group

Interview order	Preschool (age 3–4)	Kindergarten (age 5–6)	Grade 2 (age 7–8)
	Seriation tasks	*Subitizing tasks*	*Skip counting tasks*
App #1 (pre-assessment)	Pink tower—free moving	10-frame	100s chart
App #2 (learning app 1)	Pink tower—tapping	Hungry guppy	Frog number line
App #3 (learning app 2)	Red rods	Fingu	Counting beads
App #1 (post-assessment)	Pink tower—free moving	10-frame	100s chart
	Counting tasks	*Quantities tasks*	*Place value tasks*
App #4 (pre-assessment)	Base-10 blocks	Base-10 blocks	Base-10 blocks
App #5 (learning app 1)	Base-10 blocks: 1–5	Base-10 blocks: 11–20	Zoom number line
App #6 (learning app 2)	Base-10 blocks: numerals	Base-10 blocks: numerals	Place value cards
App #4 (post-assessment)	Base-10 blocks	Base-10 blocks	Base-10 blocks

As seen in Table 2.1, during each interview, children interacted with a pre-assessment app on the iPad, then interacted with two learning apps that contained a series of mathematical tasks, and finally interacted with a post-assessment app that revisited the tasks from the pre-assessment. This procedure was repeated for a second mathematics content topic for each of the age-level groups using different apps and app tasks. Our goal was to select apps so that each series of learning and assessment tasks (i.e., pre-app, learning app 1, learning app 2, post-app) focused on one specific mathematics content topic that was age-appropriate for the children in that age-level group. This ensured that children spent time interacting with multiple apps, and therefore, interacting with multiple representations of the same mathematics content topic, to support concept development of that particular topic. Apps were selected by content alignment and were not selected based upon structural alignment.

During interviews, one researcher served as the interviewer and presented the mathematics tasks on the iPad to the child. A second researcher started the recording equipment and viewed the interview from the observation booth. A real-time video capture on a laptop allowed the second researcher to record observational notes while the interview was occurring. At the end of each interview, researchers downloaded the video data from the wall-mounted camera and the GoPro camera and secured it on an external hard drive device.

2.4.5 Data Analysis

Researchers first coded the video data through frame-by-frame video analysis to interpret children' interactions with the mathematics virtual manipulative apps. Video data were analyzed and coded for learning performance (i.e., children's accuracy in completing the tasks) and efficiency (i.e., changes in the speed with which the children completed the tasks). In the quantitative analysis, we used descriptive statistics to explore the data. Because the data were not normally distributed, we used the Wilcoxon Signed Ranked Test to analyze changes in learning performance and efficiency. This non-parametric statistical test uses the median of related samples (e.g., pre- and post-assessment scores) to compare data sets and is appropriate for skewed data and small samples.

In the qualitative analysis, we analyzed and coded the video data to identify children's actions, interactions, and access to app features for each app using a process of open coding. As themes emerged, we revisited the video data using axial coding to develop major categories. We identified specific examples to summarize patterns of children's observable interactions, to note when these interactions resulted in changes in performance or efficiency, and to note the content and structure of the apps that were being used at that time. Further, researchers identified samples in the videos to highlight trends in the data and that may contribute to the discussion on app alignment.

Our results in this paper focus specifically on children's learning performance and efficiency during the pre- and post-assessment portions of the interviews and on how the alignment of the apps might explain the changes. Other papers, based on the data collected in this large research project, detail children's learning progressions, explore app affordances, and describe strategies children used during interactions with the apps (e.g., Bullock, Moyer-Packenham, Shumway, Watts, MacDonald, 2015; Moyer-Packenham et al., 2014a, 2014b, 2015a, 2015b; Tucker & Moyer-Packenham, 2014; Tucker, Moyer-Packenham, Shumway, & Jordan, 2016; Watts et al., 2016).

2.5 Results and Discussion

The research question in this study focused on how the use of content-aligned and structurally-aligned virtual manipulative mathematics apps contributed to children's mathematics learning. The results presented discuss the quantitative findings, the qualitative frame-by-frame video analysis, and the complementarity of the results to understand how app alignment may explain some of the changes in children's learning. In the first section, we present the statistical results and discuss each of these results by age group. In the second section, we present the apps children used in each age group, along with figures from the video analysis that provide a representative composite panel of the children's interactions with the apps in each part of the interview sequences (i.e., a basic storyboard that shows a view of what children were doing with the mathematics objects within each of the apps). We then merge the quantitative and qualitative data to discuss the role of app alignment.

Learning Performance and Efficiency Results for All Age Groups

A summary of the pre- and post-assessment results for each age group is presented in Table 2.2. This table focuses on the significant results for all age groups for performance and efficiency.

As Table 2.2 shows, preschool children's (age 3–4) learning performance scores on the seriation and counting sequence tasks remained relatively constant, while their efficiency scores significantly improved for seriation and counting. Improved efficiency on both sequences could be the result of improved understanding of the tasks or it could be a function of learning the technology and more comfortably

Table 2.2 Summary table of performance and efficiency outcomes for pre- and post-assessment apps

Measures	N	Mean rank Post[a]	Mean rank Pre[a]	z	*p*
Preschool seriation	35				
Performance measure					NS
Efficiency measure		16.35	16.89	−2.095	.036*
Preschool counting	35				
Performance measure					NS
Efficiency measure 1		18.52	14.00	−4.244	.000**
Efficiency measure 2		18.65	13.07	−3.522	.000**
Kindergarten subitizing	33				
Performance measure		2.67	7.25	−2.228	.026*
Efficiency measure					NS
Kindergarten quantity	33				
Performance measure					NS
Efficiency measure		18.17	12.22	−2.880	.000**
Grade 2 skip counting	32				
Performance measure 1		.00	3.50	−2.214	.27*
Performance measure 2		.00	3.50	−2.214	.27*
Efficiency measure 1		14.98	20.17	−3.539	.000**
Efficiency measure 2		14.20	15.58	−2.495	.013*
Grade 2 place value	32				
Performance measure					NS
Efficiency measure					NS

[a]Negative ranks are shown first; then positive ranks for each paired condition. *Significant at $p < .05$; **significant at $p < .001$; NS indicates that the measures were not significant. This table is a reproduction of the results which were first reported in Moyer-Packenham et al. (2015a)

working with the apps on the post-assessments. While learning performance remained constant, preschoolers seemed to learn the physical mechanics needed to complete the tasks in a more efficient manner resulting in improved overall efficiency for both seriation and counting tasks.

Kindergarteners (age 5–6) showed significant increases in learning performance for subitizing, and improved efficiency for quantity. Kindergarteners seemed to improve in learning performance while also learning to use the technology efficiently. The Kindergarten quantity task included pre- and post-assessment apps and two learning apps that were all variations of the base-10 block virtual manipulative, which may have allowed the children to become familiar with the design of this app and its features. Additionally, kindergarteners' fine motor skills may have become more refined as they interacted with each base-10 block app.

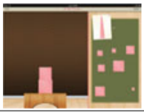
The Grade 2 (age 7–8) results in Table 2.2 showed significant increases in learning performance and efficiency for skip counting, but not for place value. Once again, these results could be due to improved skill in skip counting after working through the learning apps, greater facility with the apps, or a combination of improved mathematical understanding and efficiency with the technology. Results could have also been influenced by the similarity of the skip counting tasks because, in each task for skip counting, children were asked to count by 4s, 6s, and 9s. There seemed to be a ceiling effect on the pre-assessment for place value, with many children mastering the app tasks initially.

2.5.1 App Alignment Results for Preschool

This section presents the six apps used by preschoolers and the composite storyboard panels of typical preschoolers' interviews using video frames taken from the video data. We will use the term *video frame* throughout the paper when we are referring to the still images that were pulled from the video clips as a way to distinguish the static image (video frame) from the dynamic videos (video clip). In the sections that follow the presentation of the preschool data, we also present similar examples for kindergarten participants and Grade 2 participants. A screen shot of the six apps used by the preschool children (age 3–4) is presented in Table 2.3.

The screen shots in the left column of Table 2.3 show the counting task apps. In the Pre and Post App, children build a target number within 9 using base ten blocks. In Learning App 1, children build the sequence of numbers from 1 to 5 using base ten blocks. In Learning App 2, children count a set of base ten blocks within 9. We consider all three apps in the counting sequence to be content aligned because they all asked children to count, and we consider them structurally aligned because they all used the same mathematical objects (base ten blocks) and the same task (counting). All three apps were goal oriented (as opposed to open ended), because there was a correct response for each task.

Table 2.3 Screen shots of preschool apps

Counting tasks		Seriation tasks	
Pre/post app Montessori numbers (1–9)		Pre/post app Pink tower (free moving)	
Learning app 1 Montessori numbers (1–20: 1–5)		Learning app 1 Pink tower (Card #12)	
Learning app 2 Montessori numbers (1–9)		Learning app 2 Intro to math (red rods)	

The right column of Table 2.3 displays the seriation task apps. In the Pre and Post App, children build a tower with different sized free moving blocks from largest to smallest by dragging the blocks. In Learning App 1, children build a tower from largest to smallest with different sized static blocks by tapping the appropriate block. In Learning App 2, children order different sized rods from largest to smallest by dragging the rods. We consider all three apps in the seriation sequence to be content aligned because they all asked children to seriate similar objects from largest to smallest. We consider all three apps to be structurally aligned because they use similar mathematical objects (squares and rectangles) and the same task (seriate from largest to smallest). All three apps were goal oriented.

Preschool children's learning performance remained constant, but they experienced changes in efficiency for counting and seriation; therefore, we reviewed the video data to understand how app alignment may have contributed to changes in efficiency. Figure 2.3 shows a composite storyboard that includes video clips from six different preschool participants on the preschool seriation task. It includes four common participant errors by preschoolers on the pre-assessment, a sample of one participant using the Pink Tower learning app, and a sample of a successful participant on the post-assessment.

The top row of Fig. 2.3 shows four common participant errors made by the preschoolers on the seriation pre-assessment app. These were coded as errors because the expectation was that children would put the blocks in order from largest to smallest, building a pink tower. These four errors illustrate the variety of levels of conceptual understanding that children in the preschool interviews brought with them to the seriation task. Child #1 is an example of the first common error that children made; they stacked blocks directly on top of each other to create a short pile of blocks. Like others who built a pile of blocks, Child #1 did not stack the blocks in seriation order; rather, the blocks were stacked primarily by their proximity to the pile. Child #2 is an example of another common error where children built a misshapen tower. In this example, Child #2 builds a leaning tower with

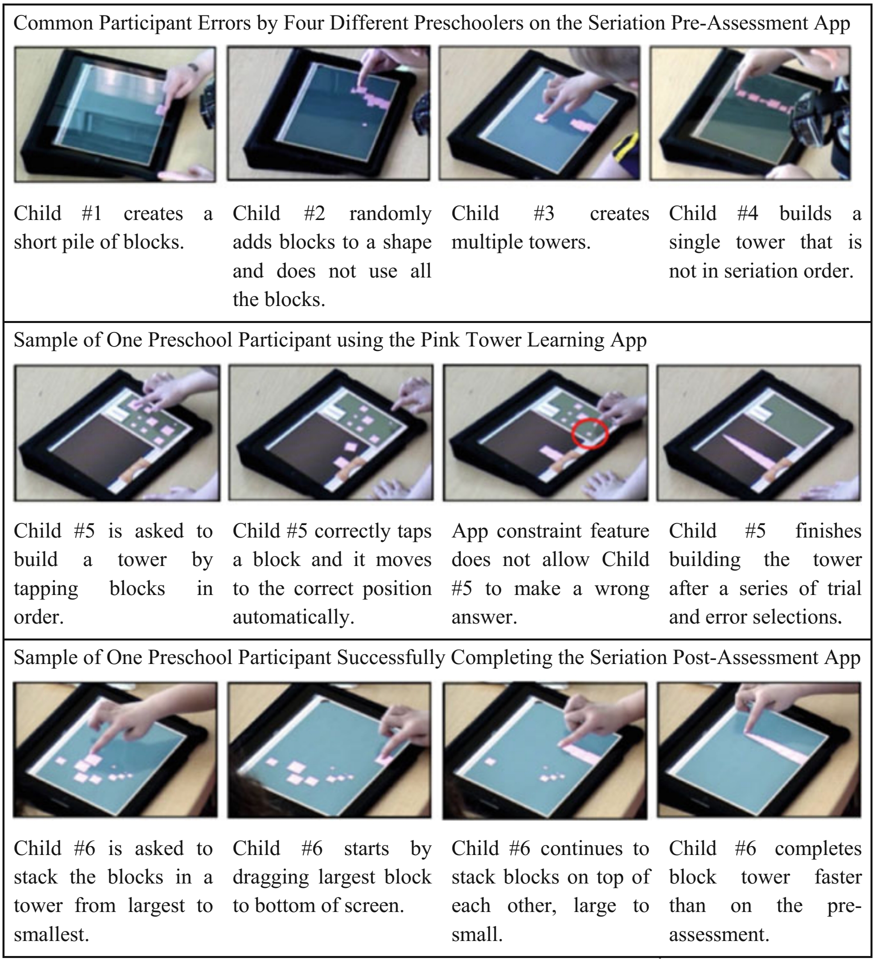


Fig. 2.3 Composite storyboard of Preschool participants' video examples from the seriation learning progression

about half of the blocks and then randomly added blocks to the middle or side of the tower. Sometimes these blocks appeared to be used to fill in gaps or curves in the shape. As seen in this video frame example, the smallest block was often left out of the odd shaped towers completely. Child #3 shows an example of a third common error where children created multiple towers. In this example, Child #3 created a short tower at the bottom of the iPad screen and then created a second tower by stacking blocks in a single pile. Other children stacked their second tower vertically, horizontally, or in a single pile. The fourth common error is shown by Child #4 where the child built a single tower, but not in seriation order from largest to smallest. Other children made similar errors such as having one or two blocks out of